DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

DISTRIBUTED FAULT TOLERANT CONTROL AND COMMUNICATION FOR INTERCONNECTED SYSTEMS

PANAGIOTIS PANAGI

A dissertation submitted to the University of Cyprus in partial fulfillment of the requirements for the degree of Doctor of Philosophy

May, 2015

VALIDATION PAGE

Panagiotis Panagi

Distributed Fault Tolerant Control and Communication for Interconnected Systems

The present Doctoral Dissertation was submitted in partial fulfillment of the requirements for the Degree of Doctor of Philosophy at the ***Department of Electrical and Computer Engineering****, and was approved on* ***25/05/2015*** *by the members of the* ***Examination Committee****.*

Committee Chair ______________________________

Christos Panayiotou, Associate Professor

Research Supervisor ______________________________

Marios Polycarpou, Professor

Committee Member ______________________________

Georgios Ellinas, Associate Professor

Committee Member ______________________________

Kostas Kyriakopoulos, Professor

Committee Member ______________________________

Michalis Michaelides, Lecturer

DECLARATION OF DOCTORAL CANDIDATE

The present doctoral dissertation was submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy of the University of Cyprus. It is a product of original work of my own, unless otherwise mentioned through references, notes, or any other statements.

Doctoral Candidate : Panagiotis Panagi

Signature : ....

Περίληψη

Ο έλεγχος πολύπλοκων συστημάτων μεγάλης κλίμακας με τη χρήση ενός κεντρικού ελεγκτή προϋποθέτει υψηλές υπολογιστικές και επικοινωνιακές απαιτήσεις, γεγονός που καθιστά απαγορευτική την υλοποίηση τους στην πράξη. Ως εκ τούτου, η επιστημονική κοινότητα στρέφεται προς την ανάπτυξη μεθόδων κατανεμημένου ελέγχου, όπου το σύστημα μεγάλης κλίμακας διαιρείται σε μικρότερα διασυνδεδεμένα υποσυστήματα, και το κάθε υποσύστημα ελέγχεται ξεχωριστά από τοπικό ελεγκτή. Το χαρακτηριστικό γνώρισμα των διασυνδεδεμένων συστημάτων είναι πως η συμπεριφορά του κάθε υποσυστήματος εξαρτάται από τη δυναμική συμπεριφορά των άλλων υποσυστημάτων. Επιπλέον, η ευστάθεια, η επίδοση και η αξιοπιστία του συστήματος είναι άρρητα συνδεδεμένη με την ποσότητα της πληροφορίας που ανταλλάσσεται μεταξύ των υποσυστημάτων.

Η παρούσα διατριβή επικεντρώνεται στην ανάπτυξη μεθόδων αξιόπιστου ελέγχου για διασυνδεδεμένα συστήματα. Θεωρούμε μη-γραμμικά υποσυστήματα με άγνωστες μη-γραμμικές αλληλεπιδράσεις μεταξύ των υποσυστημάτων, και επιπλέον θεωρούμε την περίπτωση κατά την οποία πολλαπλές βλάβες μπορούν να συμβούν σε κάθε υποσύστημα και διασύνδεση. Η ανάπτυξη της μεθόδου κατανεμημένου ελέγχου βασίζεται στη χρήση προσαρμοστικών μοντέλων προσέγγισης για την εκτίμηση των αγνώστων διασυνδέσεων και των αγνώστων αλλαγών στη δυναμική συμπεριφορά των υποσυστημάτων λόγω της εμφάνισης βλαβών. Σε πρώτο στάδιο αναπτύσσεται μέθοδος αποκεντρωμένου ελέγχου διασυνδεδεμένων υποσυστημάτων που εξασφαλίζει την ευστάθεια του συστήματος, χωρίς την ανάγκη ανταλλαγής πληροφοριών μεταξύ των υποσυστημάτων. Προς τη βελτίωση της ευρωστίας του συστήματος στην παρουσία σφαλμάτων προσέγγισης, αναπτύσσεται μέθοδος βασισμένη σε τροποποίηση νεκρής-ζώνης, σε συνδυασμό με προσαρμοστική μέθοδο για την εκτίμηση των άνω ορίων των αγνώστων σφαλμάτων προσέγγισης. Εκτός της περιοχής κάλυψης των μοντέλων προσέγγισης, η παρουσία μεγάλων σφαλμάτων προσέγγισης μπορεί να προκαλέσει προβλήματα αστάθειας στο σύστημα. Το πρόβλημα αυτό αντιμετωπίζεται με την ανάπτυξη αποκεντρωμένου συστήματος ελέγχου

ασφαλείας για την καθοδήγηση της τροχιάς του κάθε υποσυστήματος εντός της περιοχής κάλυψης. Σε επόμενο στάδιο, αναπτύσσεται μέθοδος για την κατανεμημένη ανίχνευση και αντιμετώπιση βλαβών, όπου η επικοινωνία μεταξύ των υποσυστημάτων βασίζεται στη χρήση του τοπικού σφάλματος παρακολούθησης. Το κατανεμημένο σύστημα ανίχνευσης βλαβών βασίζεται σε ένα σύνολο μη-γραμμικών εκτιμητών, ένα για κάθε υποσύστημα, και εξασφαλίζεται πως δεν συμβαίνουν άκυροι συναγερμοί ανίχνευσης βλάβης. Επιπλεόν, αποδεικνύεται πως με την προσέγγιση ενός άνω ορίου της συνάρτησης βλάβης, αντί της ίδιας της συνάρτησης βλάβης, εξασφαλίζεται η ευρωστία του συστήματος στην παρουσία σφαλμάτων προσέγγισης. Προς τη βελτιστοποίηση της ανταλλαγής πληροφοριών μεταξύ των υποσυστημάτων, αναπτύσσεται αλγόριθμος επικοινωνίας που βασίζεται στο συντονισμό μεταξύ των συστημάτων. Ο προτεινόμενος αλγόριθμος επικοινωνίας μειώνει σημαντικά το κόστος επικοινωνίας, με ελάχιστες επιπτώσεις στην απόδοση του συστήματος. Περαιτέρω, διαμορφώνεται το πρόβλημα βελτιστοποίησης της επικοινωνίας για διασυνδεδεμένα συστήματα με τη χρήση συναρτήσεων βήματος για τη προσέγγιση των αγνώστων διασυνδέσεων. Καθορίζεται η συνάρτηση βήματος που οδήγει στη βέλτιστη προσέγγιση άγνωστης συνάρτησης και με βάση αυτό το αποτέλεσμα, αναπτύσσεται ένας πιο αποδοτικός αλγόριθμος βασισμένος σε προσαρμοστικά μοντέλα προσέγγισης. Ο προτεινόμενος αλγόριθμος επικοινωνίας ελαχιστοποιεί την αβεβαιότητα για την επίδραση των διασυνδέσεων στη συμπεριφορά των υποσυστημάτων. Με τη χρήση προσομοιώσεων, παρουσιάζεται η αποτελεσματικότητα της προτεινόμενης μεθόδου για τον αξιόπιστο κατανεμημένο έλεγχο διασυνδεδεμένων συστημάτων, και των αλγορίθμων επικοινωνίας για τη βελτιστοποίηση της ανταλλαγής πληροφοριών μεταξύ των υποσυστημάτων.

Abstract

The control of complex and spatially distributed systems in a centralized architecture is computationally and communicationally intensive. Towards the development of viable solutions, research efforts are shifting towards a distributed control architecture, where the large-scale system is decomposed into smaller interconnected subsystems and controlled through a network of local decision-making modules. A key characteristic of interconnected systems is that the behavior of each subsystem is correlated not only with the local dynamics, but also with the dynamics of the other subsystems. In addition, the stability, reliability and performance properties of the overall system are often limited by the amount of information exchanged between the subsystems. A key objective is the development of energy-efficient distributed control and communication algorithms that guarantee the stability, performance and reliability of the system, in the presence of uncertain interconnections and faults.

This thesis addresses the problem of distributed fault tolerant control for a class of interconnected systems. We consider feedback linearizable nonlinear subsystems, coupled by unknown nonlinear interconnections in which multiple faults may appear in any of the subsystems as well as in the interconnection effects. The distributed fault tolerant control scheme is based on the use of adaptive approximation models for estimating the unknown interconnection effects and changes in model dynamics due to failures. At first, we consider the case of no information exchange between the subsystems and develop a decentralized fault tolerant control scheme that guarantees uniform ultimate boundedness of the tracking errors to a small region around zero. The presence of residual approximation errors is addressed using a dead-zone modification in the adaptive laws combined with an adaptive bounding method. Outside the coverage region of the approximation models, the residual approximation error is typically significantly large, such that the states of the subsystems may become unbounded. This issue is addressed with the development of a

decentralized safety control scheme for steering the trajectory back into the coverage region. Next, a distributed fault detection and accommodation scheme is presented, where the subsystems exchange information according to a self-triggering tracking-error based communication scheme. The distributed fault detection scheme is based on a set of distributed nonlinear estimators corresponding to each subsystem, and ensures that there are no false detection alarms. It is shown that by approximating the upper bound of the fault function, instead of the fault function itself, robustness to residual approximation errors is ensured. Towards optimizing the exchange of information between subsystems, a coordinated communication scheme is presented which substantially reduces the communication cost, with minimal impact on the system performance. Moreover, an optimized communication technique is developed based on the use of step functions for approximating the unknown interconnections and fault functions. Through rigorous mathematical analysis, the step function with the best approximation property is derived. Following this result, an efficient communication algorithm is presented which utilizes adaptive approximation models to minimize the uncertainty about the coupling dynamics. The effectiveness of the proposed distributed fault tolerant control and communication scheme is illustrated with simulations.

Acknowledgments

Foremost, I would like to express my gratitude to my advisor, Prof. Marios Polycarpou, for his support in every aspect of my graduate experience. His expertise, work ethic and attention to details, were inspiring both for my research, and as a professional in general.

I want to thank the members of my PhD committee for taking the time to evaluate the work in this thesis.

To my parents, wife and daughter, thank you for your support, love, and patience.

Publications

Published journal publications

1. P. Panagi and M. M. Polycarpou, "A coordinated communication scheme for distributed fault tolerant control," *Industrial Informatics, IEEE Transactions on*, vol. 9, no. 1, pp. 386–393, 2013

2. P. Panagi and M. M. Polycarpou, "Distributed fault accommodation for a class of interconnected nonlinear systems with partial communication," *Automatic Control, IEEE Transactions on*, vol. 56, no. 12, pp. 2962–2967, 2011

3. P. Panagi and M. M. Polycarpou, "Decentralized fault tolerant control of a class of interconnected nonlinear systems," *Automatic Control, IEEE Transactions on*, vol. 56, no. 1, pp. 178–184, 2011

Published conference proceedings

1. P. Panagi and M. M. Polycarpou, "Distributed fault detection and accommodation with partial communication," *Proc. of the 4th International Symposium on Communications, Control and Signal Processing*, pp. 1–5, 2010

2. P. Panagi and M. M. Polycarpou, "Decentralized fault accommodation of a class of interconnected nonlinear systems using an adaptive approximation approach," *Proc. of the 17th Mediterranean Conference on Control and Automation*, pp. 546–551, 2009

3. P. Panagi and M. M. Polycarpou, "Decentralized adaptive approximation based control with safety scheme outside the approximation region," *Proc. of the 47th IEEE Conference on Decision and Control*, pp. 92–97, 2008

4. P. Panagi and M. M. Polycarpou, "Decentralized adaptive approximation based control of a class of large-scale systems," *Proc. of the 2008 American Control Conference*, pp. 4191–4196, 2008

Contents

List of Figures

List of Tables

Chapter 1

Introduction

The emergence of large-scale engineering systems has created a paradigm shift in the way systems are analyzed, designed and implemented. Prominent examples of large-scale systems include the Internet, critical infrastructure systems and swarm robots. Characterizing a system as *large-scale* reflects the effort required to understand its behavior (e.g., commercial aircraft), but it may also reflect its spatial distribution (e.g., power grid). The inherent complexity of large-scale systems, makes the synthesis of a single centralized control system a difficult task. Moreover, the geographical distribution of the system can make the cost for centralization of information prohibitively expensive.

The common denominator of large-scale systems is that they are composed by several subsystems coupled through their dynamics (dynamically coupled systems), objectives (cooperative systems), or decision-making processes (networked control systems). Each subsystem can be modeled and controlled in isolation, based on well-understood control methodologies. However, when considering the system as a whole, the coupling between the subsystems reveals the need for a theoretical framework that combines control, communication and cooperation methodologies.

In this thesis we consider spatially distributed large-scale systems where the individual subsystems are interconnected through their dynamics. A key observation is that the behavior of each subsystem is correlated not only with the local dynamics, but also with the dynamics of other subsystems. The goal is to design local controllers that stabilize each subsystem in the presence of unknown interconnections, as well as changes in the system dynamics due to failures. To this end, a key objective is to understand how the information flow affects the stability and performance of the overall system. Information sharing involves either *a priori* knowledge of other subsystems models and goals, or *a posteriori* information exchanged between the subsystems online. In a decentralized control architecture, the subsystems do not share information, while in a distributed control architecture, the subsystems are allowed to exchange limited information online.

A key complexity in the development of decentralized control schemes is that the states of remote subsystems are unknown, and at the same time the coupling dynamics between the subsystems are at least partially unknown. Towards addressing this issue, it is typically assumed that the interconnections are weak (e.g., bounded by lower order polynomials), or satisfy a certain structure which may not be realistic in practical applications. In addition, the stability and performance of the system is often obtained at the expense of a large control effort, required for compensating for the effects of the unmodeled interconnections.

A promising extension of the decentralized control paradigm is based on the use of distributed state estimation methods for estimating the states of remote subsystems. According to this approach, the unknown states of the other subsystems are replaced by local estimators, such that it is possible to use well-known adaptive and robust control techniques for addressing the interconnections effects. In a variation of this technique, the subsystems share their desired states *a priori*, and utilize them online as estimates of the actual states. Augmenting decentralized control schemes with states estimators reduces the complexity and control effort of the local controllers, and broadens their applicability to a greater class of systems. However, in the presence of unmodeled dynamics, the states estimates can quickly deviate from the actual states, which may degrade the performance of the system, or even cause instability issues. Therefore, in the presence of faults and unknown interconnections with significant magnitude, it is important that the local controllers do not rely on distributed state estimation methods alone.

It can be said that decentralized control schemes are to distributed control schemes, as feedforward control schemes are to feedback control schemes. This analogy is justified by the fact that, the design of both feedforward and decentralized control schemes is largely based on the premise that an accurate enough model of the system is available. Feedback control schemes allow us to relax the assumption of an accurate model, by taking measurements of the system performance and adjusting the controller parameters accordingly. Similarly in a distributed control scheme, the lack of knowledge of remote subsystems models and interconnections can be replenished through the exchange of information online. Furthermore, it is natural to assume that as the communication rate increases, the performance of the system improves. However, due to high cost of communication, from a practical viewpoint it is important that the information flow is kept at a minimum. This creates an interesting trade-off where, on one hand we want to improve the stability and performance of the overall system, and on the other hand we want to limit the information that is exchanged online between the subsystems.

Towards minimizing the communication cost, a promising approach is based on the *communication-as-needed* approach: a subsystem broadcasts information only when the other subsystem needs it. A key challenge is determining when the subsystems need information, and develop communication algorithms for broadcasting information only when it benefits the stability, tracking performance, or some other property of the system. It is reasonable to assume that since the need for communication is a direct result of the coupling between the subsystems, the design of the communication algorithm should account for the structure of the interconnection effects as well as faults that occur in the system. Moreover, the decision to broadcast information should account on what is already known by the other subsystems, available through online state estimators or other *a priori* shared knowledge. Therefore, it can be said that the holy grail lies at the intersection of estimation and communication techniques, where in the absence of communication an estimator provides an accurate enough approximation of the states of remote subsystems, and the effects of inaccurate state estimates are neutralized through the exchange of information.

1.1 Motivation

This thesis focuses on the problem of distributed fault tolerant control for uncertain nonlinear interconnected systems. The main objective is to design local controllers for each subsystem that guarantee the stability of the system in the presence of uncertain nonlinear interconnections and faults. A key challenge in the control for interconnected systems is the development of techniques for dealing with uncertainties in the interconnection effects. The main difficulty is due to the fact that the states of the remote subsystems are unknown, and at the same time the interconnection effects are at least partially unknown. Ideally, each subsystem needs as much information about the other subsystems' states as possible, in order to properly estimate and address the uncertain coupling dynamics. However, in the case of spatially distributed large-scale systems (such as, power systems or satellites formation), it is unrealistic to assume continuous or almost-continuous availability of remote subsystems states. This motivates the design of decentralized local controllers for stabilizing each subsystem, in the presence of unknown interconnections, as well as changes in the system dynamics due to failures, without the need for the subsystems to exchange information. The control of large-scale systems in a decentralized control architecture is an appealing approach, as it avoids the need for implementing a communication infrastructure and, since each controller depends only on local measurements, each subsystem is more autonomous. In this thesis we study the problem of decentralized control for a class of interconnected systems and in Chapter 4 we present a decentralized fault tolerant control scheme that addresses the case of unknown interconnections and multiple faults with significantly large unknown magnitude, without exchanging state information between subsystems. However, due to the fact that no information is available about the other subsystems' states, as the strength of the interconnections increases, the local controller may generate large feedback gains to compensate for the presence of the unknown interconnection effects. In such cases, the local control input signal resembles a high-gain feedback design. Moreover, the presence of strong interconnections can lead to poor transient response and significantly increase the convergence time of the tracking errors.

Towards addressing some of the drawbacks of completely decentralized control architectures, that is, reduce the control effort for addressing the coupling dynamics and improve the performance of the overall system, a promising approach is to consider a distributed control architecture where the subsystems share *a priori* and/or limited online information. The premise for distributed control is that, provided that each subsystem has partial knowledge of the other subsystems' states, more accurate estimates of the unknown coupling dynamics can be obtained, such that it becomes possible to reduce the required control energy for canceling the effects of the coupling dynamics, as well as improve the tracking performance of the system. Moreover, it is expected that as the subsystems exchange more information, the benefits in the control effort and performance are increased. On the other hand, the geographical separation of the subsystems incurs a high energy cost for broadcasting information, often exceeding the energy required by sensors and control actuators [39, 64]. As a result, a key objective in this thesis is to answer the question of *when* should the subsystems communicate such that the benefits from the exchange from information in the performance of the system are maximized. Previous work in distributed control consider the case where the subsystems communicate based on a state level-crossing communication scheme [48, 82, 86]. It is known that such communication schemes provide better results than time-based communication schemes, where information is broadcasted periodically [5]. However, as it is shown in this thesis, in the case of interconnections with higher-order nonlinearities, a state level-crossing communication scheme leads to suboptimal utilization of the available communication resources. This motivates the development of more efficient communication algorithms, for allowing the subsystems to exchange information only when it benefits the performance or other metric of the system.

The principal difficulty in the control of interconnected systems is dealing with the coupling dynamics. Previous work in decentralized control has shown that it is feasible to address the presence of the interconnection effects, without the need for any information sharing between the subsystems. However, by considering distributed control schemes and by increasing the available information about the other subsystems states, it becomes to possible to improve the properties of the feedback control scheme and the performance of the system. In order to illustrate this, consider the tracking control problem for a simple system comprised of m interconnected subsystems, where the i-th subsystem, $i = 1, \ldots, m$, is described by

$$\dot{x}_i = a_i x_i + b_i u_i + \sum_{j=1}^{m} \theta_{ij} \phi_{ij}(x_j) \tag{1.1}$$

where $x_i \in \mathbb{R}$ is the state of the i-th subsystem, u_i is the control input, and $a_i \in \mathbb{R}$ and $b_i \in \mathbb{R} - \{0\}$ are known constants. The interconnection term $\theta_{ij}\phi_{ij}(x_j)$ represents the effect of the j-th subsystem on the i-th subsystem dynamics, where $\theta_{ij} \in \mathbb{R}$ is an unknown constant, and $\phi_{ij} : \mathbb{R} \to \mathbb{R}$ is a known function. For notational convenience, $\theta_{ii} = 0$ for $i = 1, \ldots, m$. The objective is to design a control law u_i for each i-th subsystem such that x_i follows a reference trajectory x_{d_i}, where x_{d_i} and $\dot{x}_{d_i}$ are uniformly bounded. Let the functions ϕ_{ij} be globally Lipschitz such that

$$|\phi_{ij}(x_j) - \phi_{ij}(\bar{x}_j)| \leqslant L_{ij}|x_j - \bar{x}_j|, \tag{1.2}$$

for all $x_j, \bar{x}_j \in \mathbb{R}$, where $L_{ij} > 0$ are the Lipschitz constants. From (1.2), there exist some $\sigma_{ij} > 0$ constants such that

$$|\phi_{ij}(x_j)| \leqslant L_{ij}|x_j| + \sigma_{ij}. \tag{1.3}$$

The bounding assumption (1.3) is satisfied by several mechanical systems [75]. In addition, from (1.3) and the fact that x_{d_j} is uniformly bounded, there exist some $\bar{\sigma}_{ij} > 0$ constants such that

$$|\phi_{ij}(x_j)| \leqslant L_{ij}|\tilde{x}_j| + \bar{\sigma}_{ij}. \tag{1.4}$$

It has been shown that the bounding assumption (1.4) is satisfied for a spring-connected double inverted pendulum [71], and for an inter-vehicle spacing regulation problem [72]. Consider the tracking error dynamics, $\tilde{x}_i = x_i - x_{d_i}$ of the i-th subsystem, which based on (1.1) satisfy

$$\dot{\tilde{x}}_i = a_i + b_i u_i + \delta_{ij}(x_j) - \dot{x}_{d_i} \tag{1.5}$$

The performance and robustness properties of the control scheme are closely tied to the structure and complexity of the interconnection, as well as available *a priori* and *a posteriori* information about the remote subsystems. Towards illustrating the repercussions of this dependency, consider the following cases:

Case A. The structure of the interconnections and the states of remote subsystems are both unknown.

Case B. The structure of the interconnections is available, i.e., it is known that the interconnections are described by $\theta_{ij}\phi_{ij}(x_j)$, with known globally Lipschitz functions ϕ_{ij}.

Case C. Same as in Case B., and in addition the desired state x_{d_i} for each subsystem i is available to all the subsystems for all $t > 0$.

Case D. Same as in Case B., and in addition $\hat{x}_j^i$ estimates of the remote x_j states are available to the i-th subsystem, satisfying $|x_j - \hat{x}_j^i| \leqslant d_{ij}$, where $d_{ij} > 0$ are design constants.

Therefore, at each case we progressively assume more information is available about the interconnections and the other subsystems. In the next subsections we present the control design and discuss the properties of the feedback control scheme for each case.

1.1.1 Case A. Unknown interconnections

Consider the isolated subsystems (1.1), $\delta_{ij}(x_j) = 0$ for $i = 1, \ldots, m$ and $j = 1, \ldots, m$. Then the control law u_i given by

$$u_i = -\frac{1}{b_i}\big(a_i x_i + k_i \tilde{x}_i - \dot{x}_{d_i}\big), \tag{1.6}$$

guarantees global asymptotic stability of the i-th isolated subsystem, i.e., $\tilde{x}_i \to 0$ as $t \to \infty$, where $k_i > 0$ are design constants representing the feedback gain. In the absence of any information about either the structure of the interconnections or the x_j states, the i-th subsystem cannot do better than treat the $\theta_{ij}\phi_{ij}(x_j)$ interconnections as disturbance terms. The following lemma characterizes the stability properties of the feedback control scheme in the presence of interconnection terms $\theta_{ij}\phi_{ij}(x_j)$.

Lemma 1.1. *Given that the feedback gain k_i satisfies*

$$k_i = \lambda_i + \frac{1}{2}\sum_{j=1}^{m} 1 + \theta_{ij}^2 L_{ij}^2, \tag{1.7}$$

where $\lambda_i > 0$ is a constant, the closed-loop system described by the interconnected system (1.1) and the control law (1.6) ensure that $\tilde{x}_i$ is ultimately bounded by $\frac{\theta_{ij}\bar{\sigma}_{ij}}{\lambda_i}$.

The proof of Lemma 1.1 is given in Appendix 7.2.3. Lemma 1.1 shows that even in the case where the k_i parameters are large enough such that (1.7) is satisfied, only boundedness of the tracking errors can be guaranteed. Asymptotic stability is possible only for $\sigma_{ij} = 0$, which requires that the interconnections $\theta_{ij}\phi_{ij}(x_j)$ vanish at $\tilde{x}_j = 0$. For k_i parameters not satisfying (1.7), the Lyapunov analysis can not guarantee the stability of the system. In the absence of any information about the interconnections, the only hope for stabilizing the system is to select large enough parameters k_i, such that the feedback term $-K_i\tilde{x}_i$ can partially compensate for the presence of the uncertain interconnections. However, such a design implies high-gain feedback controllers and the effort required to stabilize the system may lead the actuator to saturation.

1.1.2 Case B. Interconnections with known structure

Consider the case where the structure of the interconnections $\theta_{ij}\phi_{ij}(x_j)$ is known and that the ϕ_{ij} functions are globally Lipschitz. The following analysis shows that we can use this knowledge to design adaptive control algorithms that stabilize the feedback control scheme. Consider the sum of terms

$$A(x_1, \tilde{x}_1, \ldots, x_m, \tilde{x}_m) = \sum_{i=1}^{m}\sum_{j=1}^{m} \tilde{x}_i \theta_{ij} \phi_{ij}(x_j),$$

which based on (1.4) satisfies

$$A(\tilde{x}_i, \tilde{x}_1, \ldots, \tilde{x}_m) \leqslant \sum_{i=1}^{m}\sum_{j=1}^{m} |\tilde{x}_i| \theta_{ij} L_{ij} |\tilde{x}_j| + |\tilde{x}_i| \theta_{ij} \bar{\sigma}_{ij}.$$

Using the inequality $2\alpha\beta \leqslant \alpha^2 + \beta^2$ for $\alpha, \beta \in \mathbb{R}$

$$A(x_1, \tilde{x}_1, \ldots, x_m, \tilde{x}_m) \leqslant \sum_{i=1}^{m}\sum_{j=1}^{m} \Big(\frac{1}{2} + \frac{1}{2}\theta_{ij}^2 L_{ij}^2\Big)\tilde{x}_i^2 + |\tilde{x}_i| \theta_{ij} \bar{\sigma}_{ij} \tag{1.8}$$

Define the parameters $\theta_{ij}^a, \theta_{ij}^b$ as

$$\theta_{ij}^a = \frac{1}{2} + \frac{1}{2}\theta_{ij}^2 L_{ij}^2 \tag{1.9}$$

$$\theta_{ij}^b = \theta_{ij} \bar{\sigma}_{ij} \tag{1.10}$$

Since the θ_{ij}^a, θ_{ij}^b parameters are unknown, they are estimated online with adaptive parameters $\hat{\theta}_{ij}^a$, $\hat{\theta}_{ij}^b$. Let the control law for the i-th subsystem be given by

$$u_i = -\frac{1}{b_i}\big(a_i + k_i \tilde{x}_i - \dot{x}_{d_i} + u_{s_i}\big) \tag{1.11}$$

$$u_{s_i} = \sum_{j=1}^{m} \hat{\theta}_{ij}^a \mathrm{sgn}(\tilde{x}_i)|\tilde{x}_i| + \hat{\theta}_{ij}^b \mathrm{sgn}(\tilde{x}_i), \tag{1.12}$$

where the adaptive parameters $\hat{\theta}_{ij}^a$, $\hat{\theta}_{ij}^b$ are updated according to the following adaptive laws

$$\dot{\hat{\theta}}_{ij}^a = \gamma_{ij}^a \tilde{x}_i^2 \tag{1.13}$$

$$\dot{\hat{\theta}}_{ij}^b = \gamma_{ij}^b |\tilde{x}_i|, \tag{1.14}$$

where γ_{ij}^a, $\gamma_{ij}^b > 0$ are design constants representing the adaptive gain. The following lemma characterizes the stability properties of the feedback control scheme.

Lemma 1.2. *The closed-loop system described by the interconnected system (1.1), the control law (1.11) and the adaptation laws (1.13), (1.14) guarantee that the tracking errors $\tilde{x}_i$ converge asymptotically to zero.*

The proof of Lemma 1.2 is given in Appendix 7.2.3. Knowledge of the structure of the interconnections makes it possible to design decentralized adaptive controllers that guarantee the stability of the system, despite the fact that the states of the other subsystems are completely unknown. Notice however that due to the non-decreasing growth of the parameter estimates $\hat{\theta}^a_{ij}$ and $\hat{\theta}^b_{ij}$, the u_{s_i} control terms can produce large feedback gains. Therefore, the stability of the decentralized feedback control scheme may only be guaranteed at the expense of a large control effort. Intuitively, the adaptive bounding terms ensure that each isolated subsystem is sufficiently stable, such that the class of interconnections that satisfy (1.2) can not destabilize the system.

1.1.3 Case C. Desired states are available to all the subsystems

In this case, the desired states x_{d_j}, $j = 1, \ldots, m$ are available to all the subsystems for all $t > 0$. Provided that the actual state x_j is close enough to the desired state x_{d_j}, then x_{d_j} represents a good estimate of the unknown state x_j. This additional knowledge can be used to improve the properties of the feedback control scheme. Let the control law for the i-th subsystem be given by

$$u_i = -\frac{1}{b_i}\left(a_i + k_i\tilde{x}_i - \dot{x}_{d_i} + u_{s_i}\right) \tag{1.15}$$

$$u_{s_i} = \sum_{j=1}^{m} \hat{\theta}^a_{ij}\text{sgn}(\tilde{x}_i)|\tilde{x}_i| + \hat{\theta}_{ij}\phi_{ij}(x_{d_j}), \tag{1.16}$$

where $\hat{\theta}_{ij}$ are adaptive parameter estimates of the unknown θ_{ij} parameters, updated according to the following adaptive law

$$\dot{\hat{\theta}}_{ij} = \gamma_{ij}\tilde{x}_i\phi_{ij}(x_{d_j}), \tag{1.17}$$

where $\gamma_{ij} > 0$ are design constants. The following lemma characterizes the stability properties of the feedback control scheme.

Lemma 1.3. *The closed-loop system described by the interconnected system (1.1), the control law (1.15) and the adaptation laws (1.13), (1.17) guarantee that $\tilde{x}_i$ converge asymptotically to zero.*

The proof of Lemma 1.3 is given in Appendix 7.2.3. Note that the i-th subsystem makes an effort to address the uncertain interconnection term $\theta_{ij}\phi_{ij}(x_j)$, using a term $\hat{\theta}_{ij}\phi_{ij}(x_{d_j})$ in the feedback control term u_{s_i}. Therefore, it is useful to study the overall error $\theta_{ij}\phi_{ij}(x_j) - \hat{\theta}_{ij}\phi_{ij}(x_{d_j})$ which represents how well the feedback term

$\hat{\theta}_{ij}\phi_{ij}(x_{d_j})$ matches the uncertain interconnection term $\theta_{ij}\phi_{ij}(x_j)$. Adding and subtracting $\theta_{ij}\phi_{ij}(x_{d_j})$ we obtain

$$\begin{aligned} |\theta_{ij}\phi_{ij}(x_j) - \hat{\theta}_{ij}\phi_{ij}(x_{d_j})| &= |\theta_{ij}\phi_{ij}(x_j) - \theta_{ij}\phi_{ij}(x_{d_j}) + \theta_{ij}\phi_{ij}(x_{d_j}) - \hat{\theta}_{ij}\phi_{ij}(x_{d_j})| \\ &\leqslant |\theta_{ij}\phi_{ij}(x_j) - \theta_{ij}\phi_{ij}(x_{d_j})| + |\theta_{ij}\phi_{ij}(x_{d_j}) - \hat{\theta}_{ij}\phi_{ij}(x_{d_j})| \\ &\leqslant |\theta_{ij}|L_{ij}|\tilde{x}_j| + |\tilde{\theta}_{ij}||\phi_{ij}(x_{d_j})|, \end{aligned} \tag{1.18}$$

where $\tilde{\theta}_{ij} = \hat{\theta}_{ij} - \theta_{ij}$ is the parameter estimation error. The right-hand side of (1.18) shows that the ability to match the uncertain interconnection term $\theta_{ij}\phi_{ij}(x_j)$ is limited by two factors, the parameter estimation error $\tilde{\theta}_{ij}$ and the tracking error $\tilde{x}_j$. When $\tilde{x}_j$ is small, a large overall error reveals that the magnitude of the parameter estimation error $\tilde{\theta}_{ij}$ is large. However this does not present an issue, as the goal is to drive the tracking error x_i to zero, not parameter estimation convergence. This means that while $\tilde{\theta}_{ij}$ can be large, the estimated parameter $\hat{\theta}_{ij}$ can be such that the term $\hat{\theta}_{ij}\phi_{ij}(x_{d_j})$ is able to compensate for the presence of the uncertain interconnection. On the other hand, provided that $\tilde{\theta}_{ij}$ is small, the term $|\theta_{ij}|L_{ij}|\tilde{x}_j|$ implies that a large overall error is due to a tracking error $\tilde{x}_j$ with significant magnitude. A surrogate of this term is addressed by the bounding control term $\hat{\theta}^a_{ij}\mathrm{sgn}(\tilde{x}_i)|\tilde{x}_i|$. The stability analysis shows that all tracking errors eventually converge to zero, which implies that $|\theta_{ij}|L_{ij}|\tilde{x}_j| \to 0$ as $t \to \infty$. However, in the presence of large tracking errors during the initial stages of the system, the adaptive bounding parameters $\hat{\theta}^a_i$ may grow large, which can make the control effort unnecessarily large. The fact that an upper bound of the tracking error $\tilde{x}_j$ is not guaranteed, may force the i-th subsystem to generate large adaptive bounding terms for addressing the effects of large overall errors.

1.1.4 Case D. Remote states estimates are available

Finally, we consider the case where estimates of the states of remote subsystems are available. More specifically, an estimate $\hat{x}^i_j(t)$ of x_j is available to the i-th subsystem that satisfies

$$|x_j(t) - \hat{x}^i_j(t)| \leqslant d_{ij}, \tag{1.19}$$

for all $t > 0$, where $d_{ij} > 0$ is a constant. The $\hat{x}^i_j$ estimate is typically based on information exchange between the subsystems, or a combination of local state estimation with communication. In this formulation, d_{ij} represents the communication threshold. Based on $\hat{x}_j^i$, the i-th subsystem uses a feedback control term $\hat{\theta}_{ij}\phi_{ij}(\hat{x}_j^i)$ to address the uncertain interconnection term $\theta_{ij}\phi_{ij}(x_j)$. Based on (1.2) and adding and subtracting $\theta_{ij}\phi_{ij}(\hat{x}_j^i)$, the overall error $\theta_{ij}\phi_{ij}(x_j) - \hat{\theta}_{ij}\phi_{ij}(\hat{x}_j^i)$ satisfies

$$|\theta_{ij}\phi_{ij}(x_j) - \hat{\theta}_{ij}\phi_{ij}(\hat{x}_j^i)| \leqslant |\theta_{ij}|L_{ij}d_{ij} + |\tilde{\theta}_{ij}||\phi_{ij}(x_{d_j})|,$$

where the fact that $\hat{x}_j^i(t)$ satisfies $|x_j(t) - \hat{x}_j^i(t)| \leqslant d_{ij}$ for all $t > 0$ is used. Define the parameters θ_{ij}^c as

$$\theta_{ij}^c = |\theta_{ij}|L_{ij}d_{ij}. \tag{1.20}$$

The unknown parameters θ_{ij}^c are estimated online with adaptive parameters $\hat{\theta}_{ij}^c$. Let the control law for the i-th subsystem be given by

$$u_i = -\frac{1}{b_i}\big(a_i + k_i\tilde{x}_i - \dot{x}_{d_i} + u_{s_i}\big) \tag{1.21}$$

$$u_{s_i} = \sum_{j=1}^{m}\hat{\theta}_{ij}^c \mathrm{sgn}(\tilde{x}_i) + \hat{\theta}_{ij}\phi_{ij}(\hat{x}_j^i), \tag{1.22}$$

where the parameter estimates are $\hat{\theta}_{ij}^c$ and $\hat{\theta}_{ij}$ are updated according to the following adaptive laws

$$\dot{\hat{\theta}}_{ij}^c = \gamma_{ij}^c|\tilde{x}_i| \tag{1.23}$$

$$\dot{\hat{\theta}}_{ij} = \gamma_{ij}\tilde{x}_i\phi_{ij}(\hat{x}_j^i), \tag{1.24}$$

where $\gamma_{ij}^c > 0$ are design constants. The following lemma characterizes the stability properties of the feedback control scheme.

Lemma 1.4. *The closed-loop system described by the interconnected system (1.1), the control law (1.21) and the adaptation laws (1.23), (1.24) guarantee that $\tilde{x}_i$ converge asymptotically to zero.*

The proof of Lemma 1.4 is given in Appendix 7.2.3. The bounding control term $\hat{\theta}_{ij}^c\mathrm{sgn}(\tilde{x}_i)$ in (1.22) can be made as small as desired by selecting a smaller communication threshold d_{ij}. This demonstrates a trade-off between communication cost and performance. A smaller communication threshold can improve the performance properties of the feedback control scheme, but it can also substantially increase the amount of information that is exchanged online. In addition, the fact that the estimation error $\hat{x}_j^i - x_j$ is bounded by a certain constant d_{ij}, ensures the boundedness of the overall error $|\theta_{ij}\phi_{ij}(\hat{x}_j^i) - \hat{\theta}_{ij}\phi_{ij}(x_j)|$, such that it becomes possible to reduce the control effort for addressing its presence.

The above analysis illustrates that the performance of the system is tightly related to the amount of available information shared between subsystems. Even though in each case the control design guarantees the stability of the interconnected system, by assuming more information is available about the remote subsystems' states, it becomes possible to progressively enhance the performance properties of the system. Moreover, the exchange of information between interconnected subsystems can significantly reduce the control effort for addressing the uncertain interconnection terms. In later chapters, we consider a more general class of interconnected systems, as well as changes in the system dynamics due to the occurrence of multiple faults. As we will see, ensuring the feedback control scheme stability and performance without a large control effort becomes a challenging task, and dictates the development of more sophisticated distributed fault tolerant control and communication methodologies.

1.2 Contributions

The main goal of this thesis is the development of fault tolerant control methodologies for interconnected systems. At first, we consider the case where the subsystems do not exchange information, and develop a decentralized fault tolerant control scheme where the presence of the unknown coupling dynamics is addressed through the use of robustifying terms, designed based on an adaptive approximation framework. A dead-zone modification in the adaptive laws addresses stability and robustness issues associated with the presence of residual approximation errors, while an adaptive bounding method relaxes the assumption for a known upper bound on the residual approximation errors. However, decentralized control architectures are typically effective only for weakly interconnected systems, as the presence of strong coupling dynamics can potentially turn the control law into high gain.

A key objective of this thesis is the development of efficient communication algorithms for enhancing the stability, performance and reliability of the system. A distributed fault detection and accommodation scheme is developed where the subsystems exchange information based on a self-triggering tracking-error based communication scheme. Partial knowledge of the other subsystems' states allows for obtaining more accurate estimates of the unknown interconnections and faults, such that the control effort to address the unknown coupling dynamics is reduced. Moreover, it is demonstrated that the detectability of faults is improved as the amount of state information exchanged between the subsystems increases. A coordinated communication scheme is presented, which generalizes the self-triggering communication scheme and allows the designer to reduce the cost for communication by trading off some tracking performance. The main idea of the coordinated communication scheme is to avoid to broadcast information when the receiver subsystem performs well, i.e., its tracking error is small. Finally, an optimized communication algorithm is developed where the subsystems communicate such that the uncertainty about the coupling dynamics is minimized. In the special case of known interconnection functions, the presented communication scheme guarantees that the subsystems exchange information such that the impact on the subsystems dynamics due to inaccurate remote states information is minimized, and as a result, it leads to less conservative control gains for addressing the interconnection effects. In the more general case of unknown interconnections and faults, the decision for communication is based on the use of adaptive approximation models for learning the unknown coupling dynamics, and then using this knowledge for minimizing the uncertainty about the coupling dynamics.

1.3 Organization of the Thesis

The organization of the thesis is as follows:

Chapter 2 conducts a literature review on decentralized and distributed control methods for interconnected systems, as well as fault diagnosis methods for dynamic systems.

Chapter 3 presents a decentralized fault tolerant control scheme for the control of a class of interconnected nonlinear systems. We consider the case where changes in dynamics due to multiple failures may occur in any of the subsystems as well as in the interconnection effects. Based on linearly parameterized neural networks, the presence of uncertain dynamics is addressed by estimating unknown upper bounds of the interconnections and faults. Moreover the robustness in the presence of residual approximation errors is guaranteed with the use of a combination of a dead-zone modification in the adaptive laws, and an adaptive bounding method. The main result shows that, despite the fact that the control design is completely decentralized, boundedness of the tracking errors to a small region is guaranteed. In addition, the fact that the approximation errors can become significantly large outside the coverage region of the approximation can cause instability issues to closed-loop system.

For addressing this problem, we develop a decentralized safety control scheme, based on a decentralized sliding mode control design and adaptive bounding terms for compensating for the unknown interconnections and faults. The stability analysis shows that the decentralized safety control scheme guarantees that the time spent outside the coverage region of the approximator is finite. This chapter is based on [49–51, 53].

Chapter 4 presents a distributed fault detection and accommodation design for a class of interconnected nonlinear systems, where faults can occur in any of the subsystems as well as in the interconnections. The subsystems share their desired states *a priori*, and communicate online according to a self-triggering tracking-error based communication scheme. More specifically, the state of a subsystem is communicated to the other subsystems whenever the local tracking error exceeds a certain threshold. First, a set of distributed nonlinear estimators for each subsystem is designed for the detection of the occurrence of faults. The fault detection scheme ensures that there are no false alarms. After a fault is detected by any subsystem, a distributed fault accommodation algorithm is activated for compensating the effect of the fault. The fault accommodation algorithm is based on the adaptive approximation of an upper bound of the fault function. It is shown that by approximating the upper bound of the fault function, instead of the fault function itself, robustness to residual approximation errors is ensured. The stability analysis establishes asymptotic stability of the closed-loop system. This chapter is based on [52, 54].

Chapter 5 presents a coordinated communication scheme for the distributed fault tolerant control of interconnected nonlinear systems. The proposed communication scheme is based on the idea that it is possible to reduce the cost of communication, by avoiding to broadcast information when the receiver does not need it. According to this scheme, two subsystems communicate only when the tracking errors of both of the subsystems exceed a certain threshold. The stability analysis of the feedback control scheme show the boundedness of the tracking errors to a region around zero. The simulation results show that the communication cost is substantially reduced, when compared to the self-triggering tracking-error based communication scheme, presented in Chapter 4. The trade-off is that only boundedness of the tracking errors can be guaranteed, while in the case of a self-triggering tracking-error based communication scheme, asymptotic convergence of the tracking errors to zero is ensured. This chapter is based on [55].

Chapter 6 presents an optimized communication scheme for the distributed fault tolerant control of interconnected nonlinear systems. The main objective is the development of a communication decision algorithm, such that the benefits from communication in the performance (or other metric) of the system are maximized. Communication optimization is formulated as a problem of best approximation of a continuous function based on the use of step functions. We establish the approximation properties of step functions, and show that a partition of the range of the function leads to the best $\mathcal{L}_\infty$ step function approximation. Moreover, we consider a class of piecewise linear functions and study the approximation performance improvement when a best step function approximation is used. The distributed fault tolerant control design is based on the use of linearly parameterized neural networks for approximating the unknown interconnections. When the local tracking error is outside a small region around zero, the other subsystems broadcast their states according to a state level-crossing communication scheme. When the local tracking error is inside a small region around zero, the communication decision is based on the use of approximation models of the unknown interconnections and fault functions, such that the uncertainty about the coupling dynamics is minimized. The stability analysis shows boundedness of the tracking errors to a small region around zero.

Chapter 7 contains some concluding remarks and outlines directions for future research.

Chapter 2

Literature Review

2.1 Decentralized Control of Interconnected Systems

During the past decades there has been significant research activity in the modeling and control of large-scale and physically distributed systems. This interest is motivated by applications in such diverse areas as manufacturing, transportation, power systems and mobile robotics. While the control structure of such systems was initially considered in a centralized and/or hierarchical framework, as the complexity of large-scale systems grows, it becomes apparent that the design of a single centralized controller is a difficult task. In addition, the spatial distribution of the system requires considerable communication resources for collecting measurements and broadcasting control decisions. In general, as a result of the complexity and geographical separation of a large-scale system, the presupposition of centrality of information and computation cannot be satisfied.

A promising approach is based on a *divide and conquer* strategy, where the large-scale system is decomposed into several subsystems, and the task of controlling the overall large-scale system is divided into smaller subproblems of synthesizing local controllers for each subsystem [65]. The decomposition of the large-scale system leads to an interconnected systems structure, and the design is based on a decentralized control architecture (Fig. 2.1). The underlying idea of the decentralized control approach is that, if local controllers stabilize the subsystems in the presence of interconnections between the subsystems, then the stability of the overall large-scale system is guaranteed. Decentralized control leads to simpler models and can reduce

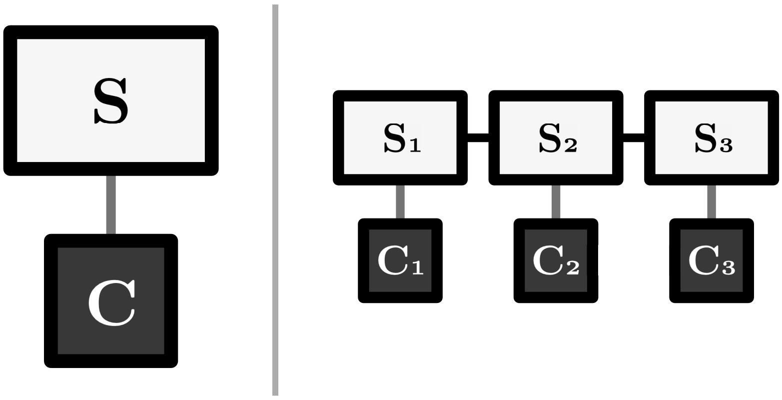


Figure 2.1: Decentralized Control Architecture. A large-scale system S is decomposed into smaller interconnected subsystems S_1, S_2 and S_3, and each subsystem is independently controlled by decentralized controllers C_1, C_2 and C_3, respectively.

the computational burden for control system analysis and design. Furthermore, the fact that the control process runs locally at each subsystem, removes the requirement for broadcasting information to and from a central distant location.

A key challenge in decentralized control is the development of control methodologies for addressing the interconnections between the subsystems. Consider the interconnected systems S_i and S_j shown in Fig. 2.2, where δ_{ij} represents the effect of the S_j dynamics onto the S_i dynamics. The goal in decentralized control is to design stabilizing controllers for each S_i and S_j subsystems. The fact that both the interconnection effect δ_{ij} is typically partially unknown and, at the same time, no online information is available about subsystem S_j, constitutes one of the core issues in decentralized control. A considerable amount of research effort is directed towards overcoming this obstacle.

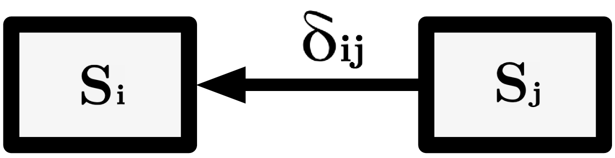


Figure 2.2: The S_j dynamics affect the S_i dynamics through the interconnection δ_{ij}.

The strength of the interconnections is a fundamental indicator of the complexity of the decentralized control problem. In general, as we consider interconnected systems with stronger couplings, it becomes increasingly difficult to stabilize the subsystems in a decentralized way. As it is shown in [29], even weak interconnections can potentially destabilize the system, if not properly addressed by the control design. A well known result that guarantees the stabilizability of interconnected systems by local state feedback is based on the M-matrix condition [33,70]. The basic idea is that if the *degree of stability* of the closed-loop isolated subsystems is sufficiently higher than the strength of the interconnections, then it is guaranteed that the stabilized isolated subsystems will remain stable when interconnected. By ensuring that the M-matrix condition is satisfied, we are able to design feedback control laws as if the subsystems were isolated. In [29,40] the authors present model reference adaptive schemes for linear interconnected systems that satisfy the M-matrix condition. The proposed design guarantee boundedness of the state errors in the presence of bounded interconnections. However, the M-matrix condition typically depends on unknown parameters, such that it may be difficult to satisfy in practice. Furthermore, such schemes can typically only guarantee boundedness of the states into some small region, and it is often not possible to control the size of the stability region. In [24], linearly interconnected systems are considered and a decentralized adaptive control method is presented that does not require the M-matrix condition. The design is based on an adaptive high-gain approach that guarantees global boundedness of the solutions. The feedback gains are adapted to whatever level is required to counteract instability caused by the presence of the interconnections effects.

The aforementioned decentralized control design methods are based on the assumption of linear interconnections or weak nonlinear interconnections. As it is demonstrated in [69], these schemes are not able to handle higher-order interconnections; the presence of interconnections with significant strength can potentially destabilize the system. The aforementioned work is the first one that considers higher-order interconnections. The authors propose a decentralized adaptive control scheme that is applicable to linear interconnected systems with interconnections bounded by unknown p-th order polynomials in states. The stability of the decentralized feedback control scheme is achieved through the use of higher-order adaptive feedback terms that are able to dominate the unknown polynomial growth interconnections. A drawback of the proposed scheme is that the interconnections need to satisfy a matching condition. In other words, the uncertain interconnections terms need to appear the same point as the control inputs. The matching condition has since been addressed with the backstepping technique [35], and in [31], the matching condition for the interconnections is relaxed by combining a high-gain feedback approach with adaptive backstepping, that guarantees global boundedness of the solutions. In [26], a robust backstepping design is presented that extends the results to the case of general nonlinear bounds of interconnections. However, although it generalizes the applicability of the design, the local controller may exhibit high-gain feedback signals in order to compensate for the effects of strong interconnections. The stability of the overall system is maintained by a sufficiently large local control effort that is able to robustly counteract the presence of the interconnections, no matter the strength of their effect.

Towards addressing some of the drawbacks of completely decentralized control architectures, a recent approach assumes common *a priori* knowledge of the desired states of the interconnected subsystems. It was introduced by the work of [43, 47] for linearly interconnected systems, and extended to nonlinear interconnections in [46]. According to this approach, the unknown states of the other subsystems, which appear in the local feedback control law, are replaced by the known reference signals. Intuitively, if the subsystems perform as expected, the reference signals provide a good estimate of the unknown actual states. The authors consider linearly interconnected systems and present an adaptive control scheme that ensures asymptotic stability of the overall system. A sufficient condition for the applicability of the methodology is that the reference signals are close enough to the actual states, which may be impossible to guarantee in practice. The presence of large initial tracking errors, disturbances, faults, or changes of the desired states during operation of the system, can considerably degrade the tracking performance or even destabilize the system. Based on this approach, [45] presents a decentralized adaptive output-feedback control scheme. The unknown interconnections are partially compensated through the use of neural networks, and the remainder terms are addressed with the use of adaptive bounding control. In [9], the results are extended to nonlinear interconnections that do not satisfy the matching condition. In addition, the proposed scheme does not impose any bounding assumptions on the interconnections. In general, the substitution of the unknown actual states with the desired states produces a replacement error into the subsystem dynamics, which is typically assumed to satisfy a Lipschitz condition and as such, it is easier to address with adaptive bounding terms. However, in the presence of unmodeled dynamics (such as faults or disturbances), these bounding terms can become quite large.

A promising approach for dealing with nonlinear uncertainties in the system dynamics is based on the use of approximation models [10, 19, 60, 66, 68, 73, 85, 87]. The universal approximation property of such structures, with the proper selection of parameters and basis functions, allows for estimating the uncertain nonlinear dynamics of the system to an arbitrary accuracy. The ability to estimate the uncertain nonlinear dynamics makes it possible to adjust the parameters feedback control schemes to the needs of the system, avoiding the requirement for large adaptive bounding terms, or ad-hoc nonlinear adaptive feedback terms. In [66], Gaussian radial basis functions are employed to adaptively compensate for the system unknown nonlinearities. A constructive procedure is presented, which directly translates the smoothness properties of the nonlinearities involved, into a specification of the network required to represent the system to a desired degree of accuracy. In [60], the inherent approximation error is adaptively estimated online using an adaptive bounding method. In [73], stable direct and indirect adaptive approximation based controllers are presented based on the principle of certainty equivalence, that provide asymptotic tracking of a reference signal. In [68], an output feedback control scheme is presented which combines adaptive approximation control with a high-gain observer to achieve semi-global boundedness of the tracking error, while the presence of approximation errors is addressed using projection modification methods. Adaptive approximation methods have been applied for the control of nonlinear interconnected systems in [28, 37, 45, 71, 77, 78, 95]. The principal difficulty in such schemes is the design of suitable adaptive laws for estimating the unknown interconnections functions, provided that the inputs of the functions (typically the states of remote subsystems) are not available.

2.2 Distributed Control of Interconnected Systems

The applicability of decentralized control schemes is typically limited by at least one of the following:

(a) Only linear, or weak nonlinear, interconnections are considered,

(b) The structure of the interconnections is known (e.g., linear growth, polynomial growth, etc.),

(c) Only boundedness of the solution is guaranteed (not asymptotic stability),

(d) A sufficiently large control effort is required,

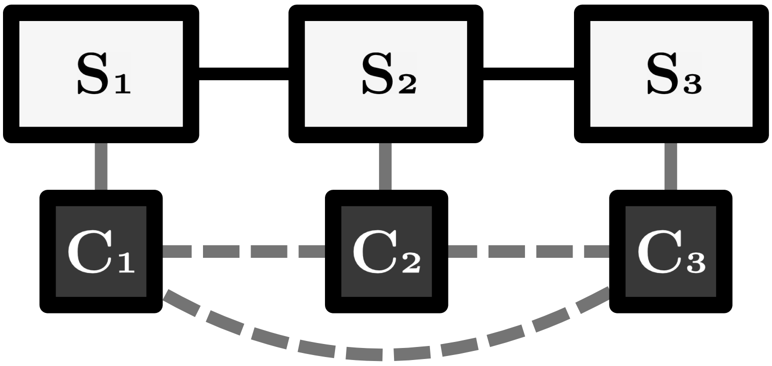


Figure 2.3: Distributed Control Architecture. The C_1, C_2 and C_3 controllers are allowed to exchange limited information online.

(e) Changes in subsystems and interconnections dynamics due to faults are not considered.

Motivated by limitations of decentralized control, a natural direction towards the development of reliable control schemes for large-scale systems is to consider distributed architectures, where the interconnected subsystems are allowed to exchange limited information online (Fig. 2.3). The underlying idea is that by incorporating online knowledge about the states of the other subsystems, it becomes possible to partially alleviate the limitations of completely decentralized control schemes.

The concept of the exchange of information between interconnected subsystems is formulated in [46]. The simulation analysis in this work demonstrates that the performance of the system is substantially improved, even when the subsystems communicate at very few instants. The authors assert that the design of the communication algorithm is completely decoupled by the stability of the system. In other words, the system remains stable even when no information is exchanged between the subsystems. It is indeed desirable to ensure the stability of the system in the absence of communication, such that the system operates normally in the case of failing communication links. At the same time however, we are interested in being able to analyze how communication affects the stability and performance of the system. It is sensible to assume that by increasing the rate of communication, some of the properties of the system (e.g., performance, control energy, tolerance to faults, etc.) will likely improve. However, an analytical framework that relates the design of the communication decision algorithm to the properties of the feedback control scheme is not present.

In practical applications of distributed interconnected systems, there is a need for maximizing the amount of useful information that is broadcasted between subsystems, while minimizing communication cost. The importance of such a goal becomes clearer in the case of spatially distributed systems, where the broadcast of information incurs large energy cost, in some cases exceeding the energy required by sensors and control actuators [39, 64]. We seek to improve the quality of communication, rather than the quantity, by designing broadcasting algorithms with low-bandwidth requirements that intelligently decide when to communicate, such that the benefits in system stability and performance are maximized. Time-based communication schemes, where each system broadcasts information periodically, leads to unnecessary high communication cost, especially in the case of slowly-developing dynamical systems. A promising approach is based on the use of internal system events for deciding when to broadcast information [2,3,14,15,23,27,59,67,81,93]. It has been shown that an event-based communication scheme leads to better performance, as compared to a time-based communication scheme, with the same amount of broadcasted information [5]. In an event-based communication scheme, communication occurs whenever some monitored signal (such as the state) crosses one or more levels. Event-based communication is also known as level-crossing sampling [34], send-on-delta algorithm [44], and Lebesgue sampling [5]. According to an event-based communication scheme, the exchange of information is more frequent when the monitored signal changes rapidly, and sparser when the signal varies slowly. In this way, communication resources of each subsystem are reserved when there is little benefit in broadcasting information, and instead being utilized in cases where more information would actually benefit the other subsystems. A special case of event-based communication schemes utilizes the local tracking error for deciding when to communicate [46, 48]. Intuitively, this communication algorithm is based on the idea that when the local tracking error is small, the desired reference trajectory provides a good approximation of the actual state. Therefore, whenever the tracking error is small and the desired trajectory of neighboring subsystems are available, then communication can be avoided and the desired trajectory can be used instead of the actual state measurement. However, as the interconnections become more complex, with higher degrees of nonlinearities, these approaches become less effective, since a small (large) local tracking error does not necessarily imply a small (large) impact on the other subsystems' dynamics. This important fact implies that the design of the communication algorithm cannot be an afterthought, but it should be tightly dependent to the structure of the interconnection dynamics, as well as the control algorithm.

Table 2.1 summarizes the features of relevant decentralized and distributed control methodologies for interconnected systems.

Table 2.1: Overview of relevant decentralized and distributed control methodologies for interconnected systems.

Publications	Interconnections	Control Design	Communication	Stability Properties	Faults
[31, 69]	bounded by unknown p-order polynomials	adaptive control	no	ultimate boundedness	no
[71]	bounded by unknown first-order polynomials	adaptive approximation control	no	asymptotic stability	no
[28]	bounded by unknown nonlinear functions	adaptive approximation control	no	ultimate boundedness	no
[47]	linear interconnections	adaptive control	share desired states	asymptotic stability	no
[46]	bounded by unknown p-order polynomials	adaptive control	share desired states	asymptotic stability	no
[45]	bounded by unknown first-order polynomials	adaptive approximation control	share desired states	ultimate boundedness	no
[9]	no assumption	adaptive approximation control	share desired states	ultimate boundedness	no
[48]	linear interconnections	adaptive control	self-triggering tracking-error based	asymptotic stability	no
[82]	Lipschitz interconnections	adaptive control	state level-crossing	ultimate boundedness	no

2.3 Fault Diagnosis Methods for Dynamic Systems

A key objective in engineering is the design of reliable systems that are able to operate within certain performance margins even in the presence of faults [6,61,74,76, 80,88,89,91,94]. Although there has been significant research activity in decentralized adaptive control for interconnected systems, the problem of fault tolerance for such systems is not well studied and it presents many theoretical challenges.

A large amount of adaptive control techniques that have been developed in the past were based on the assumption that the model of the system is sufficiently accurate, with only small perturbations from the nominal model. In practice, critical changes in the system dynamics may appear which lead to unsatisfactory performance or even instability, if not addressed correctly. In general, fault tolerant control systems (FTCS) schemes can be classified into two types: passive FTCS and active FTCS. In a passive FTCS scheme, the controller is designed to be robust to a certain class of faults [18, 84]. On the other hand, in an active FTCS scheme the system reacts actively to failures by reconfiguring control actions so that the stability and performance properties of the system are retained [7, 11, 56, 76, 92]. In general, an active FTCS performs three tasks: (a) fault detection, for detecting the occurrence of a fault, (b) fault isolation and identification, for determining the location, type and magnitude of the fault, and (c) fault accommodation, for reconfiguring the control law in order to accommodate the effect of the fault.

The model-based approach for detecting faults was introduced in [32,42]. According to this approach, a mathematical model of the healthy system is constructed and, by comparing the healthy behavior with the system behavior, a diagnostic residual signal is generated that is sensitive to faults [41]. The fault detection algorithm declares a fault occurrence typically by comparing a residual signal to a certain threshold. One of the key challenges of this approach is the detection of faults when other unknown dynamics are present (such as, disturbances, measurement errors, etc.). The fault detection algorithm needs to be able to distinguish between the presence of faults and the presence of other unknown dynamics, in order to avoid false alarms. In the structured residual approach (see, for example, [22]), a residual signal is generated such that it is sensitive to faults and intangible to other unknown dynamics. Various methods have been introduced for generating such residual signals, including parity equations, diagnostic observers and Kalman filtering [25]. However, these ap-

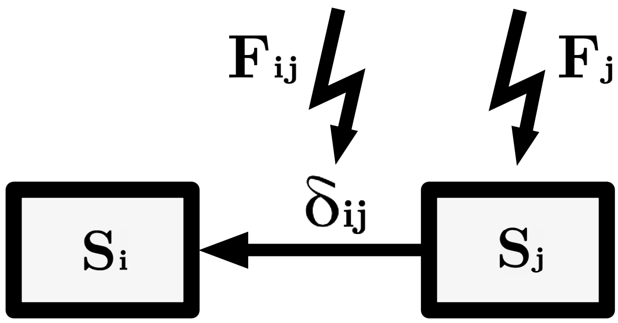


Figure 2.4: The F_{ij} fault affects the S_i subsystem dynamics. The F_j fault may have an impact on the S_i subsystem dynamics through the δ_{ij} interconnection.

proaches involve the risk that slowly developing faults may not be detected, because the enhancement of robustness to other unknown dynamics is associated with an accompanying decrease of the sensitivity of the fault detection algorithm to slowly developing faults. To overcome this difficulty, adaptive fault detection schemes have been proposed, where an adaptive estimator of the healthy system is constructed [16]. By incorporating adaptation, the robustness of the residual with respect to other unknown dynamics is enhanced and as a consequence, it is made possible to detect slowly developing faults.

In the case of nonlinear systems with considerable modeling uncertainty, the construction of a mathematical model of the healthy system can be a difficult task. An alternative approach is based on the use of adaptive approximators for modeling uncertain parts of the system, and for increasing the robustness of the fault detection algorithm to unknown dynamics [61, 63, 79]. Through the use of adaptive approximation models, the controller is able, not only to identify the occurrence of a fault, but it can also provide an approximation of the effect of the fault in the system dynamics. Based on the use of approximation models, the feedback control law is reconfigured to accommodate the effect of the fault.

In the case of interconnected systems, a fault may affect the interconnections, while a fault occurring in any of the subsystems may have an impact on other subsystems (Fig. 2.4). A decentralized fault tolerant control system needs to be able to automatically compensate the effects of faults in the system dynamics, without necessarily exchanging of information between the subsystems [38, 57, 58]. In [20], a distributed fault detection scheme for large-scale systems with overlapping decompositions is presented, where a local fault detector is designed for each subsystem.

Through the use of consensus filters, it is demonstrated that the detectability of faults that are affecting state variables shared among different subsystems can be improved. In [90], the authors consider a distributed fault detection scheme for a class of interconnected nonlinear uncertain systems, where each subsystem communicates its local state estimate to all the other subsystems. In [83], a fault detection design for networked control systems is presented, where each subsystem periodically communicates with a centralized fault detector.

Chapter 3

Decentralized Fault Tolerant Control

3.1 Introduction

In this chapter, we address the problem of the decentralized fault tolerant control for a class of interconnected feedback linearizable systems. We consider interconnected nonlinear subsystems that are exactly feedback linearizable, coupled by unknown nonlinear interconnections in which multiple faults may appear in any of the subsystems as well as in the interconnection effects. The decentralized control law of each subsystem is designed in an adaptive approximation framework and through rigorous stability analysis, uniform ultimate boundedness of the tracking errors to a small region around zero is proved. The presence of residual approximation errors are addressed using a dead-zone modification in the adaptive laws combined with an adaptive bounding method. Outside the coverage region of the approximators, a decentralized safety control scheme is designed to steer back the trajectory by using a sliding mode approach with adaptive bounds. A key contribution of the proposed control scheme for interconnected systems is that it addresses the case of unknown interconnections and multiple faults with significantly large unknown magnitude, without exchanging state information between subsystems.

The chapter is organized as follows. In Section 3.2 we formulate the problem and in Section 3.3 we present the decentralized fault tolerant control design. In Section 3.4, system stability is established through Lyapunov analysis. Section 3.5 presents a decentralized safety control scheme for the case where the trajectories go outside the coverage region. In Section 3.6, a simulation example is presented to illustrate the fault tolerant control methodology. Finally, Section 3.7 contains some concluding remarks.

3.2 Problem Formulation

We consider a system comprised of m interconnected subsystems, which may be subject to multiple faults occurring at unknown times. The i-th subsystem, $i = 1, \ldots, m$, is described by

$$\begin{aligned} \dot{x}_{ij} &= x_{i(j+1)}, \qquad j = 1, \ldots, n_i - 1 \\ \dot{x}_{in_i} &= f_i(x_i) + g_i(x_i)u_i + \Delta_i(x_1, \ldots, x_m) + \sum_{k=1}^{\kappa_i} \beta(t - T_{ik}) h_{ik}(x_1, \ldots, x_m) \end{aligned} \tag{3.1}$$

where $x_i = [x_{i1}, \ldots, x_{in_i}]^\top \in \mathbb{R}^{n_i}$ is the state of the i-th subsystem, $u_i \in \mathbb{R}$ is the input, $y_i = x_{i1} \in \mathbb{R}$ is the output of the i-th subsystem, $f_i : \mathbb{R}^{n_i} \mapsto \mathbb{R}$ and $g_i : \mathbb{R}^{n_i} \mapsto \mathbb{R}$ are known functions representing the nominal local dynamics of the i-th subsystem and $\Delta_i : \mathbb{R}^n \mapsto \mathbb{R}$ (where $n = \sum_{i=1}^m n_i$) represents the unknown interconnection effects between the i-th subsystem and the remaining subsystems. The term $h_{ik} : \mathbb{R}^n \mapsto \mathbb{R}$ denotes the change in the i-th subsystem dynamics due to the k-th fault, while $\beta(t - T_{ik})$ represents the corresponding time profile of the fault that occurs at some unknown time T_{ik}. In this chapter, we consider the possibility of multiple faults in each subsystem, where $T_{i(q-1)} < T_{iq}$ for $q = 2, 3, \ldots, \kappa_i$, with κ_i denoting the number of faults occuring in the i-th subsystem (if there are no faults in the i-th subsystem, κ_i is simply set to $\kappa_i = 0$).

In the fault diagnosis literature, two main categories of faults have been considered: (i) abrupt faults, where the time profile satisfies $\beta(t - T_{ik}) = 0$ for $t < T_{ik}$, and $\beta(t - T_{ik}) = 1$ for $t \geqslant T_{ik}$; (ii) incipient faults where $\beta(t - T_{ik}) = 0$ for $t < T_{ik}$ and $\beta(t - T_{ik})$ increases monotonically from 0 to 1 for $t \geqslant T_{ik}$. In this work, we consider a general class of faults with time profiles that satisfy $0 \leqslant \beta(t - T_{ik}) \leqslant 1$, for $t \geqslant T_{ik}$. As we will see, the rate of growth of the fault as described by $\beta(t - T_{ik})$, does not affect the fault tolerant control design. The fault tolerant control objective is to synthesize decentralized adaptive approximation based control laws u_i such that each y_i follows a smooth reference trajectory y_{d_i} in the presence of the unknown interconnection terms Δ_i and fault functions h_{ik}. It is assumed that each input gain function, g_i, is bounded away from zero in order to guarantee the controllability of the feedback control scheme. Moreover, the desired reference trajectory vector $Y_{d_i} = \left[y_{d_i}, \ldots, y_{d_i}^{n_i}\right]$ for each i-th subsystem is assumed to be available and uniformly bounded.

3.3 Decentralized Fault Tolerant Control Design

Consider the tracking error dynamics, $\tilde{x}_{ij} = x_{ij} - y_{d_i}^{(j-1)}$, of the i-th subsystem, which, based on (3.1), satisfy:

$$\begin{aligned}
\dot{\tilde{x}}_{ij} &= \tilde{x}_{i(j+1)}, \qquad\qquad j = 1, 2, \ldots, n_i - 1 \\
\dot{\tilde{x}}_{in_i} &= f_i(x_i) + g_i(x_i)u_i + \Delta_i(x) + \sum_{k=1}^{\kappa_i} \beta(t - T_{ik})h_{ik}(x) - y_{d_i}^{(n_i)},
\end{aligned}$$

where $x = \left[x_1^\top,\ x_2^\top,\ \ldots, x_n^\top\right]^\top$ is the state vector of the overall system. The tracking error dynamics can be written in matrix state-space form as

$$\dot{\tilde{x}}_i = A\tilde{x}_i + B\left(f_i(x_i) + g_i(x_i)u_i + \Delta_i(x) + \sum_{k=1}^{\kappa_i} \beta(t - T_{ik})h_{ik}(x) - y_{d_i}^{(n_i)}\right), \tag{3.2}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}.$$

Let the decentralized adaptive approximation based control law u_i be given by:

$$u_i = u_i^* + u_{F_i}, \tag{3.3}$$

where u_i^* is the nominal control law that stabilizes the i-th subsystem in the absence of interconnection effects ($\Delta_i = 0$) and faults ($h_{ik} = 0$ for all k), and u_{F_i} is an augmented control component for addressing the interconnection effects Δ_i and the change in dynamics due to faults. The nominal control law u_i^* of the i-th subsystem is defined as

$$u_i^* = \frac{-K_i^\top \tilde{x}_i - f_i(x_i) + y_{d_i}^{(n_i)}}{g_i(x_i)}, \tag{3.4}$$

where the vector $K_i = [k_{i1}, \ldots, k_{in_i}]^\top \in \mathbb{R}^{n_i}$ is chosen such that $A - BK_i^\top$ is a Hurwitz matrix. Since $A - BK_i^\top$ is Hurwitz, for any $Q_i > 0$ there exists $P_i > 0$ satisfying the

Lyapunov equation, $P_i(A - BK_i^\top) + (A - BK_i^\top)^\top P_i = -Q_i$. Define the scalar training error $e_i = B^\top P_i \tilde{x}_i$. We impose the following assumptions on the interconnection terms Δ_i and fault functions h_{ik}.

Assumption 3.1. *The interconnection terms Δ_i and fault functions h_{ik} are bounded by*

$$|\Delta_i(x_1, x_2, \ldots, x_m)| \leqslant \sum_{j=1}^{m} \gamma_{ij}^{\Delta}(|e_j|) \tag{3.5}$$

$$|h_{ik}(x_1, x_2, \ldots, x_m)| \leqslant \sum_{j=1}^{m} \gamma_{ijk}^{h}(|e_j|), \tag{3.6}$$

where $\gamma_{ij}^{\Delta} : \mathbb{R}^+ \mapsto \mathbb{R}^+$ and $\gamma_{ijk}^{h} : \mathbb{R}^+ \mapsto \mathbb{R}^+$ are unknown analytic functions.

From (3.5) and (3.6) we deduce that, since $0 \leqslant \beta(t - T_{ik}) \leqslant 1$ for all $t > 0$,

$$|\Delta_i(x)| + \sum_{k=1}^{\kappa_i} \beta(t - T_{ik}) \, |h_{ik}(x)| \leqslant \sum_{j=1}^{m} \gamma_{ij}^{\Delta}(|e_j|) + \sum_{k=1}^{\kappa_i} \sum_{j=1}^{m} \gamma_{ijk}^{h}(|e_j|),$$

Therefore, there exists an unknown analytic function γ_{ij} such that:

$$|\Delta_i(x)| + \sum_{k=1}^{\kappa_i} \beta(t - T_{ik}) \, |h_{ik}(x)| \leqslant \sum_{j=1}^{m} \gamma_{ij}(|e_j|), \tag{3.7}$$

where $\gamma_{ij} = \gamma_{ij}^{\Delta} + \sum_{k=1}^{\kappa_i} \gamma_{ijk}^{h}$. Assumption 3.1 allows us to consider interconnections effects, Δ_i, and fault functions, h_i, of significantly large unknown magnitude. Moreover, according to (3.7), the fault tolerance problem for the decentralized control scheme is reduced to the handling of the unknown bounding functions γ_{ij}. As we will see later, a surrogate of the bounding functions γ_{ij} (denoted by $s_i(e_i)$) is adaptively approximated for use in the feedback control law, using adaptive approximation models such as sigmoidal neural networks, radial basis functions (RBF), wavelets, etc. [19].

Remark 3.1. A similar bound on the unknown interconnection effects and fault functions, as the one described in Assumption 3.1, has been used in [28] and [71]. Assume that the interconnection effects satisfy,

$$|\Delta_i(x)| \leqslant \sum_{j=1}^{m} \bar{\gamma}_{ij}^{\Delta}(|z_j|), \tag{3.8}$$

where $z_i = B^\top P_i x_i$ and $\bar{\gamma}_{ij}^{\Delta}(|z_j|) = \sum_{k=0}^{p} \bar{a}_{ijk} |z_j|^k$, where $\bar{a}_{ijk} \in \mathbb{R}$, $k = 0, 1, \ldots, p$ are constants and $0 \leqslant p < \infty$. We continue to show that the interconnection effect Δ_i satisfying (3.8) is equivalent to Δ_i satisfying (3.5). From (3.5), since the $\gamma_{ij}^{\Delta}(|e_j|)$ function is analytic, using Taylor's theorem it can be represented as

$$\gamma_{ij}^{\Delta}(|e_j|) = \sum_{k=0}^{\infty} a_{ijk} \, |e_j|^k, \tag{3.9}$$

where $a_{ijk} \in \mathbb{R}$, $k = 1, 2, \ldots, p$ are constants. We define $d_i = B^\top P_i \left[y_{d_i}, \dot{y}_{d_i} \ldots y_{d_i}^{n_i - 1}\right]$, which allows the scalar training error e_i to be expressed as $e_i = z_i - d_i$ and $\bar{\gamma}_{ij}^{\Delta}(|z_j|)$ can be written as

$$\bar{\gamma}_{ij}^{\Delta}(|z_j|) = \sum_{k=0}^{p} \bar{a}_{ijk} |e_j + d_j|^k.$$

Using the binomial theorem (see, for example, [12]), $\bar{\gamma}_{ij}^{\Delta}$ can be written as,

$$\bar{\gamma}_{ij}^{\Delta}(|z_j|) = \sum_{k=0}^{p} \bar{a}_{ijk} \sum_{p=0}^{k} \left| \binom{k}{p} d_j^{k-p} e_j^p \right|. \tag{3.10}$$

In order to establish that, when the interconnection effect Δ_i satisfies (3.8), it necessarily satisfies (3.5), we need to show that there exists some a_{ijk}, such that

$$\gamma_{ij}^{\Delta}(|e_j|) \geqslant \bar{\gamma}_{ij}^{\Delta}(|z_j|). \tag{3.11}$$

Using (3.9) and (3.10), and after some mathematical manipulations we deduce that if we choose a_{ijk} as,

$$a_{ijk} = \begin{cases} \sum_{l=k}^{p} \binom{l-k}{k} \bar{a}_{ijl} \left|\bar{d}_j\right|^{l-k} & k = 0, 1, \ldots, p \\ 0 & k = p+1, p+2, \ldots, \infty, \end{cases}$$

where $\bar{d}_j = \max\limits_t \{d_j(t)\}$, we ensure that (3.11) holds. Therefore the class of interconnected systems with interconnection effects Δ_i that are bounded by (3.5), includes interconnected systems with interconnection effects that are bounded by polynomials of the states, of an arbitrary unknown order $0 \leqslant m < \infty$:

$$|\Delta_i(x)| \leqslant \sum_{j=1}^{m} \sum_{k=0}^{p} \bar{a}_{ijk} |z_j|^k.$$

The same argument holds for the fault functions h_{ik}. □

In this work, for simplicity we consider linearly parameterized approximators, which allow each $s_i(e_i)$ to be represented as

$$s_i(e_i) = \phi_{s_i}(e_i)^\top \theta_{s_i} + \mu_{s_i}(e_i), \tag{3.12}$$

where $\phi_{s_i}(e_i)$ is a set of basis functions, θ_{s_i} is a set of unknown constant weights and $\mu_{s_i}(e_i)$ is the residual approximation error. Typically, the residual approximation error is small within a certain region of coverage, while it may become large outside of this region. It is noted that, since $s_i(e_i)$ is by definition a surrogate of the bounding functions γ_{ij}, the residual approximation error can be made as small as desired in the region of coverage, by choosing a smoother $s_i(e_i)$ that a given approximation model ($\phi_{s_i}(e_i)^\top\theta_{s_i}$) can approximate with better accuracy. However, the presence of, even small, approximation errors within the region of coverage may cause instability issues to the feedback control scheme due to parameter drift. In order to address this issue we use a dead-zone modification in the adaptive laws combined with an adaptive bounding method [62]. The approximation of the unknown functions $s_i(e_i)$ given by (3.12) is assumed to hold for a certain compact set. Within this compact set, there exists an upper bound $\bar{\mu}_{s_i}$ for the residual approximation error $\mu_{s_i}(e_i)$; i.e. $|\mu_{s_i}(e_i)| \leqslant \bar{\mu}_{s_i}$. The unknown $\bar{\mu}_{s_i}$ is estimated on-line by an adaptive estimate which is denoted by $\hat{\mu}_{s_i}$. The augmented control component u_{F_i} of the i-th subsystem is defined as follows:

$$u_{F_i} = \frac{-\phi_{s_i}(e_i)^\top\hat{\theta}_{s_i} - u_{c_i}}{g_i(x_i)} \tag{3.13}$$

$$u_{c_i} = \begin{cases} \hat{\mu}_{s_i} sgn(e_i) & \text{if} \quad \tilde{x}_i^\top P_i \tilde{x}_i > \bar{\lambda}_{P_i}\epsilon_i^2 \\ 0 & \text{if} \quad \tilde{x}_i^\top P_i \tilde{x}_i \leqslant \bar{\lambda}_{P_i}\epsilon_i^2, \end{cases} \tag{3.14}$$

where $\epsilon_i > 0$ is a design constant and $\bar{\lambda}_{P_i}$ is the maximum eigenvalue of P_i, respectively. The parameter estimates of the adaptive approximator, $\hat{\theta}_{s_i}$ and the adaptive bounding parameter $\hat{\mu}_{s_i}$ are updated according to the following adaptive laws

$$\dot{\hat{\theta}}_{s_i} = \Gamma_{s_i}\phi_{s_i}(e_i)q_i(e_i, \tilde{x}_i, \epsilon_i) \tag{3.15}$$

$$\dot{\hat{\mu}}_{s_i} = \gamma_{s_i} \left| q_i(e_i, \tilde{x}_i, \epsilon_i) \right|, \tag{3.16}$$

where Γ_{s_i} is a positive definite matrix and γ_{s_i} is a positive constant representing the adaptation gains of the parameter estimates and $q_i\left(e_i, \tilde{x}_i, \epsilon_i\right)$ is a dead-zone, defined as

$$q_i(e_i, \tilde{x}_i, \epsilon_i) = \begin{cases} 0 & \tilde{x}_i^\top P_i \tilde{x}_i \leqslant \bar{\lambda}_{P_i}\epsilon_i^2 \\ e_i & \tilde{x}_i^\top P_i \tilde{x}_i > \bar{\lambda}_{P_i}\epsilon_i^2 \end{cases} \tag{3.17}$$

The overall decentralized control law of the i-th subsystem is given by

$$u_i = \frac{-K_i^\top\tilde{x}_i - f_i(x_i) - \phi_{s_i}(e_i)^\top\hat{\theta}_{s_i} - u_{c_i} + y_{d_i}^{(n_i)}}{g_i(x_i)}. \tag{3.18}$$

Remark 3.2. Assumption 3.1 can be satisfied for any continuous interconnection function and fault function. However in the case of interconnections with significantly large magnitude, as e_j goes to zero, the Δ_i and h_{ik} functions are bounded by large constant terms. In this case, the local controller may generate large control signals in order to compensate for the unknown interconnections and fault functions, possibly leading to high-gain feedback. In practical applications, large feedback gains may lead the control signal to saturation, which can cause instability issues to the feedback control scheme. The necessity for imposing Assumption 3.1 on the interconnections and fault functions can be explained by the fact that the states of the remote subsystems are completely unknown. In the case of strong interconnections, and in order to enhance the applicability of non-centralized control architectures, it is necessary that the subsystems share state information online. In the next chapters we consider distributed control schemes and show that, by introducing the ability for the subsystems to communicate, it becomes possible to considerably reduce the control effort for addressing the unknown interconnections and fault functions. □

Remark 3.3. The decentralized control law of the i-th subsystem, given by (3.18), utilizes adaptive approximation models for establishing fault tolerance to all faults satisfying (3.6). Typically, in adaptive approximation based control law, the unknown functions are compensated for, through the use of approximation models that adaptively approximate these unknown functions. However, in this work, in order to preserve the decentralizability of the feedback control scheme, the unknown interconnection effects and fault functions are compensated for, by adaptively approximating a surrogate of the bounding functions γ_{ij}. □

Remark 3.4. The introduction of the adaptive bounding term $\hat{\mu}_{s_i}$ allows us to use arbitrary small ϵ_i, that characterizes the dead-zone width. Moreover, it avoids the need for a known upper bound on the residual approximation error ($\bar{\mu}_{s_i}$) to be available. Although the non-decreasing growth of $\hat{\mu}_{s_i}$ could cause large feedback signals, as we will see later, it can only increase over a finite interval due to the presence of the dead-zone. □

In the following section, we consider the stability properties of the above decentralized fault tolerant control scheme.

3.4 Stability Analysis

Although the interconnection effects Δ_i and the fault functions h_{ik} are not only functions of local states x_i but also functions of states of remote subsystems $x_j,\ j \neq i$, as we prove next, each subsystem's output is able to asymptotically track the reference signal within a small error, without exchanging state information between subsystems.

Lemma 3.1. *The closed-loop system described by the interconnected system (3.1), the decentralized control law (3.18) and the adaptation laws (3.15), (3.16), guarantee that* $\|\tilde{x}_i(t)\|$ *is uniformly ultimately bounded by* ϵ_i*; i.e., the total time such that* $\tilde{x}_i^\top P_i \tilde{x}_i > \bar{\lambda}_{P_i}\epsilon_i^2$ *is finite.*

Proof. Let the Lyapunov function of the i-th subsystem be given by $V_i = V_{i1} + V_{i2}$ where

$$V_{i1} = \frac{1}{2}\tilde{x}_i^\top P_i \tilde{x}_i\ , \qquad V_{i2} = \frac{1}{2}\tilde{\theta}_{s_i}^\top \Gamma_{s_i}^{-1}\tilde{\theta}_{s_i} + \frac{1}{2\gamma_{s_i}}(\hat{\mu}_{s_i} - \bar{\mu}_{s_i})^2,$$

where $\tilde{\theta}_{s_i} = \hat{\theta}_{s_i} - \theta_{s_i}$ is the parameter estimation error vector. By substituting the control law (3.18) into the tracking error dynamics (3.2), we obtain the following expression for the closed-loop tracking error dynamics

$$\dot{\tilde{x}}_i = \left(A - BK_i^\top\right)\tilde{x}_i + B\left(\Delta_i(x) + \sum_{k=1}^{\kappa_i}\beta(t-T_{ik})h_{ik}(x) - \phi_{s_i}(e_i)^\top\hat{\theta}_{s_i} - u_{c_i}\right).$$

The time derivative of V_{i1} satisfies

$$\begin{aligned}\dot{V}_{i1} =& -\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i + e_i\left(\Delta_i(x) + \sum_{k=1}^{\kappa_i}\beta(t-T_{ik})h_{ik}(x) - \phi_{s_i}(e_i)^\top\hat{\theta}_{s_i} - u_{c_i}\right)\\ \leqslant& -\ \frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i - e_i\phi_{s_i}(e_i)^\top\hat{\theta}_{s_i} - e_i u_{c_i}\\ &+ |e_i|\left(|\Delta_i(x)| + \sum_{k=1}^{\kappa_i}\beta(t-T_{ik})\,|h_{ik}(x)|\right).\end{aligned}$$

Using (3.7),

$$|e_i|\left(|\Delta_i(x)| + \sum_{k=1}^{\kappa_i}\beta(t-T_{ik})\,|h_{ik}(x)|\right) \leqslant |e_i|\sum_{j=1}^{m}\gamma_{ij}(|e_j|).$$

Since γ_{ij} are analytic functions, their derivative exists to any degree required. Therefore, using Taylor's Theorem (see, for example, [12]), there exist smooth functions ξ_{ij} such that $\gamma_{ij}(|e_j|) = \gamma_{ij0} + |e_j|\xi_{ij}(|e_j|)$, where $\gamma_{ij0} = \gamma_{ij}(0)$ is a constant. Therefore, defining $\gamma_{i0} = \sum_{j=1}^{m} \gamma_{ij0}$ and using the inequality $2\alpha\beta \leqslant \alpha^2 + \beta^2$ for $\alpha, \beta \in \mathbb{R}$, we obtain

$$\begin{aligned}\dot{V}_{i1} &\leqslant -\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i - e_i \phi_{s_i}(e_i)^\top \hat{\theta}_{s_i} - e_i u_{c_i} + \gamma_{i0}|e_i| + |e_i| \sum_{j=1}^{m} |e_j| \xi_{ij}(|e_j|) \\ &\leqslant -\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i - e_i \phi_{s_i}(e_i)^\top \hat{\theta}_{s_i} - e_i u_{c_i} + \gamma_{i0}|e_i| + \frac{1}{2} n e_i^2 + \frac{1}{2} \sum_{j=1}^{m} e_j^2 \xi_{ij}^2(|e_j|).\end{aligned}$$

Hence, after some re-ordering of terms,

$$\begin{aligned}\sum_{i=1}^{m} \dot{V}_{i1} \leqslant \sum_{i=1}^{m} \Bigg[&-\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i - \phi_{s_i}(e_i)^\top \hat{\theta}_{s_i} e_i - e_i u_{c_i} \\ &+ \gamma_{i0}|e_i| + \frac{1}{2} m e_i^2 + \frac{1}{2} e_i^2 \sum_{j=1}^{m} \xi_{ji}^2(|e_i|) \Bigg].\end{aligned}$$

Let $s_i(e_i) = \gamma_{i0}\mathrm{sgn}(e_i) + \frac{1}{2} m e_i + \frac{1}{2} e_i \sum_{j=1}^{m} \xi_{ji}^2(|e_i|)$. Using (3.12) we have

$$\sum_{i=1}^{m} \dot{V}_{i1} \leqslant \sum_{i=1}^{m} \left[-\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i - e_i \phi_{s_i}(e_i)^\top \tilde{\theta}_{s_i} + e_i \mu_{s_i} - e_i u_{c_i} \right].$$

Let the Lyapunov function of the overall system br given by $V = \sum_{i=1}^{m} V_{i1} + V_{i2}$. The time derivative of V satisfies,

$$\begin{aligned}\dot{V} \leqslant \sum_{i=1}^{m} \Bigg[&-\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i - e_i \phi_{s_i}(e_i)^\top \tilde{\theta}_{s_i} + e_i \mu_{s_i} - e_i u_{c_i} \\ &+ \tilde{\theta}_{s_i}^\top \Gamma_{s_i}^{-1} \dot{\hat{\theta}}_{s_i} + \frac{1}{\gamma_{s_i}} \left(\hat{\mu}_{s_i} - \bar{\mu}_{s_i} \right) \dot{\hat{\mu}}_{s_i} \Bigg].\end{aligned}$$

Subtituting u_{c_i} from (3.14), for $\tilde{x}_i^\top P_i \tilde{x}_i > \bar{\lambda}_{P_i} \epsilon_i^2$ we obtain,

$$\begin{aligned}\dot{V} \leqslant \sum_{i=1}^{m} \Bigg[&-\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i + \tilde{\theta}_{s_i}^\top \Gamma_{s_i}^{-1} \left(\dot{\hat{\theta}}_{s_i} - \Gamma_{s_i} \phi_{s_i}(e_i) e_i \right) \\ &+ \hat{\mu}_{s_i} \left(\frac{1}{\gamma_{s_i}} \dot{\hat{\mu}}_{s_i} - |e_i| \right) + e_i \mu_{s_i} - \frac{1}{\gamma_{s_i}} \bar{\mu}_{s_i} \dot{\hat{\mu}}_{s_i} \Bigg].\end{aligned}$$

Substituting the adaptive laws (3.15), (3.16) for $\tilde{x}_i^\top P_i \tilde{x}_i > \bar{\lambda}_{P_i} \epsilon_i^2$, the Lyapunov function satisfies

$$\dot{V} \leqslant \sum_{i=1}^{m} \left[-\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i + e_i \mu_{s_i} - |e_i| \bar{\mu}_{s_i} \right] \leqslant -\frac{1}{2} \sum_{i=1}^{m} \tilde{x}_i^\top Q_i \tilde{x}_i$$

which shows that $\tilde{x}_i(t)$ will go into the set $\mathcal{W}_i = \left\{ \tilde{x}_i \in \mathbb{R}^{n_i} \middle| \, \tilde{x}_i^\top P_i \tilde{x}_i \leqslant \bar{\lambda}_{P_i} \epsilon_i^2 \right\}$. Moreover, since adaptation stops inside the dead-zone, $\tilde{x}_i, \; \hat{\theta}_{s_i}, \; \hat{\mu}_{s_i} \in \mathcal{L}_\infty$, for all $t > 0$. However, the fact that $\hat{\mu}_{s_i}$ is non-decreasing, (3.16), shows that each subsystem enter the dead-zone in finite time, i.e., there exists a $t_0 < \infty$ such that $\tilde{x}_i \in \mathcal{W}_i$, for all $t > t_0$. This result not only shows the uniformly ultimately boundedness of $\|\tilde{x}_i\|$ by ϵ_i, but also that parameter drift in the presence of residual approximation errors is avoided. □

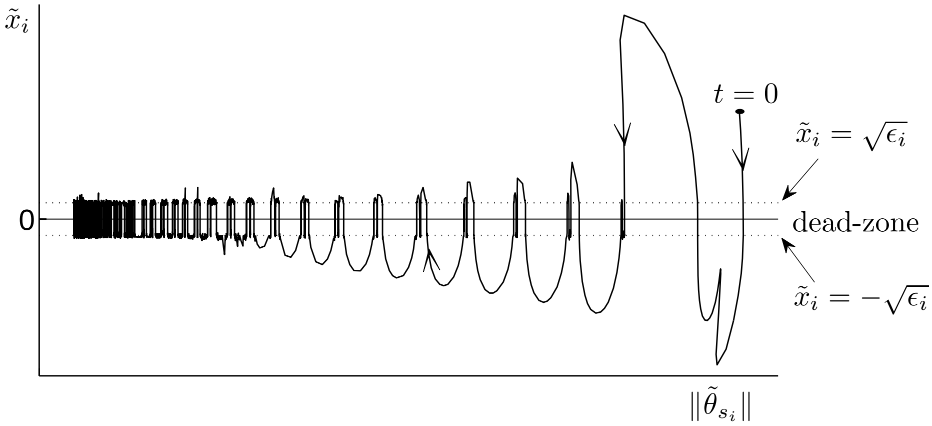


Figure 3.1: Illustration of a possible error trajectory of a scalar system

Fig. 3.1 shows the type of error trajectory, $\left\{\tilde{x}_i,\ \|\tilde{\theta}_{s_i}\|\right\}$, that could occur for a scalar system, where for illustration purposes the μ_{s_i} coordinate is ommited.

3.5 Decentralized Safety Control Scheme

In the previous analysis, we assumed that the states of each subsystem are restricted within a certain compact coverage region. Within this region, the residual approximation error μ_{s_i} can be arbitrarily reduced by enhancing the approximation capabilities of the $\hat{s}_i$ approximator. However, outside the coverage region, the size of μ_{s_i} is typically significantly large, such that the states of the subsystems may become unbounded. Even in the case that the initial state conditions are inside the coverage region, due to large initial parameter estimation errors, the states may still leave the coverage region. Therefore, in order to address this problem, in this section we consider the design of a decentralized safety control scheme based on sliding mode control with adaptive bounds. The state space x_i of each subsystem i is divided into the following subset,

$$A_{D_i} = \left\{x_i \mid \|x_i - x_{0_i}\|_{p,\beta_i} \leqslant 1\right\} \ ,$$

where x_{0_i} is a fixed vector in the state space of subsystem i and $\|x\|_{p,\beta}$ is the weighted p-norm, $\|x\|_{p,\beta} = \left[\sum_{j=1}^{k} \left(\frac{|x_j|}{\beta_j}\right)^p\right]^{\frac{1}{p}}$. Through the use of the weighted p-norm, for different values of p and β, it is possible to specify subsets with arbitrary dimensions in different coordinates (e.g., ellipse, rectangle) and not necessarily of equal dimensions in different coordinates. The sliding manifold of the i-th subsystem is defined as $e_i = B^\top P_i \tilde{x}_i = 0$. We impose the following assumption.

Assumption 3.2. *In the region* $\mathbb{R}^n - A_{D_1} \times A_{D_2} \times \ldots \times A_{D_m} \equiv \mathbb{R}^n - A_D$,

$$|\Delta_i(x)| + \sum_{k=1}^{\kappa_i} \beta(t - T_{ik}) |h_{ik}(x)| \leqslant \sum_{j=1}^{m} w_{s_{ij}} |e_j| M_{s_{ij}}(x_j) + w_{s_{i0}}, \tag{3.19}$$

where $M_{s_{ij}} : \mathbb{R}^{n_j} \mapsto \mathbb{R}$ *are known functions, and* $w_{s_{ij}}$, $w_{s_{i0}}$ *are unknown positive parameters.*

It is noted that, in general, we do not need to know the functions $M_{s_{ij}}(x_j)$ since theoretically, they can be set to one. However, it is best to incorporate as much prior knowledge as possible into the design to avoid unnecessary large feedback gains. Define the vector $w_{s_i} = [w_{s_{1i}}, \ldots, w_{s_{mi}}]^\top$ and let $\hat{w}_{s_i}$ and $\hat{w}_{s_{i0}}$ be the estimates of w_{s_i} and $w_{s_{i0}}$ respectively. The corresponding parameter estimation errors are defined as $\tilde{w}_{s_i} = \hat{w}_{s_i} - w_{s_i}$ and $\tilde{w}_{s_{i0}} = \hat{w}_{s_{i0}} - w_{s_{i0}}$. The decentralized sliding mode control law is given by,

$$u_{s_i} = \frac{-K_i^\top x_i - f_i(x_i) + \text{sgn}(e_i)\Pi_i}{g_i(x_i)} \tag{3.20}$$

$$\Pi_i = -\frac{m}{2}|e_i| - \frac{|e_i|}{2} \sum_{j=1}^{m} \hat{w}_{s_{ji}}^2 M_{s_{ji}}^2(x_i) - \hat{w}_{s_{i0}}. \tag{3.21}$$

The parameter estimates $\hat{w}_{s_i}$ and $\hat{w}_{s_{i0}}$ are updated according to the following adaptive laws,

$$\dot{\hat{z}}_{s_i} = \Gamma_{w_i} L_{s_i}(x_i) e_i^2 \tag{3.22}$$

$$\dot{\hat{w}}_{s_{i0}} = \gamma_{w_{i0}} |e_i|, \tag{3.23}$$

where Γ_{w_i} is positive definite matrix and $\gamma_{w_{i0}}$ is a positive constant corresponding to the adaptive rates of the parameter estimates, $\hat{z}_{s_i} = \left[\hat{w}_{s_{1i}}^2 \ \hat{w}_{s_{2i}}^2 \ \ldots \ \hat{w}_{s_{mi}}^2\right]^\top$ and $L_{s_i} = \left[M_{s_{1i}}^2(x_i) \ M_{s_{2i}}^2(x_i) \ \ldots \ M_{s_{mi}}^2(x_i)\right]^\top$.

Lemma 3.2. *The decentralized sliding mode control law (3.20) and the adaptation laws (3.22)-(3.23) guarantee that each time* x_i *leaves* A_{D_i} *it returns to it in finite time.*

Proof. Let the Lyapunov function of the i-th subsystem be given by $V_i = V_{i1} + V_{i2}$, where

$$V_{i1} = \frac{1}{2}\tilde{x}_i^\top P_i \tilde{x}_i, \qquad V_{i2} = \frac{1}{2}\tilde{z}_{s_i}^\top \Gamma_{w_i}^{-1} \tilde{z}_{s_i} + \frac{1}{2\gamma_{w_{i0}}} \tilde{w}_{s_{i0}}^2,$$

where $\tilde{z}_{s_i} = \left[\tilde{w}_{s_{1i}}^2 \ \tilde{w}_{s_{2i}}^2 \ \dots \ \tilde{w}_{s_{mi}}^2\right]^\top$. By substituting the control law u_{s_i}, (3.20), into $\dot{\tilde{x}}_i$ we obtain that the time derivative of V_{i1} satisfies

$$\begin{aligned}\dot{V}_{i1} \leqslant{} & \tilde{x}_i^\top P_i\left[(A - BK_i^\top)\tilde{x}_i + B\left(\operatorname{sgn}(e_i)\Pi_i + \Delta_i(x) + \sum_{k=1}^{\kappa_i}\beta(t-T_k)h_{ik}(x)\right)\right] \\ \leqslant{} & -\frac{1}{2}\tilde{x}_i^\top Q_i\tilde{x}_i + |e_i|\left(\Pi_i + |\Delta_i(x)| + \sum_{k=1}^{\kappa_i}\beta(t-T_{ik})\,|h_{ik}(x)|\right).\end{aligned}$$

Substituting Π_i from (3.21) and using the bound in equation (3.19), we obtain

$$\begin{aligned}\dot{V}_{i1} \leqslant{} & -\frac{1}{2}\tilde{x}_i^\top Q_i\tilde{x}_i + |e_i|\left[-\frac{m}{2}|e_i| - \frac{|e_i|}{2}\sum_{j=1}^{m}\hat{w}_{s_{ji}}^2 M_{s_{ji}}^2(x_i) - \tilde{w}_{s_{i0}}\right. \\ & \left. + \sum_{j=1}^{m} w_{s_{ij}}\,|e_j|\,M_{s_{ij}}(x_j)\right].\end{aligned}$$

Therefore, using the inequality $2\alpha\beta \leqslant \alpha^2 + \beta^2$ for $\alpha, \beta \in \mathbb{R}$,

$$\dot{V}_{i1} \leqslant -\frac{1}{2}\tilde{x}_i^\top Q_i\tilde{x}_i + \frac{1}{2}\sum_{j=1}^{m} e_j^2 w_{s_{ij}}^2 M_{s_{ij}}^2(x_j) - \frac{1}{2}e_i^2\sum_{j=1}^{m}\hat{w}_{s_{ji}}^2 M_{s_{ji}}^2(x_i) - |e_i|\,\tilde{w}_{s_{i0}}.$$

After some reordering of terms,

$$\sum_{i=1}^{m}\dot{V}_{i1} \leqslant \sum_{i=1}^{m}\left[-\frac{1}{2}\tilde{x}_i^\top Q_i\tilde{x}_i - \frac{1}{2}e_i^2\sum_{j=1}^{m}\tilde{w}_{s_{ji}}^2 M_{s_{ji}}^2(x_i) - |e_i|\,\tilde{w}_{s_{i0}}\right].$$

The time derivative of the Lyapunov function of the overall system satisfies

$$\begin{aligned}\dot{V} \leqslant \sum_{i=1}^{m} & \left[-\frac{1}{2}\tilde{x}_i^\top Q_i\tilde{x}_i - \frac{1}{2}e_i^2\sum_{j=1}^{m}\tilde{w}_{s_{ji}}^2 M_{s_{ji}}^2(x_i) - |e_i|\,\tilde{w}_{s_{i0}}\right. \\ & \left. + \tilde{z}_{s_i}^\top\Gamma_{w_i}^{-1}\dot{\tilde{z}}_{s_i} + \frac{1}{\gamma_{w_{i0}}}\tilde{w}_{s_{i0}}\dot{\tilde{w}}_{s_{i0}}\right].\end{aligned}$$

By grouping terms we obtain

$$\dot{V} \leqslant \sum_{i=1}^{m}\left[-\frac{1}{2}\tilde{x}_i^\top Q_i\tilde{x}_i + \tilde{z}_{s_i}^\top\Gamma_{w_i}^{-1}\left(\dot{\tilde{z}}_{s_i} - \Gamma_{w_i}L_{s_i}(x_i)e_i^2\right) + \gamma_{w_{i0}}^{-1}\tilde{w}_{s_{i0}}\left(\dot{\tilde{w}}_{s_{i0}} - \gamma_{w_{i0}}|e_i|\right)\right].$$

By substituting the adaptive laws (3.22)-(3.23), the Lyapunov function derivative satisfies $\dot{V} \leqslant -\frac{1}{2}\sum_{i=1}^{m}\tilde{x}_i^\top Q_i\tilde{x}_i$, which shows that the state vector x_i of each subsystem enter A_{D_i} in finite time. □

3.6 Simulation Example

To illustrate the design methodology for the decentralized adaptive approximation based fault tolerant control, consider the following interconnected uncertain system:

$$
\begin{aligned}
\Sigma_1 \ : \ \ & \dot{x}_{11} = x_{12}, \\
& \dot{x}_{12} = x_{11}^4 + \Delta_1(x) + \left(1 + x_{11}^2 x_{12}^2\right) u_1 + \beta(t - T_1) h_1(x) \\
\Sigma_2 \ : \ \ & \dot{x}_{21} = x_{22}, \\
& \dot{x}_{22} = x_{22}^3 + \Delta_2(x) + \left(1 + |x_{21} x_{22}|\right) u_2 + \beta(t - T_2) h_2(x) \\
\Sigma_3 \ : \ \ & \dot{x}_{31} = x_{32}, \\
& \dot{x}_{32} = x_{32}^2 + \Delta_3(x) + \left(2 + \sin(x_{31})\right) u_3 + \beta(t - T_3) h_3(x)
\end{aligned}
$$

where $x = \left[x_1^\top, x_2^\top, x_3^\top\right]^\top$, $y_i = x_{i1}$ is the output of Σ_i subsystem, $\Delta_1(x_1, x_2, x_3) = x_{21} - x_{22} - x_{31} + x_{32}$, $\Delta_2(x_1, x_2, x_3) = x_{11} + \frac{1}{2}x_{12} + x_{31} + x_{32}$ and $\Delta_3(x_1, x_2, x_3) = x_{21} + \frac{1}{2}x_{22}$. The matrix P_i satisfying the Lyapunov equation, for $Q = I_{2\times 2}$, is given by,

$$
P_i = \begin{bmatrix} 1.5 & 0.5 \\ 0.5 & 1 \end{bmatrix} \qquad i = 1, 2, 3,
$$

where $K_1 = K_2 = K_3 = \begin{bmatrix} 1 & 1 \end{bmatrix}^\top$. The desired trajectory vector $Y_{d_i} = [y_{d_i}, \dot{y}_{d_i}]^\top$ and the signal $\ddot{y}_{d_i}$ are generated using a third order filter with a bandwidth of 5 (rad/sec) and unity gain below this frequency. The filter input is chosen as a square wave of zero mean, 1.5 amplitude and a frequency of 0.4 Hz (the same for all three subsystems). The coverage regions A_{D_i} are chosen as $A_{D_i} = \max\left\{\frac{|x_{i1}|}{3}, \ \frac{|x_{i2}|}{9}\right\} \leqslant 1, \quad i = 1, 2, 3$. Within this region, a lattice of equally spaced radial basis functions are designed for the approximation of the unknown interconnection effects and the faults. We consider the case in which abrupt faults occur in Σ_1 at $T_1 = 4$ *sec* and simultaneously in Σ_2 and Σ_3 at $T_2 = T_3 = 10$ *sec*. For simulation purposes, the unknown fault functions h_1, h_2 and h_3 are chosen as, $h_1 = (x_{31} + 0.5x_{32})^3 + x_{12}x_{22} + |x_{11}|$, $h_2 = x_{31}^3 + x_{32}^2$ and $h_3 = x_{21}^2 x_{32} + 5\,|x_{22}|\cos(0.01x_{31})$.

In Fig. 3.2 we plot the output tracking error of each subsystem, $y_i - y_{d_i}$, $i = 1, 2, 3$, indicating the time occurrence of the faults. As illustrated by the plot, through the use of adaptive approximation of the interconnection effects and the faults, the subsystems are able to follow the corresponding reference trajectories. In Fig. 3.3 we plot the parameter estimates, $\hat{\theta}_{s_1}$, $\hat{\theta}_{s_2}$ and $\hat{\theta}_{s_3}$ of the approximation of the $s_1(e_1)$, $s_2(e_2)$ and $s_3(e_3)$ functions, respectively. As shown, a change in the dynamics of

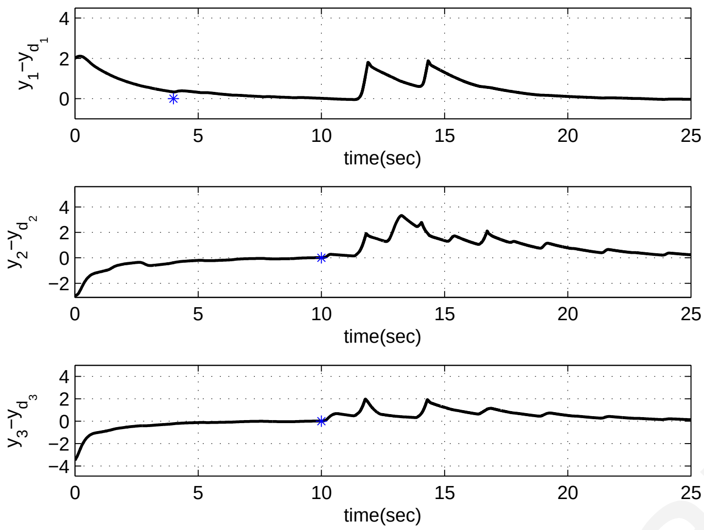


Figure 3.2: Time evolution of the output tracking error.

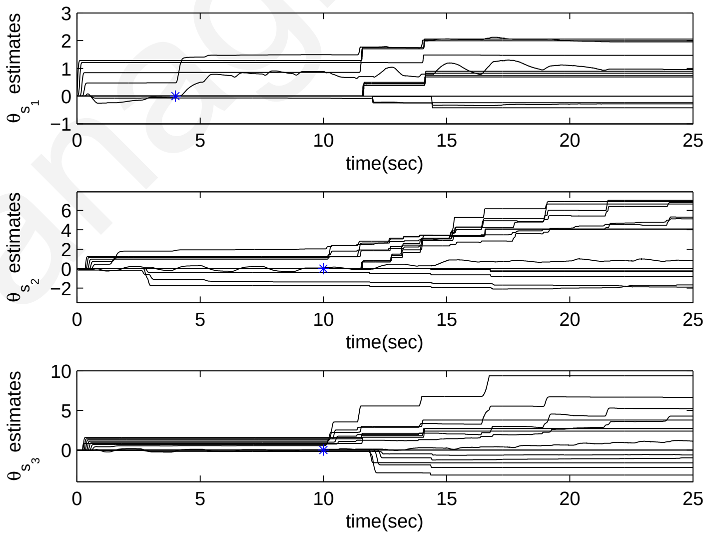


Figure 3.3: Time evolution of the adaptive parameter estimates.

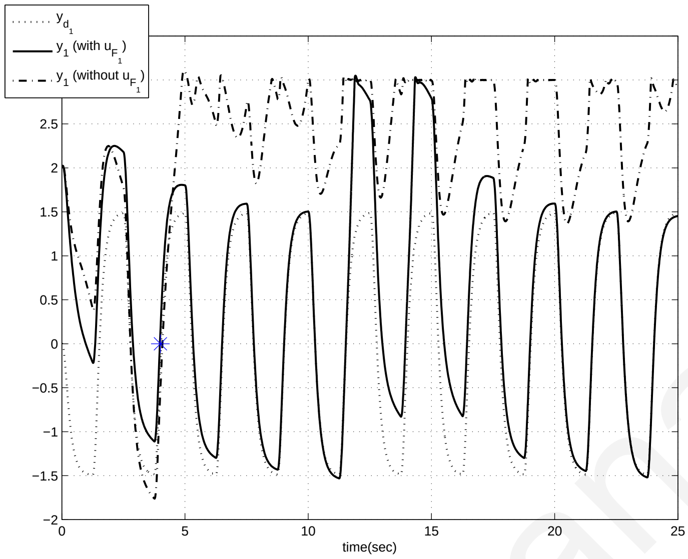


Figure 3.4: Tracking performance of Σ_1, with and without adaptive approximation.

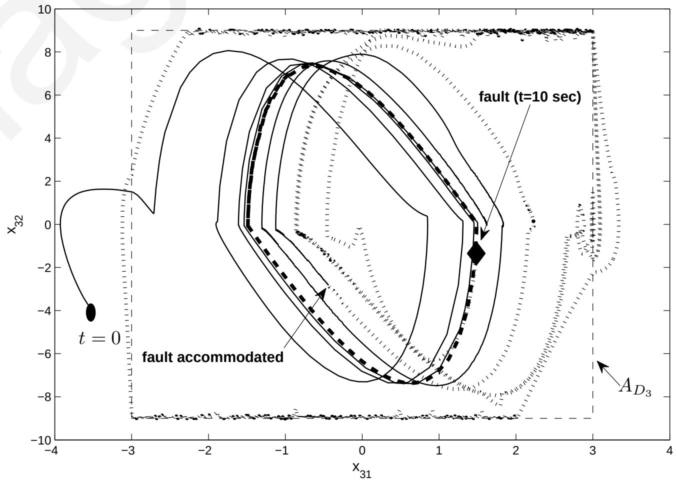


Figure 3.5: Phase plane plot of x_{31} versus x_{32}.

each subsystem due to the faults, causes the parameter estimates to adapt so as to accommodate the faults. This is also the case when the change in dynamics is due to a remote fault (in a different subsystem), as can be seen in the plot of $\hat{\theta}_{s_1}$ after approximately $t = 12$ *sec*. Although the h_2 and h_3 faults, in Σ_2 and Σ_3 respectively, do not directly affect the Σ_1 dynamics, the effects of the faults are propagated to Σ_1 through the Δ_1 interconnection. More specifically, the h_2 and h_3 faults cause a large change in the Δ_1 interconnection and as a result the scalar error e_1 is increased (see Fig. 3.2). However, the parameter estimates, $\hat{\theta}_{s_1}$, are self-adapted to correct this effect. Fig. 3.4 demonstrates the effectiveness of the use of adaptive approximation of the interconnections effects and faults. More specifically, it illustrates the tracking performance of Σ_1 in the case where adaptive approximation is used, together with the case where adaptive approximation is switched off (i.e., $u_{F_1}(t) = 0$ for all t). Although the presence of the safety control scheme ensures the boundedness of the tracking error, as shown by the plot, the absence of adaptive approximation results in a severe degrade of the tracking performance, especially after the occurrence of the h_1 fault at $t = 4$ *sec*. Fig. 3.5 shows the phase plane plot of the states of Σ_3. The desired trajectory is shown as a thick dashed line inside the coverage region. For illustration purposes, the trajectory between the fault occurrence time $t = T_3 = 10$ *sec* of the h_3 fault and the time at which the fault is accommodated, is shown as a dotted line. As can be seen, the fault causes the trajectory to leave the coverage region, however the decentralized sliding mode control with adaptive bounds is able to steer the states of the subsystem back to the coverage region.

3.7 Conclusion

In this chapter we have presented a decentralized adaptive approximation design for the fault tolerant control of a class of nonlinear uncertain interconnected systems. We have considered multiple faults that occur not only in the local subsystem dynamics but also in the interconnections between the subsystems. Using Lyapunov analysis we have derived stable adaptive laws for compensating the effects of the unknown interconnections and the unknown fault functions. It has been shown that a class of interconnections and fault functions can be adaptively approximated locally, without the need of state information exchange between subsystems. The introduction of a dead-zone modification in the adaptive laws combined with adaptive bounding parameters, addresses stability and robustness issues associated with the presence of residual approximation errors. By combining a dead-zone modification in the adaptive laws with an adaptive bounding method, it becomes possible to relax the assumption of a known upper bound on the residual approximation errors. Moreover, the presence of the dead-zone prevents the adaptive bounding term from drifting to infinity. Through the development of a safety scheme outside the coverage region, we have addressed stability problems associated with the case where the trajectory leaves the coverage region. It is important to note that in cases where the interconnections functions have very large magnitude, then this may lead to saturation of the control signal. In such cases, one may consider distributed control approaches, such as the ones presented in later chapters.

Chapter 4

Distributed Fault Detection and Accommodation

4.1 Introduction

In this chapter we present a distributed fault detection and accommodation scheme for a class of feedback linearizable uncertain interconnected systems. We consider faults that occur in the subsystems local dynamics, as well as in the interconnections. The subsystems are allowed to exchange state information according to a self-triggering tracking-error based communication algorithm, where state information is shared with other subsystems only if the tracking error exceeds a certain bounding threshold, which is a design variable. Intuitively, the distributed fault detection and accommodation design is based on utilizing the *a priori* available desired reference trajectory if the tracking error is within a certain threshold, while using the transmitted state information if the tracking error exceeds the threshold. The distributed fault detection scheme is based on a set of distributed nonlinear estimators corresponding to each subsystem. Each estimator utilizes local, as well as, remote communicated state information. A nominal control law is designed that ensures asymptotic stability of the tracking errors in the absence of faults. After a fault is detected in any subsystem, a fault accommodation algorithm based on the adaptive approximation approach [19] is activated for compensating the effect of the fault. It is shown that by approximating the upper bound of the fault function, instead of the fault function itself, robustness to residual approximation errors is ensured. Through rigorous stability analysis, asymptotic stability of all tracking errors, in the presence of faults, is established.

This chapter is organized as follows. In Section 4.2 we formulate the problem and in Section 4.3 we present the design of the distributed fault detection scheme. Section 4.4 presents a distributed nominal control design, whose stability properties are established through the use of Lyapunov analysis. In Section 4.5, a distributed fault accommodation scheme is presented and analyzed. In Section 4.6, a simulation example is presented to illustrate the effectiveness of the proposed distributed fault detection and accommodation scheme. Finally, Section 4.7 contains some concluding remarks.

4.2 Problem Formulation

We consider a system comprised of m interconnected subsystems, which may be subject to faults occurring at unknown times T_i. The i-th subsystem, $i = 1, \ldots, m$, is described by

$$\dot{x}_{ij} = x_{i(j+1)} \qquad j = 1, \ldots, n_i - 1 \tag{4.1}$$

$$\dot{x}_{in_i} = f_i(x_i) + g_i(x_i)u_i + \sum_{j=1}^{m} \delta_{ij}(x_j) + \beta(t - T_i)h_i(x) \tag{4.2}$$

where $x_i = [x_{i1}, x_{i2}, \ldots, x_{in_i}]^\top \in \mathbb{R}^{n_i}$ is the state of the i-th subsystem, $x = \left[x_1^\top, \ldots, x_m^\top\right]^\top \in \mathbb{R}^n$ (where $n = \sum_{i=1}^m n_i$) is the state of the overall system; $u_i \in \mathbb{R}$ is the control input of the i-th subsystem; $f_i : \mathbb{R}^{n_i} \mapsto \mathbb{R}$ and $g_i : \mathbb{R}^{n_i} \mapsto \mathbb{R}$ are partially known functions representing the local dynamics of the i-th subsystem; and $\delta_{ij} : \mathbb{R}^{n_j} \mapsto \mathbb{R}$ is an unknown interconnection function representing the effect of the j-th subsystem onto the i-th subsystem dynamics (δ_{ij} is subject to some constraints defined later on). For notational convenience, we define $\delta_{ii} = 0$, for $i = 1, \ldots, m$. The term $h_i : \mathbb{R}^n \mapsto \mathbb{R}$ denotes the unknown change in the i-th subsystem dynamics due to a fault, while $\beta(t - T_i) : \mathbb{R}^+ \mapsto \mathbb{R}$ represents the corresponding time profile of the fault that occurs at some unknown time T_i.

In this chapter we consider abrupt faults, where the time profile satisfies $\beta(t - T_i) = 0$ for $t < T_i$, and $\beta(t - T_i) = 1$ for $t \geqslant T_i$, and incipient faults, where $\beta(t - T_i) = 0$ for $t < T_i$ and $\beta(t - T_i)$ increases monotonically from 0 to 1 for $t \geqslant T_i$. Incipient faults are typically slowly developing faults that may be difficult to detect. The i-th subsystem described by (4.1), (4.2) can be written in matrix form as

$$\dot{x}_i = Ax_i + B\Big[f_i(x_i) + g_i(x_i)u_i + \sum_{j=1}^{m} \delta_{ij}(x_j) + \beta(t - T_i)h_i(x)\Big] \tag{4.3}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}.$$

It is assumed that a local nominal (known) model of the i-th subsystem is described by $\dot{x}_{N_i} = Ax_{N_i} + B\big[f_{N_i}(x_{N_i}) + g_{N_i}(x_{N_i})u_i\big]$, where

$$|f_i(x_i) - f_{N_i}(x_i)| \leqslant f_{0_i}(x_i), \qquad \forall x_i \in \mathbb{R}^{n_i} \tag{4.4}$$

$$|g_i(x_i) - g_{N_i}(x_i)| \leqslant g_{0_i}(x_i), \qquad \forall x_i \in \mathbb{R}^{n_i} \tag{4.5}$$

and $f_{0_i}(x_i)$, $g_{0_i}(x_i)$ are known local bounding functions representing the bound on the modeling uncertainty for f_i and g_i, respectively. To avoid any stabilizability problems and without loss of generality, we assume that $g_{N_i}(x_i) - g_{0_i}(x_i)$ is bounded away from zero and positive for all $x_i \in \mathbb{R}^{n_i}$, to guarantee that $g_i(x_i) > 0$ for all $x_i \in \mathbb{R}^{n_i}$. We also impose a bounding assumption on the interconnection function δ_{ij}. Specifically, we assume that δ_{ij}, for all $i \neq j$, satisfies

$$|\delta_{ij}(x_j)| \leqslant L_{ij}|x_j| + \sigma_j, \qquad \forall x_j \in \mathbb{R}^{n_j}, \tag{4.6}$$

where L_{ij} and σ_j are known constants, and $|x_j| = \sqrt{x_{j1}^2 + \ldots + x_{jn_j}^2}$ is the Euclidean norm.

The objective of this chapter is to develop a distributed fault detection and accommodation scheme for feedback linearizable nonlinear interconnected subsystems described by (4.1) and (4.2), such that each x_i follows a smooth reference trajectory vector $x_{d_i} = [x_{d_{i1}}, \ldots, x_{d_{in_i}}]^\top$. We assume that each x_{d_i} and $\dot{x}_{d_{in_i}}$ is uniformly bounded and available to all the subsystems. Let $\tilde{x}_{ij} = x_{ij} - x_{d_{ij}}$ be the tracking error for the j-th state of the i-th subsystem. The tracking error vector of the i-th subsystem is defined by $\tilde{x}_i = [\tilde{x}_{i1}, \tilde{x}_{i2}, \ldots, \tilde{x}_{in_i}]^\top$.

We consider the case in which the subsystems are allowed to exchange state information under certain conditions. Specifically, the i-th subsystem provides its state

$x_i(t)$ to all the other subsystems whenever the norm of its local tracking error $\tilde{x}_i(t)$ exceeds a certain threshold d_i, where $d_i > 0$ is a design constant. Otherwise, the other subsystems utilize the known desired state x_{d_i} instead. Define t^a_{jk} as the time instant at which the j-th subsystem starts communicating its state to the other subsystems, for the k-th time, and t^b_{jk} as the time instant at which the j-th subsystem stops communicating its state to the other subsystems, for the k-th time. Without loss of generality, we assume that $t^a_{jk} < t^b_{jk}$. We define the vector $\bar{x}^i = [\bar{x}^i_1, \ldots, \bar{x}^i_n] \in \mathbb{R}^n$ where $\bar{x}^i_j$ is given by,

$$\bar{x}^i_j \triangleq \begin{cases} x_j & \text{if} \quad t \in \left[t^a_{jk},\ t^b_{jk}\right) \\ x_{d_j} & \text{if} \quad t \in \left[t^b_{jk},\ t^a_{j(k+1)}\right) \end{cases}$$

Remark 4.1. In many practical applications of large-scale systems, there is a need to minimize communication between controllers while maintaining a high level of performance. A key motivation of this work is to reduce the communication exchange between subsystems based on a *communication-as-needed* approach. In this framework, the state $x_i(t)$ is communicated to the other subsystems only when the tracking error is above a certain constant value d_i. Therefore, the fact that the state of the j-th subsystem is known to the i-th subsystem $(j \neq i)$ for all $t \in \left[t^a_{jk},\ t^b_{jk}\right)$ and that the desired reference trajectories x_{d_j}, $j = 1, 2, \ldots, n$ are available to all the subsystems, ensures that

$$|x_j - \bar{x}^i_j| = 0 \quad \text{if} \quad t \in \left[t^a_{jk},\ t^b_{jk}\right) \tag{4.7}$$

$$|x_j - \bar{x}^i_j| \leqslant d_j \quad \text{if} \quad t \in \left[t^b_{jk},\ t^a_{j(k+1)}\right). \tag{4.8}$$

□

4.3 Distributed Fault Detection Scheme

In this section we consider the design of a distributed fault detection scheme based on a nonlinear estimator for each subsystem. The estimator for the i-th subsystem is described by

$$\dot{\chi}_i = \lambda_i \left(x_{in_i} - \chi_i\right) + f_{N_i}(x_i) + g_{N_i}(x_i)u_i + \text{sgn}(\epsilon_i) \sum_{j=1}^{m} L_{ij}|\bar{x}^i_j| + \sigma_j, \tag{4.9}$$

where $\chi_i \in \mathbb{R}$ is the estimated n_i-th state of the i-th subsystem, satisfying $\chi_i(0) = x_{in_i}(0)$, $-\lambda_i < 0$ is the estimator pole and $\epsilon_i = x_{in_i} - \chi_i$ is the estimation error, which is used for fault detection. For each subsystem i, we define a detection threshold $R_i(t)$ as follows:

$$R_i(t) = \sqrt{\frac{1}{\lambda_i}\int_0^t e^{-\frac{\lambda_i}{2}(t-\tau)} p_i\big(x_i(\tau)\big) d\tau}, \tag{4.10}$$

where p_i is given by

$$p_i(x_i) = \left(f_{0_i}(x_i) + g_{0_i}(x_i)|u_i| + \sum_{j=1}^{m} L_{ij} r_j\right)^2 \tag{4.11}$$

and r_j is given by

$$r_j = \begin{cases} 0 & \text{if} \quad t \in \left[t_{jk}^a,\ t_{jk}^b\right) \\ d_j & \text{if} \quad t \in \left[t_{jk}^b,\ t_{j(k+1)}^a\right). \end{cases}$$

A fault is declared at time t_d^i if $|\epsilon_i(t_d^i)| = R_i(t_d^i)$. The following theorem shows that in the absence of any fault the state estimation error satisfies $|\epsilon_i(t)| < R_i(t)$, therefore it is ensured that there are no false alarms.

Theorem 4.1. *The local fault detection estimator described by (4.9) satisfies*

$$|\epsilon_i(t)| < R_i(t) \qquad \forall t < T_i \tag{4.12}$$

Proof. Let the Lyapunov function for the i-th subsystem be given by $V_i = \frac{1}{2}\epsilon_i^2$. Based on (4.2) and (4.9), before the occurrence of a fault, i.e., for $t < T_i$, the time derivative of V_i satisfies:

$$\begin{aligned}
\dot{V}_i = &- \lambda_i \epsilon_i^2 + \epsilon_i\Big[f_i(x_i) - f_{N_i}(x_i)\Big] + \epsilon_i u_i\Big[g_i(x_i) - g_{N_i}(x_i)\Big] \\
&+ \epsilon_i\left[\sum_{j=1}^{m}\delta_{ij}(x_j) - \operatorname{sgn}(\epsilon_i)\sum_{j=1}^{m} L_{ij}|\bar{x}_j^i| + \sigma_j\right] \\
\leqslant &- \lambda_i \epsilon_i^2 + |\epsilon_i|\big|f_i(x_i) - f_{N_i}(x_i)\big| + |\epsilon_i u_i|\big|g_i(x_i) - g_{N_i}(x_i)\big| \\
&+ |\epsilon_i|\left|\sum_{j=1}^{m}|\delta_{ij}(x_j)| - \sum_{j=1}^{m} L_{ij}|\bar{x}_j^i| + \sigma_j\right|.
\end{aligned}$$

Using (4.4), (4.5) and (4.6) we obtain

$$\dot{V}_i \leqslant -\lambda_i \epsilon_i^2 + |\epsilon_i| f_{0_i}(x_i) + |\epsilon_i u_i| g_{0_i}(x_i) + |\epsilon_i|\left|\sum_{j=1}^{m} L_{ij}|x_j| - L_{ij}|\bar{x}_j^i|\right|.$$

Using (4.7) and (4.8) we obtain

$$\begin{aligned}\dot{V}_i \leqslant& -\lambda_i\epsilon_i^2 + |\epsilon_i| f_{0_i}(x_i) + |\epsilon_i u_i| g_{0_i}(x_i) + |\epsilon_i| \sum_{j=1}^{m} L_{ij} r_j \\ \leqslant& -\frac{\lambda_i}{2}\epsilon_i^2 + \frac{\lambda_i}{2}\Bigg[-\epsilon_i^2 + \frac{2}{\lambda_i}|\epsilon_i| \left(f_{0_i}(x_i) + g_{0_i}(x_i)|u_i| + \sum_{j=1}^{m} L_{ij} r_j \right) \Bigg].\end{aligned}$$

By completing the squares we obtain that

$$\begin{aligned}-\epsilon_i^2 + \frac{2}{\lambda_i}|\epsilon_i| &\left(f_{0_i}(x_i) + g_{0_i}(x_i)|u_i| + \sum_{j=1}^{m} L_{ij} r_j \right) \\ &\leqslant \frac{1}{\lambda_i^2} \left(f_{0_i}(x_i) + g_{0_i}(x_i)|u_i| + \sum_{j=1}^{m} L_{ij} r_j \right)^2\end{aligned}$$

Therefore the time derivative of the Lyapunov function satisfies

$$\dot{V}_i \leqslant -\frac{\lambda_i}{2}\epsilon_i^2 + \frac{1}{2\lambda_i} \left(f_{0_i}(x_i) + g_{0_i}(x_i)|u_i| + \sum_{j=1}^{m} L_{ij} r_j \right)^2$$

Integrating both sides we obtain

$$V_i(t) \leqslant \frac{1}{2\lambda_i} \int_0^t e^{-\frac{\lambda_i}{2}(t-\tau)} \left[f_{0_i}(x_i(\tau)) + g_{0_i}(x_i(\tau))|u_i(\tau)| + \sum_{j=1}^{m} L_{ij} r_j \right]^2 d\tau \tag{4.13}$$

Therefore, based on the definition of V_i, $|\epsilon_i(t)| = \sqrt{2V_i(t)} \leqslant \sqrt{\frac{1}{\lambda_i} \int_0^t e^{-\frac{\lambda_i}{2}(t-\tau)} p_i\big(x_i(\tau)\big) d\tau}$

□

The distributed fault estimator, given by (4.9), guarantees that whenever (4.12) is not satisfied, there is a fault in the i-th subsystem. However, the reverse is not true, i.e., it is not guaranteed that when $|\epsilon_i(t)| < R_i(t)$, the i-th subsystem is fault free. In order for a fault to be detectable, the fault function $h_i(x)$ needs to satisfy a certain fault detectability condition. The following lemma characterizes the class of faults that are detectable by the proposed distributed fault detection scheme.

Lemma 4.1. *A fault in the i-th subsystem is detected at some $t_d^i > T_i$, if the fault function h_i satisfies*

$$\int_{T_i}^{t_d^i} e^{-\frac{\lambda_i}{2}(t_d^i-\tau)} \beta^2(t-T_i) h_i^2(x(\tau)) d\tau > \frac{3}{2} \int_0^{t_d^i} e^{-\frac{\lambda_i}{2}(t-\tau)} p_i(x_i(\tau)) d\tau \tag{4.14}$$

Proof. Consider the Lyapunov the function for the i-th subsystem, $V_i = \frac{1}{2}\epsilon_i^2$. Before fault detection, i.e., for $t \in (0, t_d^i)$, the time derivative of V_i satisfies

$$\begin{aligned}
\dot{V}_i =& -\lambda_i \epsilon_i^2 + \epsilon_i \Big[f_i(x_i) - f_{N_i}(x_i) \Big] + \epsilon_i u_i \Big[g_i(x_i) - g_{N_i}(x_i) \Big] \\
&+ \epsilon_i \left[\sum_{j=1}^{m} \delta_{ij}(x_j) - \mathrm{sgn}(\epsilon_i) \sum_{j=1}^{m} L_{ij} |\bar{x}_j^i| + \sigma_j \right] + \epsilon_i \beta(t - T_i) h_i(x) \\
\geqslant& -\lambda_i \epsilon_i^2 - |\epsilon_i| \big| f_i(x_i) - f_{N_i}(x_i) \big| - |\epsilon_i u_i| \big| g_i(x_i) - g_{N_i}(x_i) \big| \\
&- |\epsilon_i| \left| \sum_{j=1}^{m} \delta_{ij}(x_j) - \sum_{j=1}^{m} L_{ij} |\bar{x}_j^i| + \sigma_j \right| + \epsilon_i \beta(t - T_i) h_i(x).
\end{aligned}$$

Using (4.4), (4.5), (4.6), (4.7) and (4.8) we obtain

$$\begin{aligned}
\dot{V}_i \geqslant& -\lambda_i \epsilon_i^2 - |\epsilon_i| \left[f_{0_i}(x_i) + g_{0_i}(x_i) |u_i| + \sum_{j=1}^{m} L_{ij} r_j \right] + \epsilon_i \beta(t - T_i) h_i(x) \\
\geqslant& -\frac{\lambda_i}{2} \epsilon_i^2 - \frac{\lambda_i}{4} \left[\epsilon_i^2 + \frac{4}{\lambda_i} \left(f_{0_i}(x_i) + g_{0_i}(x_i) |u_i| + \sum_{j=1}^{m} L_{ij} r_j \right) |\epsilon_i| \right] \\
&- \frac{\lambda_i}{4} \left[\epsilon_i^2 - \frac{4}{\lambda_i} \epsilon_i \beta(t - T_i) h_i(x) \right].
\end{aligned}$$

Completing the squares we obtain

$$\dot{V}_i \geqslant -\frac{\lambda_i}{2} \epsilon_i^2 - \frac{1}{\lambda_i} p_i(x_i) + \frac{1}{\lambda_i} \beta^2 (t - T_i) h_i^2(x).$$

Integrating both sides over $t \in [0, t_i^d]$ we obtain

$$V_i(t_d^i) \geqslant \frac{1}{\lambda_i} \int_{T_i}^{t_d^i} e^{-\frac{\lambda_i}{2}(t_d^i - \tau)} \beta^2 (t - T_i) h_i^2(x(\tau)) d\tau - \frac{1}{\lambda_i} \int_0^{t_d^i} e^{-\frac{\lambda_i}{2}(t_d^i - \tau)} p_i(x_i(\tau)) d\tau.$$

Using (4.13) we obtain

$$\begin{aligned}
\frac{1}{2\lambda_i} \int_0^{t_d^i} e^{-\frac{\lambda_i}{2}(t_d^i - \tau)} p_i(x_i(\tau)) d\tau \geqslant& \frac{1}{\lambda_i} \int_{T_i}^{t_d^i} e^{-\frac{\lambda_i}{2}(t_d^i - \tau)} \beta^2 (t - T_i) h_i^2(x(\tau)) d\tau \\
&- \frac{1}{\lambda_i} \int_0^{t_d^i} e^{-\frac{\lambda_i}{2}(t_d^i - \tau)} p_i(x_i(\tau)) d\tau.
\end{aligned}$$

Rearranging we obtain

$$\frac{3}{2} \int_0^{t_d^i} e^{-\frac{\lambda_i}{2}(t - \tau)} p_i(x_i(\tau)) d\tau \geqslant \int_{T_i}^{t_d^i} e^{-\frac{\lambda_i}{2}(t_d^i - \tau)} \beta^2 (t - T_i) h_i^2(x(\tau)) d\tau.$$

Therefore, if for some $t_d^i > T_i$, (4.14) is satisfied, it is guaranteed that the fault in the i-th subsystem will be detected. □

The detectability condition given by Lemma (4.1) is a sufficient condition for a fault to be detectable. However, it is not a necessary condition for a fault to be detected, and therefore the class of detectable faults may be significantly larger.

4.4 Distributed Nominal Control Design

Based on (4.3), the tracking error dynamics, $\tilde{x}_i = x_i - x_{d_i}$, of the i-th subsystem, satisfy:

$$\dot{\tilde{x}}_i = A\tilde{x}_i + B\Big[f_i(x_i) + g_i(x_i)u_i + \sum_{j=1}^{m}\delta_{ij}(x_j) + \beta(t-T_i)h_i(x) - \dot{x}_{d_{in_i}}\Big]. \qquad (4.15)$$

Let the distributed control law u_i be given by $u_i = u_{N_i} + u_{F_i}$, where u_{N_i} denotes the nominal control law and u_{F_i} is the augmented fault accommodation control law for addressing the change in dynamics due to the occurrence of a fault in the i-th subsystem. Prior to the detection of a fault, $u_{F_i} = 0$. In this section, we present the nominal control design and in the next section we investigate the design of the fault accommodation scheme.

The nominal control law of the i-th subsystem is defined as

$$u_{N_i} = \frac{1}{g_{N_i}(x_i) + \operatorname{sgn}(e_i u_{a_i})g_{0_i}(x_i)}u_{a_i} \qquad (4.16)$$

$$\begin{aligned} u_{a_i} = & -K_i^\top\tilde{x}_i + \dot{x}_{d_{in_i}} - f_{N_i}(x_i) - \operatorname{sgn}(e_i)f_{0_i}(x_i) \\ & -\operatorname{sgn}(e_i)\sum_{j=1}^{m}\left(L_{ij}|\bar{x}_j^i| + \sigma_j + r_j\right), \end{aligned} \qquad (4.17)$$

where the vector $K_i = [k_{i1}, \cdots, k_{in_i}]^\top \in \mathbb{R}^{n_i}$ is chosen such that $A - BK_i^\top$ is a Hurwitz matrix. Since $A - BK_i^\top$ is Hurwitz, for any $Q_i > 0$ there exists $P_i > 0$ satisfying the Lyapunov equation, $P_i(A - BK_i^\top) + (A - BK_i^\top)^\top P_i = -Q_i$. Based on P_i, the scalar tracking error e_i is defined as $e_i \triangleq B^T P_i\tilde{x}_i$.

Theorem 4.2. *Prior to the occurrence of a fault in the i-th subsystem, the distributed nominal control law u_{N_i}, given by (4.16) and (4.17), guarantees that $\tilde{x}_i(t) \to 0$ as $t \to \infty$ for $i = 1, 2, \ldots, m$.*

Proof. Let the Lyapunov function for the i-th subsystem be given by $V_i = \frac{1}{2}\tilde{x}_i^\top P_i\tilde{x}_i$. Using (4.15), (4.16) and (4.5), the time derivative of V_i satisfies

$$\dot{V}_i \leqslant \frac{1}{2}\tilde{x}_i^\top\left(A^\top P_i + P_iA\right)\tilde{x}_i + e_i\left[f_i(x_i) + u_{a_i} + \sum_{j=1}^{m}\delta_{ij}(x_j) - \dot{x}_{d_{in_i}}\right].$$

Substituting u_{a_i} and using (4.4) we obtain

$$\begin{aligned} \dot{V}_i \leqslant & -\frac{1}{2}\tilde{x}_i^\top Q_i\tilde{x}_i + e_i\left[\sum_{j=1}^{m}\delta_{ij}(x_j) - \operatorname{sgn}(e_i)\sum_{j=1}^{m}\left(L_{ij}|\bar{x}_j^i| + \sigma_j + r_j\right)\right] \\ \leqslant & -\frac{1}{2}\tilde{x}_i^\top Q_i\tilde{x}_i + |e_i|\left[\sum_{j=1}^{m}|\delta_{ij}(x_j)| - \sum_{j=1}^{m}\left(L_{ij}|\bar{x}_j^i| + \sigma_j + r_j\right)\right]. \end{aligned}$$

Using (4.6) we obtain

$$\dot{V}_i \leqslant -\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i + |e_i| \left[\sum_{j=1}^{m} L_{ij}|x_j| + \sigma_j - \sum_{j=1}^{m} \left(L_{ij}|\bar{x}_j^i| + \sigma_j + r_j \right) \right]$$
$$\leqslant -\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i + |e_i| \left[\sum_{j=1}^{m} \left(L_{ij}|x_j - \bar{x}_j^i| - r_j \right) \right].$$

Using (4.7) and (4.8) we obtain $\dot{V}_i \leqslant -\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i$. Therefore, using Barbalat's Lemma [30], it can be shown that $\tilde{x}_i(t) \to 0$ as $t \to \infty$, for $i = 1, \ldots, m$. □

4.5 Distributed Fault Accommodation Scheme Design

In this Section, we deal with reconfiguration of the control law u_i of the i-th subsystem, after a fault is detected (i.e., $t \geqslant t_d^i$). In this framework, we use adaptive approximation models for counteracting the change in dynamics due to faults. The following lemma shows that for any given set of basis functions $\phi_s(x)$, there exists an upper bound $\bar{s}(x)$ of the function $|s(x)|$ that can be represented exactly by $\theta_s^{*\top}\phi_s(x)$ within a compact set $\mathcal{D}$.

Lemma 4.2. *Given a compact set $\mathcal{D} \subset \mathbb{R}^p$, let $\phi_s(x) = \left[\phi_{s_1}(x), \ldots, \phi_{s_q}(x)\right]^\top : \mathbb{R}^p \mapsto \mathbb{R}^q$ be a set of basis functions such that for all $x \in \mathcal{D}$ at least one basis function is nonzero. Then for any bounded function $s(x)$ there exists a set of bounded parameters $\theta_s^* = \left[\theta_{s_1}^*, \ldots, \theta_{s_q}^*\right]^\top \in \mathbb{R}^q$, such that*

$$|s(x)| \leqslant \theta_s^{*\top}\phi_s(x) = \bar{s}(x), \qquad x \in \mathcal{D}.$$

Proof. Consider the function $|s(x)|$, which can be represented within a compact set $\mathcal{D}$ as,

$$|s(x)| = \theta_s^\top \phi_s(x) + \mu_s(x), \qquad x \in \mathcal{D}, \tag{4.18}$$

where $\mu_s(x)$ is the residual approximation error. We continue to show that, there exists a set of bounded constant parameters θ_s^*, such that $\bar{s}(x) = \theta_s^{*\top}\phi_s(x)$ is an upper bound of $|s(x)|$ for all $x \in \mathcal{D}$. Let the k-th parameter of θ_s^*, $k = 1, 2, \ldots, q$, be given by

$$\theta_{s_k}^* = \theta_{s_k} + \frac{1}{q}\operatorname{sgn}\left(\phi_{s_k}(x)\right) a_k, \tag{4.19}$$

where $a_k = \left\| \frac{\mu_s(x)}{\phi_{s_k}(x)} \right\|_{\mathcal{D},\infty} \triangleq \max_{x \in \mathcal{D}} \left| \frac{\mu_s(x)}{\phi_{s_k}(x)} \right|$. Substituting (4.19) into $\bar{s}(x) = \theta_s^{*\top} \phi_s(x)$ we obtain

$$\bar{s}(x) = \Big[\theta_{s_1} + \frac{1}{q}\text{sgn}\left(\phi_{s_1}(x)\right) a_1, \ldots, \theta_{s_q} + \frac{1}{q}\text{sgn}\left(\phi_{s_q}(x)\right) a_q\Big]\phi_s(x)$$

Using (4.18) we obtain that for all $x \in \mathcal{D}$.

$$\begin{aligned}
\bar{s}(x) &= \left[\frac{1}{q}\text{sgn}\left(\phi_{s_1}(x)\right) a_1, \ldots, \frac{1}{q}\text{sgn}\left(\phi_{s_q}(x)\right) a_q\right] \phi_s(x) + |s(x)| - \mu_s(x) \\
&= \frac{1}{q}\sum_{k=1}^{q}\left(|\phi_{s_k}(x)| \left\| \frac{\mu_s(x)}{\phi_{s_k}(x)} \right\|_{\mathcal{D},\infty}\right) + |s(x)| - \mu_s(x) \\
&\geqslant \frac{1}{q}\sum_{k=1}^{q}\left(|\phi_{s_k}(x)| \left| \frac{\mu_s(x)}{\phi_{s_k}(x)} \right|\right) + |s(x)| - \mu_s(x) \\
&\geqslant |s(x)|.
\end{aligned}$$

Therefore, the function $\bar{s}(x) = \theta_s^{*\top}\phi_s(x)$, with θ_s^* given by (4.19), satisfies $\bar{s}(x) \geqslant |s(x)|$ for all $x \in \mathcal{D}$. □

Remark 4.2. Lemma 4.2 is an existence result showing that for $|s(x)|$ there exists an upper bound $\bar{s}(x)$ that can be exactly represented by the adaptive approximator $\theta_s^{*\top}\phi_s(x)$. The parameters θ_s^* given by (4.19) are not necessarily optimal in terms of minimizing $\big|\bar{s}(x) - |s(x)|\big|$. □

Remark 4.3. If for some $x^0 \in \mathcal{D}$, $\phi_{s_k}(x^0) = 0$ for all $k = 0, 1, \ldots, q$, it is not guaranteed that Lemma 4.2 holds. In order to ensure that Lemma 4.2 holds, the designer needs to choose overlapping basis functions, such that at least one basis function is non-zero for all $x \in \mathcal{D}$. □

Now consider the fault function $h_i(x)$. According to Lemma 4.2, there exists a bound $\bar{h}_i(x)$ of the function $|h_i(x)|$ that can be represented within a compact set $\mathcal{X} \subset \mathbb{R}^n$ by an approximation model as follows,

$$\bar{h}_i(x) = \theta_i^\top \phi_i(x), \qquad x \in \mathcal{X}, \tag{4.20}$$

where $\theta_i \in \mathbb{R}^q$ is a set of unknown bounded constant parameters and $\phi_i(x) \in \mathbb{R}^q$ is a set of basis functions that covers $\mathcal{X}$. The augmented distributed fault accommodation control law u_{F_i} is defined by

$$u_{F_i} = \frac{1}{g_{N_i}(x_i) + \text{sgn}(e_i u_{c_i}) g_{0_i}(x_i)} u_{c_i} \tag{4.21}$$

$$u_{c_i} = -\,\text{sgn}(e_i)\hat{\theta}_i^\top\left(\phi_i(\bar{x}^i) + M_i \sum_{j=1}^{m} r_j\right), \tag{4.22}$$

where $\hat{\theta}_i(t)$ are the parameter estimates and $M_i = [m_{i1} \dots m_{iq}]^\top$ represents the Lipschitz vector such that

$$\left|\phi_{ik}(x) - \phi_{ik}(\bar{x}^i)\right| \leqslant m_{ik}|x - \bar{x}^i|, \qquad k = 1, 2, \dots, q; \quad x \in \mathcal{X} \tag{4.23}$$

The parameter estimates $\hat{\theta}_i$ are updated according to,

$$\dot{\hat{\theta}}_i = \Gamma_i |e_i| \left(\phi_i(\bar{x}^i) + M_i \sum_{j=1}^{m} r_j \right) \tag{4.24}$$

Theorem 4.3. *Given that the coverage set $\mathcal{X}$ is large enough such that $x(t) \in \mathcal{X}$ for all $t > 0$, the adaptive approximation based control law for the i-th subsystem, given by (4.16), (4.17), (4.21), (4.22), (4.24) guarantees that $\tilde{x}_i(t) \to 0$ as $t \to \infty$ for $i = 1, \dots, m$.*

Proof. Let the Lyapunov function for the i-th subsystem be given by $V_i = V_{i1} + V_{i2}$ where $V_{i1} = \frac{1}{2}\tilde{x}_i^\top P_i \tilde{x}_i$, $V_{i2} = \frac{1}{2}\tilde{\theta}_i^\top \Gamma_i^{-1} \tilde{\theta}_i$, and $\tilde{\theta}_i = \hat{\theta}_i - \theta_i$ is the parameter estimation error vector. Consider the time interval $t > \tau_d^i$. Using the result of Theorem 4.2, the time derivative of V_{i1} satisfies,

$$\dot{V}_{i1} \leqslant -\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i + e_i \left[\beta(t - T_i) h_i(x) + g_i(x_i) u_{F_i}\right].$$

Substituting u_{F_i} and using (4.5) we obtain

$$\dot{V}_{i1} \leqslant -\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i + e_i \left[\beta(t - T_i) h_i(x) + u_{c_i}\right].$$

Substituting u_{c_i} we obtain

$$\dot{V}_{i1} \leqslant -\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i + |e_i| \Big[\bar{h}_i(x) - \hat{\theta}_i^\top \phi_i(\bar{x}^i) - \hat{\theta}_i^\top M_i \sum_{j=1}^{m} r_j\Big].$$

Using (4.20),

$$\dot{V}_{i1} \leqslant -\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i + |e_i| \Big[\theta_i^\top \phi_i(x) - \hat{\theta}_i^\top \phi_i(\bar{x}^i) - \hat{\theta}_i^\top M_i \sum_{j=1}^{m} r_j\Big].$$

Adding and subtracting $\theta_i^\top \phi_i(\bar{x}^i)$ we obtain

$$\dot{V}_{i1} \leqslant -\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i + |e_i| \left[-\tilde{\theta}_i^\top \phi_i(\bar{x}^i) - \hat{\theta}_i^\top M_i \sum_{j=1}^{m} r_j\right] + |e_i| \theta_i^\top \left(\phi_i(x) - \phi_i(\bar{x}^i)\right).$$

Using (4.23) and the fact that $|x - \bar{x}^i| \leqslant \sum_{j=1}^m r_j$ we obtain

$$\dot{V}_{i1} \leqslant -\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i + |e_i| \left[-\tilde{\theta}_i^\top \phi_i(\bar{x}^i) - \tilde{\theta}_i^\top M_i \sum_{j=1}^m r_j \right].$$

The time derivative of V_i satisfies $\dot{V}_i = \dot{V}_{i1} + \tilde{\theta}_i^\top \Gamma_i^{-1} \dot{\tilde{\theta}}_i$. By grouping terms we obtain

$$\dot{V}_i \leqslant -\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i + \tilde{\theta}_i^\top \Gamma_i^{-1} \left[\dot{\tilde{\theta}}_i - \Gamma_i |e_i| \left(\phi_i(\bar{x}^i) + M_i \sum_{j=1}^m r_j \right) \right].$$

Substituting the adaptive law (4.24), we obtain $\dot{V}_i \leqslant -\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i$. Therefore, using Barbalat's Lemma [30], it can be shown that $\tilde{x}_i(t) \to 0$ as $t \to \infty$, for $i = 1, \ldots, m$. □

Remark 4.4. Intuitively, the parameter adaptive law (4.24) consists of two components. The first component (i.e., $\Gamma_i |e_i| \phi_i(\bar{x}^i)$) is used to approximate the function $\bar{h}_i(\bar{x}^i)$, while the second component (i.e., $\Gamma_i |e_i| M_i \sum_{j=1}^m d_j$) is used to compensate for the replacement error, described by the difference $\bar{h}_i(x) - \bar{h}_i(\bar{x}^i)$, which arises due to the tracking-error based communication algorithm where no state is information is shared if the tracking error is smaller than a certain threshold d_j. □

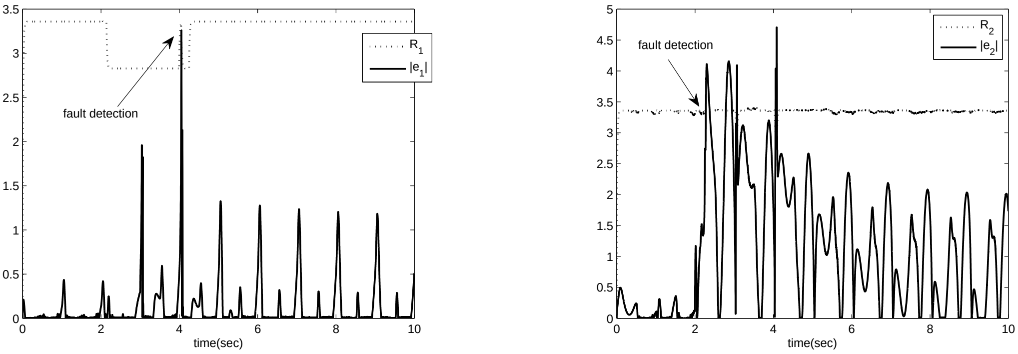
Remark 4.5. Theorem 4.3 assumes that after the occurrence of the fault, the trajectory $x(t)$ remains within the coverage region $\mathcal{X}$. If an upper bound is available on the fault function $h_i(x)$, then a safety control scheme (such as the one presented in Section 3.5) can be designed to bring the trajectory back within $\mathcal{X}$, in case that it leaves the coverage region. Moreover, in practical applications it may be necessary to use dead-zone modification to the adaptive law (4.24) in order to avoid parameter drift in the presence of measurement noise and disturbances. □

4.6 Simulation Example

In this section, we consider a simple simulation example based on two inverted pendulums connected by a spring. The i-th, $i = 1, 2$, subsystem is described by

$$\begin{aligned} \Sigma_i : \quad \dot{x}_{i1} &= x_{i2}, \\ \dot{x}_{i2} &= \left(\frac{m_i g r}{J_i} - \frac{k r^2}{4 J_i} \right) sin(x_{i1}) + \frac{k r}{2 J_i}(l - b) + \frac{1}{J_i} u_i \\ &\quad + \sum_{j=1}^m \delta_{ij}(x_j) + \beta(t - T_i) h_i(x_1, x_2) \end{aligned}$$

where $x_i = [x_{i1}, x_{i2}]^\top$ are the state vectors, $\theta_i = x_{i1}$ are the angular displacements of the pendulums from vertical, $m_1 = 5$kg and $m_2 = 6.5$kg are the pendulum end



(a) Σ_1 subsystem. The fault is detected at $t = 4.05$.

(b) Σ_2 subsystem. The fault is detected at $t = 2.27$.

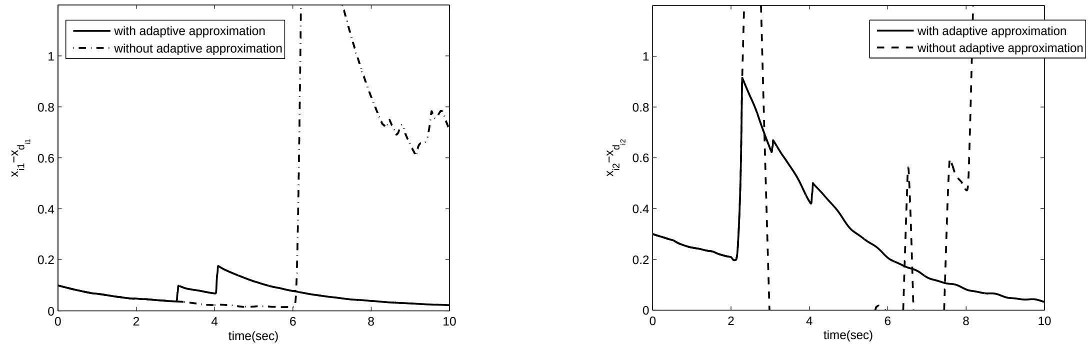
Figure 4.1: Time evolution of the ditributed fault detectors.

masses, $J_1 = 0.4$kg and $J_2 = 0.9$kg are the moments of inertia, $k = 105$N/m is the spring constant, $r = 0.8$m is the pendulum height, $l = 0.6$m is the natural length of the spring, $b = 0.5$m is the distance between the pendulums and $g = 9.81m/s^2$ is the gravitational acceleration. The interconnections are given by $\delta_{12}(x_2) = \frac{kr^2}{4J_1}\sin(x_{21})$ and $\delta_{21}(x_1) = \frac{kr^2}{4J_2}\sin(x_{21})$. The matrix P_i satisfying the Lyapunov equation, for $Q = I_{2\times2}$, is given by,

$$P_i = \begin{bmatrix} 1.5 & 0.5 \\ 0.5 & 1 \end{bmatrix} \qquad i = 1, 2, 3,$$

where $K_1 = K_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}^\top$. The desired trajectory vector $x_{d_i} = [x_{d_{i1}}, x_{d_{i2}}]^\top$ and the signal $\dot{x}_{d_{i2}}$ are generated using a third order filter with a bandwidth of 5 (rad/sec) and unity gain below this frequency. The filter input is chosen as a sine wave of zero mean, 0.7 amplitude and a frequency of 1 Hz. A lattice of equally spaced radial basis functions are designed for compensating the effects of faults. The design constants d_1 and d_2 are chosen as $d_1 = d_2 = 0.5$. We consider the case in which abrupt faults occur in Σ_1 at $T_1 = 3$ *sec* and in Σ_2 at $T_2 = 2$ *sec*. For simulation purposes, the unknown fault functions h_1 and h_2 are chosen as, $h_1 = k(x_{21}x_{12} + 0.8)\cos(x_{21}x_{22})$ and $h_2 = 3k(x_{21}x_{12} + 1.5x_{11})\cos(x_{11}x_{12})$.

Fig. 4.1a and Fig. 4.1b shows the time evolution of the distributed fault detection scheme of Σ_1 and Σ_2, respectively. As illustrated by the plots, no false alarms occur in the subsystems and the faults are detected soon after their occurrence. We have investigated the relationship between the amount of available information from remote subsystems and the fault detection time. It has been found that although



(a) Σ_1 subsystem.

(b) Σ_2 subsystem.

Figure 4.2: Time evolution of the tracking errors.

communication is substantially reduced, as compared to a centralized design, there is only a minor increase on the detection time of the faults. More specifically, with $d_1 = d_2 = 0.5$ the communication is reduced by approximately 80%, while the detection time increases only by approximately 10%.

In Fig. 4.2a and Fig. 4.2b we plot the tracking error, $x_{i1} - x_{d_{i1}}$, $i = 1, 2$, of each subsystem respectively, with and without fault accommodation (via adaptive approximation based control). As illustrated by the plot, through the use of adaptive approximation, the subsystems remain stable in the presence of faults and are able to follow the corresponding reference trajectories. One would expect that a fault that has occurred in one of the subsystems would have an impact on the dynamics of the other subsystem. However, as can be seen by the plot, this phenomenon is not present. The reason is that, the increase of the tracking error in the faulty subsystem is compensated by the exchange of state information whenever the tracking error exceeds the threshold $d_i = 0.5$.

It is important to note that the performance of the distributed fault detection and accommodation scheme is highly dependent on the choice of the design constant d_i. In general, for small d_i the distributed fault detection and accommodation scheme is fault tolerant to a larger class of faults and is able to detect faults faster and accommodate them with less control effort. As d_i increases, the distributed fault detection scheme becomes less sensitive to faults and the fault accommodation scheme requires larger control effort in order to compensate for the effects of the faults. Therefore, it becomes apparent that there is an inherent tradeoff between communication cost and fault tolerance which can be addressed with the selection of the communication threshold d_i.

4.7 Conclusion

In this chapter, we have presented a methodology for distributed fault detection and accommodation of a class of feedback linearizable interconnected uncertain nonlinear systems. The subsystems exchange state information according to a self-triggering tracking-error based communication algorithm, where each subsystem transmits its state information only when its tracking error exceeds a certain threshold. In the absence of remote state information, the desired reference trajectory, which is assumed to be available *a priori*, is used instead. The distributed fault detection scheme, based on a nonlinear estimator for each subsystem, ensures that there are no false detection alarms. A distributed fault accommodation scheme is designed based on adaptive approximation models for accommodating faults. The robustness to residual approximation errors is ensured by adaptively approximating the upper bound of the fault function, instead of the fault function itself. The stability of the proposed scheme is established through Lyapunov analysis. The simulation results demonstrated that, although the amount of available information from remote subsystems is substantially reduced, as compared to a centralized design, the impact on the performance of the fault detection and accommodation scheme is minimal. Moreover, the simulation results demonstrated that the control performance of the fault accommodation scheme as well as the ability to detect faults is closely tied to the choice of the design constants d_i. In the next chapters, we focus on the problem of communication for interconnected systems, and develop more efficient communication decision algorithms.

Chapter 5

A Coordinated Communication Scheme for Distributed Fault Tolerant Control

5.1 Introduction

In this chapter a distributed fault tolerant control scheme for a class of feedback linearizable uncertain interconnected systems is presented. We consider faults that occur in the subsystems local dynamics, as well as in the interconnections. The exchange of state information between subsystems is based on a coordinated communication scheme. More specifically, two subsystems exchange information when both of the subsystems tracking errors exceed a certain constant threshold. The distributed fault tolerant control law is designed in an adaptive approximation framework [19]. Through rigorous stability analysis, uniform ultimate boundedness of the tracking errors to a region around zero is proved.

This chapter is organized as follows. In Section 5.2 we formulate the problem and in Section 5.3 we present the distributed fault tolerant control design. In Section 5.4, we establish the stability of the distributed fault tolerant control scheme through Lyapunov analysis. Simulation results are presented in Section 5.5 and Section 5.6 contains some concluding remarks.

5.2 Problem Formulation

We consider a large-scale system comprised of m interconnected subsystems, where the i-th subsystem is described by

$$\dot{x}_{ik} = x_{i(k+1)} \qquad k = 1, 2, \ldots, n_i - 1 \tag{5.1}$$

$$\dot{x}_{in_i} = f_i(x_i) + g_i(x_i)u_i + \sum_{j \in \mathcal{P}_i} \delta_{ij}(x_j) + \beta(t - T_i)h_i(x) \tag{5.2}$$

where $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}$ is the state and control input of the i-th subsystem respectively, $x = \left[x_1^\top, \ldots, x_m^\top\right]^\top \in \mathbb{R}^n$ (where $n = \sum_{j=1}^m n_i$) is the state of the overall system, $f_i : \mathbb{R}_i^n \mapsto \mathbb{R}$ and $g_i : \mathbb{R}^{n_i} \mapsto \mathbb{R}$ are in general partially known functions representing the local dynamics of the i-th subsystem. The term $\delta_{ij} : \mathbb{R}^{n_j} \mapsto \mathbb{R}$, $j \in \mathcal{P}_i$ is an unknown interconnection function representing the effect of the j-th subsystem onto the i-th subsystem dynamics, with $j \neq i$. The set $\mathcal{P}_i$ represents all (neighboring) subsystems that are interconnected to the i-th subsystem. The term $h_i : \mathbb{R}^n \mapsto \mathbb{R}$ denotes the unknown change in the i-th subsystem dynamics due to a fault, while $\beta(t - T_i) : \mathbb{R}^+ \mapsto \mathbb{R}$ represents the corresponding time profile of the fault that occurs at some unknown time T_i.

A fault occurring in the i-th subsystem may have an impact on the i-th subsystem dynamics, but also have an impact on the interconnections effects δ_{ij}, $j \in \mathcal{P}_i$. Both of these changes in dynamics, due to the occurrence of a fault, are represented by the $h_i(x)$ function. The distributed fault tolerant control scheme presented in this chapter can be extended to the case where multiple faults occur in the local subsystems dynamics and in the interconnection. For the sake of simplicity and notational convenience, in this chapter we consider a single fault within each subsystem. Additionally, we consider abrupt faults (where the time profile satisfies $\beta(t - T_i) = 0$ for $t < T_i$, and $\beta(t - T_i) = 1$ for $t \geqslant T_i$), as well as incipient faults (where $\beta(t - T_i) = 0$ for $t < T_i$ and $\beta(t - T_i)$ increases monotonically from 0 to 1 for $t \geqslant T_i$).

The i-th subsystem described by (5.1), (5.2) can be written in matrix form as

$$\dot{x}_i = Ax_i + B\left[f_i(x_i) + g_i(x_i)u_i + \sum_{j=1}^{m} \delta_{ij}(x_j) + \beta(t - T_i)h_i(x)\right] \tag{5.3}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}.$$

It is assumed that a local nominal (known) model of the i-th subsystem is described by

$$\dot{x}_{N_i} = Ax_{N_i} + B\big[f_{N_i}(x_{N_i}) + g_{N_i}(x_{N_i})u_i\big], \tag{5.4}$$

where

$$|f_i(x_i) - f_{N_i}(x_i)| \leqslant f_{0_i}(x_i), \qquad \forall x_i \in \mathbb{R}^{n_i} \tag{5.5}$$

$$|g_i(x_i) - g_{N_i}(x_i)| \leqslant g_{0_i}(x_i), \qquad \forall x_i \in \mathbb{R}^{n_i} \tag{5.6}$$

and $f_{0_i}(x_i)$, $g_{0_i}(x_i)$ are known local bounding functions representing the bound on the modeling uncertainty for f_i and g_i, respectively. To avoid any stabilizability problems and without loss of generality, we assume that $g_i(x_i) > 0$ for all $x_i \in \mathbb{R}^{n_i}$.

The control objective is for each $x_i(t)$ to follow a desired trajectory $x_{d_i}(t)$ prior to the presence of a fault as well as after any possible faults. We assume that the desired trajectory vector $x_{d_i}(t)$ is bounded and available to all the interconnected subsystems $j \in \mathcal{P}_i$. Let $\tilde{x}_{ik} = x_{ik} - x_{d_{ik}}$ be the tracking error for the k-th state of the i-th subsystem. The tracking error vector of the i-th subsystem is defined by $\tilde{x}_i = [\tilde{x}_{i1}, \ldots, \tilde{x}_{in_i}]^\top$.

In Chapter 4, the broadcast of information between subsystems is based on the local tracking error $\tilde{x}_i$, i.e., each subsystem broadcasts its state x_i to the other subsystems whenever the norm of its local tracking error exceeds a certain threshold, denoted by d_i. When no information is available from a certain subsystem i (i.e., the tracking error is less than the threshold), its desired state x_{d_i} is used instead of the measured state x_i. It is easy to see that with this simple communication scheme each subsystem is aware that the states of interconnected subsystems are within a prescribed region, even when no information is received. This communication scheme is based on the idea that a large local tracking error can have a significant impact on the other subsystem dynamics, due to inaccurate information, while a relatively small local tracking error can have little impact on the other subsystem dynamics. However, in the case of complex interconnections, with higher degrees of nonlinearities, the aforementioned conclusions may not hold. More specifically, a large local tracking error may have little impact on the interconnected subsystems, while a small tracking error, may have a significant impact on its interconnected subsystems. More generally, the impact of subsystem j onto subsystem i is determined by the interconnection function δ_{ij}, which in general is unknown.

To illustrate this, consider the special case of scalar subsystems and analytic interconnections δ_{ij}, such that the interconnection $\delta_{ij}(x_j)$ can be represented by a Taylor series around x_{d_j} as follows:

$$\begin{aligned}\delta_{ij}(x_j) &= \delta_{ij}(x_{d_j}) + \delta_{ij}^{'}(x_{d_j})(x_j - x_{d_j}) + \frac{\delta_{ij}^{''}(x_{d_j})}{2}(x_j - x_{d_j})^2 + \ldots \\ &= \sum_{k=0}^{\infty} \frac{\delta_{ij}^{(k)}(x_{d_j})}{k!}(\tilde{x}_j)^k\end{aligned}$$

where $\delta_{ij}^{(k)}(x_{d_j})$ denotes the k-th derivative of δ_{ij} evaluated at x_{d_j}. By reordering terms we obtain

$$\begin{aligned}\delta_{ij}(x_j) - \delta_{ij}(x_{d_j}) &= \delta_{ij}^{'}(x_{d_j})\tilde{x}_j + \frac{\delta_{ij}^{''}}{2}(x_{d_j})\tilde{x}_j^2 + \ldots \\ &= \tilde{x}_j + (\delta_{ij}^{'}(x_{d_j}) - 1)\tilde{x}_j + \frac{\delta_{ij}^{''}(x_{d_j})}{2}\tilde{x}_j^2 + \ldots \\ &= \tilde{x}_j + \lambda_{ij} \qquad (5.7)\end{aligned}$$

where

$$\lambda_{ij} = (\delta_{ij}^{'}(x_{d_j}) - 1)\tilde{x}_j + \frac{\delta_{ij}^{''}(x_{d_j})}{2}\tilde{x}_j^2 + \ldots \qquad (5.8)$$

On the right-hand side of (5.7), the term λ_{ij} characterizes mostly the higher order terms of the approximation. The impact of the lack of communication is described by the replacement error $\delta_{ij}(x_j) - \delta_{ij}(x_{d_j})$. In the special case where λ_{ij} is small, then the replacement error can be approximated by the tracking error. Hence, the scheme presented in Chapter 4 where communication between subsystems is triggered when the tracking error exceeds a certain threshold works reasonably well, in the sense that it guarantees that exchange of information occurs when needed.

However, if λ_{ij} is not relatively small, then it is possible for exchange of information to occur when not needed, and vice versa, no exchange of information takes place when it may actually be useful.

To better illustrate this, consider an interconnected system composed by two scalar subsystems, described by:

$$\begin{aligned}\dot{x}_1 &= -5x_1 + u_1 + \delta_{12}(x_2) = -5x_1 + u_1 + x_2^4 \\ \dot{x}_2 &= x_2 + u_2 + \delta_{21}(x_1) = x_2 + u_2 + e^{-x_1}cos(x_1)\end{aligned}$$

Let the communication threshold be given by $d_1 = d_2 = 0.5$ and the states of the subsystems at some time $t_c \in \mathbb{R}$ be given by $x_1(t_c) = 6$ and $x_2(t_c) = 2$. Moreover, assume that the desired states of the subsystems at t_c are given by $x_{d_1}(t_c) = 5.5$ and $x_{d_2}(t_c) = 1.5$. Since both subsystems reach the communication threshold at time t_c, i.e., $|\tilde{x}_1(t_c)| = |\tilde{x}_2(t_c)| = 0.5$, both subsystems broadcast state information at time t_c. Although the tracking errors of both subsystems have reached the communication threshold, the impact of lack of communication on the subsystem dynamics is not the same, as illustrated by computing the magnitude of the replacement error:

$$\begin{aligned}|\delta_{12}(x_2(t_c)) - \delta_{12}(x_{d_2}(t_c))| &= 10.9375 \\ |\delta_{21}(x_1(t_c)) - \delta_{21}(x_{d_1}(t_c))| &= 0.000516.\end{aligned}$$

Therefore, a communication algorithm that is based only on local tracking error, does not guarantee that communication occurs when it is really needed. Similar systems, where local tracking-error communication schemes perform poorly, can be found in real-world applications. For example, in the case of a double inverted pendulum connected by a spring, where the interconnections (spring) are given by $\delta_{ij}(x_j) = \alpha_i sin(x_j)$, $\alpha_i > 0$. In this system, the replacement error satisfies $\delta_{ij}(x_j) - \delta_{ij}(x_{d_j}) = 0$ for all $x_j = x_{d_j} + \pi$. This shows that while the tracking error may be large, the replacement error can be negligible.

In order to optimize the benefits from communication, in this chapter we consider a coordinated communication scheme, where communication between interconnected subsystems is coordinated by a higher-level communication coordinator. More specifically, the i-th and j-th subsystems exchange state information, if the tracking errors of both subsystems exceed a certain threshold. If x_j is not available to the i-th subsystem, then its desired state x_{d_j} is used instead. Let d_i denote the communication threshold for the i-th subsystem and define the indicator function C_i as follows

$$C_i(t) = \begin{cases} 0 & \|\tilde{x}_i(t)\|_2 \leqslant d_i \\ 1 & \|\tilde{x}_i(t)\|_2 > d_i, \end{cases}$$

where $\|\tilde{x}_i\|_2 = \sqrt{\tilde{x}_{i1}^2 + \ldots + \tilde{x}_{in_i}^2}$ is the Euclidean norm. The communication coordinator computes a matrix $\mathcal{C}$, defined by

$$\mathcal{C}(t) = \begin{bmatrix} 0 & C_1C_2 & \cdots & \cdots & C_1C_m \\ C_2C_1 & 0 & \cdots & \cdots & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \cdots & \cdots & \cdots & 0 & C_{m-1}C_m \\ C_mC_1 & \cdots & \cdots & C_mC_{m-1} & 0 \end{bmatrix}$$

The communication coordinator receives the indicator function C_i from the i-th subsystem only whenever it has changed, in other words, whenever $\|\tilde{x}_i\|$ crosses the communication threshold d_i. The communication coordinator computes $C_i(t)C_j(t)$ which allows it to decide whether j-th subsystem should start or stop communicating with the i-th subsystem. More specifically in this coordinated communication scheme, the j-th subsystem transmits its state $x_j(t)$ to each subsystem i for which $j \in \mathcal{P}_i$, whenever:

$$C_i(t)C_j(t) = 1 \tag{5.9}$$

The communication coordinator transmits its communication decision to the involved subsystems, only upon change of the decision. Note that $\mathcal{C}$ is symmetrical ($C_i(t)C_j(t) = C_j(t)C_i(t)$), which means that communication between the subsystems is always bidirectional. Fig. 5.1 illustrates the decision logic of the coordinated communication scheme.

The coordinated communication scheme substantially decreases the cost for communication, by avoiding the transmission of information when not needed by the receiver subsystem. More specifically it addresses the case where, for example, the tracking error of the j-th subsystem is large, but the i-th subsystem performs well, and therefore information from the j-th subsystem is not needed. As we will see later, the tradeoff is that only boundedness of the tracking errors to a region around zero can be guaranteed, while in the case of a self-triggering communication scheme (such as the one presented in Chapter 4), asymptotic convergence of the tracking errors to zero is ensured.

Remark 5.1. In the case where a fault occurs in the communication coordinator, which would prevent it from coordinating information exchange between the subsystems, communication falls back to a local tracking-error based scheme. More specifically, whenever the communication coordinator becomes unavailable, each subsystem

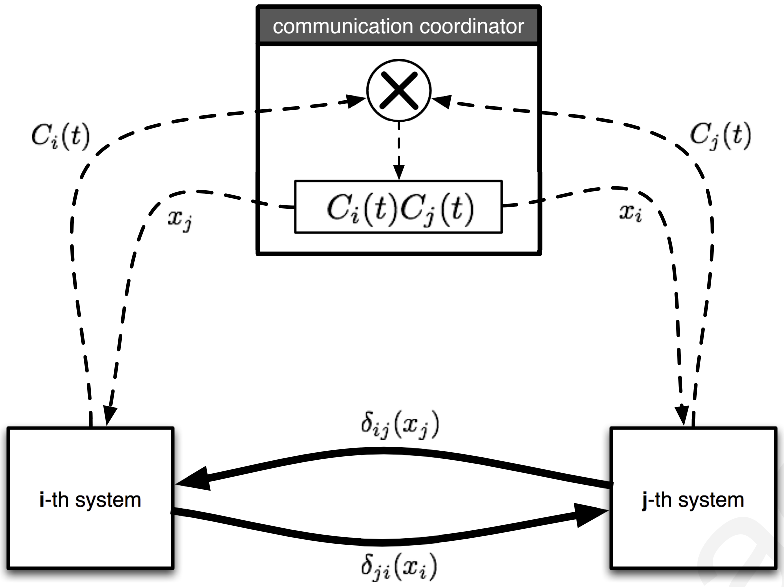


Figure 5.1: Decision logic of coordinated communication scheme.

i sends its state to the j-th subsystem, $j \in \mathcal{P}_i$, whenever $C_i(t) = 1$. Therefore, even in the case of a faulty communication coordinator, the system is able to maintain some performance margins. □

Remark 5.2. The proposed coordinated communication scheme reduces the required communication resources, by addressing the case where the local tracking error $\tilde{x}_j$ is large, but the replacement error, $\delta_{ij}(x_j) - \delta_{ij}(x_{d_j})$, may be small. This is achieved, by allowing information exchange only when both of the tracking errors, $\tilde{x}_j$ and $\tilde{x}_i$ of the j-th and i-th subsystem, exceed certain thresholds. The coordinated communication scheme does not address the case where $\tilde{x}_j$ is small, but the replacement error in the i-th subsystem is large. In order to address this case, the designer needs additional information about the interconnection, more specifically, knowledge of the term λ_{ij}. A methodology for addressing this case is presented in the next chapter. □

5.3 Distributed Fault Tolerant Control Design

Consider the tracking error dynamics, $\tilde{x}_i = x_i - x_{d_i}$, of the i-th subsytem, which, based on (5.3), satisfy

$$\dot{\tilde{x}}_i = A\tilde{x}_i + B\left[f_i(x_i) + g_i(x_i)u_i + \sum_{j=1}^{m}\delta_{ij}(x_j) + \beta(t - T_i)h_i(x) - \dot{x}_{d_{in_i}}\right]. \tag{5.10}$$

Let the distributed fault tolerant control law u_i be given by

$$u_i = u_{N_i} + u_{F_i} \tag{5.11}$$

where u_{N_i} denotes the local nominal control law for addressing the local dynamics of the i-th subsystem, and u_{F_i} is the augmented fault tolerant control law for addressing the unknown interconnections and the change in dynamics due to the occurrence of a fault in the i-th subsystem.

The local nominal control law of the i-th subsystem is defined as

$$u_{N_i} = \frac{1}{g_{N_i}(x_i) + sgn(e_i u_{a_i}) g_{0_i}(x_i)} u_{a_i} \tag{5.12}$$

$$u_{a_i} = -K_i^\top \tilde{x}_i + \dot{x}_{d_{in_i}} - sgn(e_i)(f_{N_i}(x_i) + f_{0_i}(x_i)), \tag{5.13}$$

where the vector $K_i = [k_{i1}, \ldots, k_{in_i}]^\top \in \mathbb{R}^{n_i}$ is chosen such that $A - BK_i^\top$ is a Hurwitz matrix. Since $A - BK_i^\top$ is Hurwitz, for any $Q_i > 0$ there exists $P_i > 0$ satisfying the Lyapunov equation, $P_i(A - BK_i^\top) + (A - BK_i^\top)^\top P_i = -Q_i$. Based on P_i, the scalar tracking error e_i is defined as $e_i \triangleq B^\top P_i \tilde{x}_i$.

In this chapter, we use adaptive approximation methods [19,36] for counteracting the unknown effects of the interconnections δ_{ij} on the subsystems dynamics, and the change in the subsystems dynamics due to faults, h_i. Define the function

$$s_i(x) = \sum_{j=1}^{m} \delta_{ij}(x_j) + \beta(t - T_i) h_i(x). \tag{5.14}$$

Lemma 4.2 shows that there exists an upper bound $\bar{s}_i(x)$ of the function $|s_i(x)|$ which can be represented exactly by an adaptive approximator within a compact set $\mathcal{D}$. More specifically, given a compact set $\mathcal{D} \subset \mathbb{R}^n$, and a set of basis functions $\phi_{s_i}(x) = \left[\phi_{s_{i1}}(x), \ldots, \phi_{s_{iq}}(x)\right]^\top : \mathbb{R}^n \mapsto \mathbb{R}^{q_i}$ such that for all $x \in \mathcal{D}$ at least one basis function is nonzero, then for any bounded function $s_i(x)$ there exists a set of bounded parameters $\theta_{s_i} = \left[\theta_{s_{i1}}, \ldots, \theta_{s_{iq}}\right]^\top \in \mathbb{R}^{q_i}$, such that

$$|s_i(x)| \leqslant \theta_{s_i}^\top \phi_{s_i}(x) = \bar{s}_i(x), \qquad x \in \mathcal{D}.$$

Therefore an upper bound of the unknown function $s_i(x)$ can be represented within a compact set $\mathcal{D} \subset \mathbb{R}^n$ by an approximation model as follows,

$$\bar{s}_i(x) = \theta_{s_i}^\top \phi_{s_i}(x), \tag{5.15}$$

where $\theta_{s_i} \in \mathbb{R}^{q_i}$ is a set of unknown bounded constant parameters and $\phi_{s_i}(x) \in \mathbb{R}^{q_i}$ is a set of basis functions that covers $\mathcal{D}$. The augmented fault tolerant control law u_{F_i}

is defined by

$$u_{F_i} = \frac{1}{g_{N_i}(x_i) + \operatorname{sgn}(e_i u_{c_i}) g_{0_i}(x_i)} u_{c_i} \tag{5.16}$$

$$u_{c_i} = -\operatorname{sgn}(e_i)\hat{\theta}_{s_i}^{\top}\Big(\phi_{s_i}(\bar{x}^i) + M_i \sum_{j=1}^{m} \big(1 - C_i(t)C_j(t)\big) d_j\Big), \tag{5.17}$$

where $\hat{\theta}_{s_i}(t)$ are the parameter estimates of the adaptive approximation model. The vector $\bar{x}^i$ is defined as $\bar{x}^i = [\bar{x}^i_1, \ldots, \bar{x}^i_n] \in \mathbb{R}^n$ where $\bar{x}^i_j$ is given by,

$$\bar{x}^i_j \triangleq \begin{cases} x_j & \text{if } \; C_i(t)C_j(t) = 1 \\ x_{d_j} & \text{if } \; C_i(t)C_j(t) = 0. \end{cases}$$

The vector $M_i = [m_{i1}, \ldots, m_{iq}]^{\top}$ represents the Lipschitz constant for the basis functions such that

$$\big|\phi_{ik}(x) - \phi_{ik}(\bar{x}^i)\big| \leqslant m_{ik}|x - \bar{x}^i|, \quad k = 1, 2, \ldots, q; \quad x \in \mathcal{D} \tag{5.18}$$

The parameter estimates $\hat{\theta}_{s_i}$ are updated according to,

$$\dot{\hat{\theta}}_{s_i} = C_i(t)\Gamma_{s_i}|e_i| \left(\phi_{s_i}(\bar{x}^i) + M_i \sum_{j=1}^{m} \big(1 - C_i(t)C_j(t)\big) d_j\right), \tag{5.19}$$

where Γ_{s_i} is a positive definite matrix representing the adaptation gain of the learning. The adaptation gain Γ_{s_i} affects the transient performance of the overall system. The introduction of the indicator function C_i inside the adaptive law stops the adaptation of the parameter estimates $\hat{\theta}_{s_i}$ when $\|\tilde{x}_i(t)\|_2 \leqslant d_i$. As we will see later, each $\tilde{x}_i$ converges inside the set $\|\tilde{x}_i(t)\|_2 \leqslant d_i$ in finite time, and therefore parameter drift in the presence of measurement noise and disturbances is avoided.

5.4 Stability Analysis

In this section we show that the distributed fault tolerant control scheme guarantees that the states of the subsystems track the reference signals within a small error, and that the tracking error $\tilde{x}_i$ converges within the set $\|\tilde{x}_i(t)\|_2 \leqslant d_i$ in finite time.

Lemma 5.1. *The closed-loop control system described by the interconnected system (5.1), the distributed fault tolerant control law defined by (5.11), (5.12), (5.13), (5.16) and (5.17), and the adaptation law (5.19), guarantee that $\|\tilde{x}_i(t)\|$ is uniformly ultimately bounded by d_i.*

Proof. Let the Lyapunov function for the i-th subsystem be given by $V_i = V_{i1} + V_{12}$ where

$$V_{i1} = \frac{1}{2}\tilde{x}_i^\top P_i \tilde{x}_i,$$
$$V_{i2} = \frac{1}{2}\tilde{\theta}_{s_i}^\top \Gamma_{s_i}^{-1} \tilde{\theta}_{s_i},$$

where $\tilde{\theta}_{s_i} = \hat{\theta}_{s_i} - \theta_{s_i}$ is the parameter estimation error vector. Substituting u_{N_i} and using (5.10) and (5.6), the time derivative of V_i satisfies

$$\dot{V}_{i1} \leqslant \frac{1}{2}\tilde{x}_i^\top \left(A^\top P_i + P_i A\right)\tilde{x}_i + e_i\Bigg[f_i(x_i) + g_i(x_i)u_{F_i} + u_{a_i} + \sum_{j=1}^{m}\delta_{ij}(x_j) + \beta(t-T_i)h_i(x) - \dot{x}_{d_{in_i}}\Bigg].$$

Substituting u_{a_i} and using (5.5) we obtain

$$\dot{V}_{i1} \leqslant -\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i + e_i\Bigg[g_i(x_i)u_{F_i} + \sum_{j=1}^{m}\delta_{ij}(x_j) + \beta(t-T_i)h_i(x)\Bigg].$$

Substituting u_{F_i} and using (5.6) we obtain

$$\dot{V}_{i1} \leqslant -\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i + e_i\Bigg[u_{c_i} + \sum_{j=1}^{m}\delta_{ij}(x_j) + \beta(t-T_i)h_i(x)\Bigg].$$

Substituting u_{c_i} and using (5.14) we obtain

$$\dot{V}_{i1} \leqslant -\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i + |e_i|\Bigg[\hat{\theta}_{s_i}^\top\bigg(\phi_{s_i}(\bar{x}^i) + M_i\sum_{j=1}^{m}\big(1 - C_i(t)C_j(t)\big)d_j\bigg) + |s_i(x)|\Bigg].$$

Based on (5.15),

$$\dot{V}_{i1} \leqslant -\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i + |e_i|\Bigg[-\hat{\theta}_{s_i}^\top\bigg(\phi_{s_i}(\bar{x}^i) + M_i\sum_{j=1}^{m}\big(1 - C_i(t)C_j(t)\big)d_j\bigg) + \theta_{s_i}^\top\phi_{s_i}(x)\Bigg].$$

Adding and subtracting $\theta_{s_i}^\top\phi_{s_i}(\bar{x}^i)$ we obtain

$$\dot{V}_{i1} \leqslant -\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i + |e_i|\Bigg[-\tilde{\theta}_{s_i}^\top\phi_{s_i}(\bar{x}^i) - \hat{\theta}_{s_i}^\top M_i\sum_{j=1}^{m}\big(1 - C_i(t)C_j(t)\big)d_j + \theta_{s_i}^\top\left(\phi_{s_i}(x) - \phi_{s_i}(\bar{x}^i)\right)\Bigg].$$

Using (5.18) and the fact that $|x - \bar{x}^i| \leqslant \sum_{j=1}^{m} d_j$ for all $\|\tilde{x}_i\| > d_i$, the time derivative of V_i for $\|\tilde{x}_i\| > d_i$ satisfies

$$\dot{V}_{i1} \leqslant -\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i + |e_i|\Bigg[-\tilde{\theta}_{s_i}^\top \phi_{s_i}(\bar{x}^i) - \hat{\theta}_{s_i}^\top M_i \sum_{j=1}^{m} \big(1 - C_i(t)C_j(t)\big) d_j + \theta_{s_i}^\top M_i \sum_{j=1}^{m} \big(1 - C_i(t)C_j(t)\big) d_j \Bigg]$$
$$\leqslant -\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i - \tilde{\theta}_{s_i}^\top |e_i| \Bigg(\phi_{s_i}(\bar{x}^i) + M_i \sum_{j=1}^{m} \big(1 - C_i(t)C_j(t)\big) d_j \Bigg).$$

The time derivative for the Lyapunov function of the i-th subsystem, $V_i = V_{i1} + V_{i2}$, satisfies

$$\dot{V}_i \leqslant -\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i + |e_i|\Bigg[-\tilde{\theta}_{s_i}^\top \Bigg(\phi_{s_i}(\bar{x}^i) + M_i \sum_{j=1}^{m} \big(1 - C_i(t)C_j(t)\big) d_j \Bigg) \Bigg] + \tilde{\theta}_{s_i}^\top \Gamma_{s_i}^{-1} \dot{\tilde{\theta}}_{s_i}$$
$$\leqslant -\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i + \tilde{\theta}_{s_i}^\top \Gamma_{s_i}^{-1} \Bigg(\dot{\tilde{\theta}}_{s_i} - \Gamma_{s_i} |e_i| \Bigg(\phi_{s_i}(\bar{x}^i) + M_i \sum_{j=1}^{m} \big(1 - C_i(t)C_j(t)\big) d_j \Bigg) \Bigg).$$

Substituting the adaptive law (5.19), we obtain

$$\dot{V}_i \leqslant -\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i$$

which shows that $\tilde{x}_i(t)$ converges in the set $\mathcal{W}_i = \{\tilde{x}_i \in \mathbb{R}^{n_i} \big| \|\tilde{x}_i\| \leqslant d_i\}$. Additionally, since adaptation of the parameter estimates $\hat{\theta}_{s_i}$ stops for $\|\tilde{x}_i\| \leqslant d_i$, $\tilde{x}_i$, $\hat{\theta}_{s_i} \in \mathcal{L}_\infty$, for all $t > 0$. However, since $\hat{\theta}_{s_i}$ is non-decreasing, (5.19), shows that $\tilde{x}_i(t)$ converges in the set $\mathcal{W}_i$, i.e., there exists a t_{o_i} such that $\tilde{x}_i(t) \in \mathcal{W}_i$, for all $t > t_{o_i}$. □

The above result shows that through the approximation of an upper bound of the unknown function $|s_i(x)|$, instead of $s_i(x)$, the robustness of the feedback control scheme to inherent approximation errors is guaranteed. The tradeoff is that since a function with a larger magnitude is approximated, the control effort may become larger. Moreover, due to the fact that adaptation is stopped inside $\mathcal{W}_i$, parameter drift of the adaptive parameter estimates $\hat{\theta}_{s_i}$ is avoided, even in the presence of measurement noise or disturbances.

Remark 5.3. In addition to the indicator function C_i, define the indicator function C_i^l as

$$C_i^l(t) = \begin{cases} 0 & |\tilde{x}_i(t)\|_2 \leqslant d_i^l \\ 1 & \|\tilde{x}_i(t)\|_2 > d_i^l, \end{cases}$$

where $d_i^l > 0$ is a design constant representing the communication threshold for the i-th receiver subsystem. Consider the case where the j-th subsystem transmits its state $x_j(t)$ to each subsystem i for which $j \in \mathcal{P}_i$, whenever:

$$C_i^l(t)C_j(t) = 1 \tag{5.20}$$

The communication algorithm described by (5.9) is a special case of (5.20), with $d_i^l = d_i$ for $i = 1, \ldots, m$. According to the communication algorithm described by (5.20), the designer is able to reduce the size of the convergence region, by selecting a smaller constant d_i^l. Moreover, the self-triggering tracking-error based communication algorithm presented in Chapter 4 is a special case of (5.20), with $d_i^l = 0$ for $i = 1, \ldots, m$. □

5.5 Simulation Example

In this section, we illustrate the design methodology for the distributed fault tolerant control, using a simple simulation example. Consider the following interconnected system:

$$\begin{aligned}
\Sigma_1: \quad \dot{x}_{11} &= x_{12} \\
\dot{x}_{12} &= x_{11}^2 + x_{12}^3 + (1 + x_{11}^2)u_1 + x_{22}^4 + \beta(t - T_1)h_1(x) \\
\Sigma_2: \quad \dot{x}_{21} &= x_{21} \\
\dot{x}_{22} &= x_{21}^2 + 5x_{22}^3 + (2 + 0.5x_{21}^2)u_2 + e^{x_{11}}\cos(x_{11}) + \beta(t - T_2)h_2(x),
\end{aligned}$$

where $x_i = [x_{i1}, x_{i2}]^\top$ is the state vector of the i-th subsystem ($i = 1, 2$). The matrix P_i satisfying the Lyapunov equation, for $Q = I_{2\times 2}$, is given by,

$$P_i = \begin{bmatrix} 1.5 & 0.5 \\ 0.5 & 1 \end{bmatrix} \qquad i = 1, 2,$$

where $K_1 = K_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}^\top$. The desired trajectory vector $x_{d_i} = [x_{d_{i1}}, x_{d_{i2}}]^\top$ and the signal $\dot{x}_{d_{i2}}$ are generated using a third order filter with a bandwidth of 5 (rad/sec) and unity gain below this frequency. The filter input is chosen as a square wave of zero mean, 1.5 amplitude and a frequency of 0.4 Hz. A lattice of equally spaced radial basis functions are designed for compensating the effects of the unknown interconnections and faults. The communication thresholds d_1 and d_2 are chosen as $d_1 = d_2 = 0.2$. We consider the case in which abrupt faults occur in Σ_1 at $T_1 = 10$ sec and in Σ_2

at $T_2 = 15$ sec. For simulation purposes, the unknown fault functions h_1 and h_2 are chosen as $h_1 = |x_{11}x_{12}| + x_{21}x_{22}$ and $h_2 = 0.4x_{11}^3 + x_{12}$.

In Fig. 5.2 the Euclidean norm of the tracking error vector for each subsystem, $\|\tilde{x}_i\|_2$, is shown. The time occurrences of the fault T_1 and T_2 are also indicated. As depicted by the plot, through the use of adaptive approximation of the bound of the interconnections and fault functions, each subsystem is able to follow the corresponding reference trajectories. For illustration purposes we show only the first 30 sec, however the subsystems converge inside $\|\tilde{x}_i\| \leqslant d_i$ and therefore parameter drift is avoided.

Fig. 5.3 shows the time evolution of the parameter estimates $\hat{\theta}_{s_1}$ and $\hat{\theta}_{s_2}$ for the approximation of the $\bar{s}_1(x)$ and $\bar{s}_2(x)$ functions, respectively. As illustrated by the plot, the change in dynamics due to faults causes the parameter estimates to increase in order to accommodate the faults.

Fig. 5.4 shows the communication cost for the coordination-based communication algorithm, as compared to the self-triggering tracking-error based communication algorithm presented in Chapter 4. According this communication algorithm, the i-th subsystem transmits its state to the other subsystems whenever its local tracking error exceeds d_i. The communication cost for the i-th subsystem is defined as $\int_0^t C_i(\tau)d\tau$. As shown by the plot, a coordination-based communication algorithm results in significant reduction of the amount of information that is exchanged between subsystems. The tradeoff of using a coordinated communication algorithm is that only ultimately boundedness of the tracking errors to a region around zero can be guaranteed, while in the case of the self-triggering tracking-error based communication algorithm, asymptotic stability is ensured.

5.6 Conclusion

This chapter presented a distributed fault tolerant control scheme for a class of interconnected nonlinear uncertain systems. A coordinated communication scheme increases the benefits from communication as compared to a self-triggering tracking-error based communication scheme. The unknown interconnections and fault functions are compensated for, using linearly parameterized approximation models. Robustness to residual approximation errors is guaranteed by approximating upper bounds of the unknown interconnections and fault functions. Through Lyapunov anal-

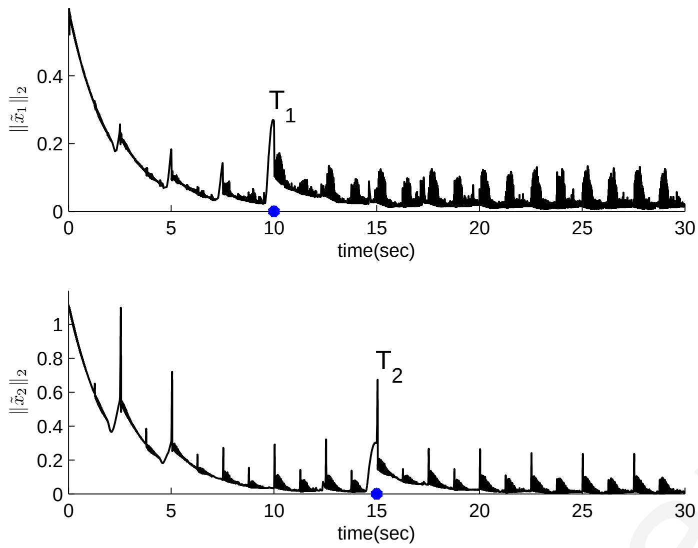


Figure 5.2: Time evolution of the tracking error vector norm $\|\tilde{x}_i\|_2$.

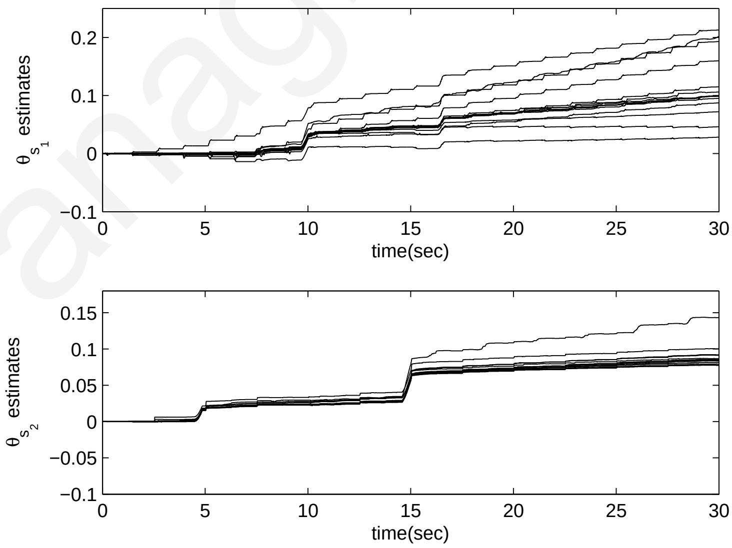


Figure 5.3: Time evolution of the adaptive parameter estimates.

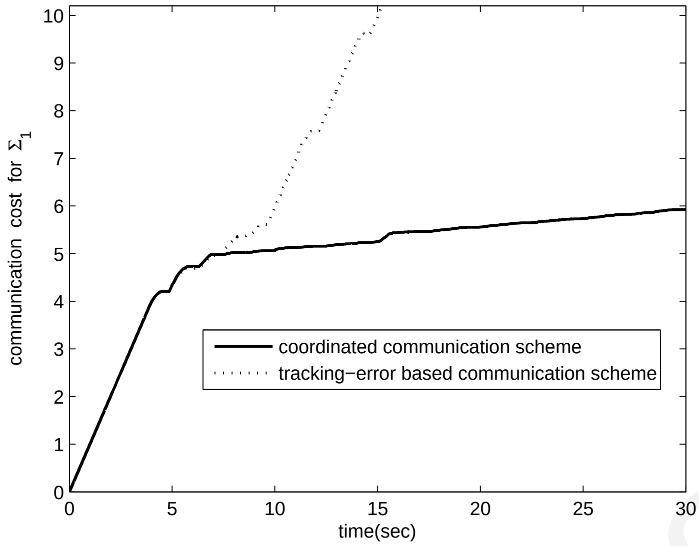


Figure 5.4: Comparison of communication costs for coordinated and self-triggering tracking-error based communication schemes.

ysis, uniform ultimate boundedness of the tracking errors to a small region around zero is shown. Moreover, the stability analysis shows that parameter drift in the presence of residual approximation errors is avoided, and that communication between subsystems stops after a certain time instant.

Chapter 6

An Optimized Communication Scheme for Distributed Fault Tolerant Control

6.1 Introduction

Communication plays a crucial role in the control of interconnected systems. An important characteristic in interconnected systems is that the behavior of each subsystem is correlated not only with the local dynamics, but also with the dynamics of other subsystems. Due to this characteristic, the performance of the system is often limited by communication. All other variables being equal, as the communication rate increases, the performance obtained with a distributed control architecture, tends to the performance obtained by a centralized control architecture. A key challenge is the design of low-energy efficient communication schemes, for improving the performance of the distributed control scheme, while keeping the communication cost constant. Previous work has shown that event-driven state-based communication schemes achieve better results as compared to time-driven schemes, with the same cost for communication. Moreover, in the presence of interconnections with significantly large magnitude, one can not expect satisfactory performance when the subsystems do not exchange information [46]. By allowing the interconnected subsystems to exchange information, it becomes possible to consider large-scale systems with strong nonlinear interconnections, and at the same time improve performance of the system.

Towards optimizing the exchange of information between subsystems, in Chapter 5 we have presented a coordinated communication scheme that reduces the cost for communication by avoiding to broadcast state information when a large uncertainty about the remote states has a relatively small impact on the local subsystem dynamics. This is achieved by utilizing the tracking error of the receiver subsystem in the communication decision. However, the coordinated communication scheme fails to address the case where a relatively small uncertainty about the states of the other subsystems can have a significant impact on the local subsystem dynamics. The goal in this chapter is the design of a communication algorithm for addressing both of these cases. More specifically, the optimized communication decision algorithm presented in this chapter, aims to minimize the replacement error, that arises due to the uncertainty about the states of remote subsystems. The problem of optimizing communication for interconnected systems is formulated as a problem of obtaining the best approximation of a continuous function with step functions. The decision for communication is such that the approximation of the interconnections based on the received samples of the remote states is optimal. A rigorous analysis shows that step functions are universal approximators. In addition, the step function with the best approximation property is derived. Following these results, a distributed fault tolerant control for a class of uncertain interconnected systems is presented, where multiple faults may occur in the interconnections. The decision for communication is based on the use of adaptive approximation models of the unknown interconnections and fault functions. More specifically, when the local tracking error is larger than a certain constant threshold, each subsystem receives information from other subsystems based on a state level-crossing communication scheme. When the local tracking error becomes smaller than this threshold, each subsystem transmits the estimated parameters of the approximation model to the others subsystems, and the broadcast of information is based on an approximation-model level-crossing scheme. It is assumed that an estimate of the remote subsystems' states is available to each subsystem, especially during the learning phase of the unknown coupling dynamics. The presence of inherent residual approximation errors and replacement errors are addressed using a dead-zone modification in the adaptive laws combined with an adaptive bounding method. Through rigorous stability analysis, uniform ultimate boundedness of the tracking errors to a region around zero is proved.

This chapter is organized as follows. In Section 6.2 we formulate the problem and in Section 6.3 we present a mathematical analysis for function approximation based on the use of step functions. Section 6.4 presents the communication scheme design, and Section 6.5 presents the distributed fault tolerant control design. In Section 6.6, we establish the stability of the distributed fault tolerant control scheme through Lyapunov analysis. Simulation results are presented in Section 6.7 and Section 6.8 contains some concluding remarks.

6.2 Problem Formulation

Consider a system described by a collection of m nonlinear uncertain subsystems, where the i-th subsystem is described by:

$$\begin{aligned} \dot{x}_{ik} &= x_{i(k+1)} \qquad k = 1, 2, \ldots, n-1 \\ \dot{x}_{in} &= f_i(x_i) + g_i(x_i)u_i + \sum_{j=1}^{m} \delta_{ij}(x_j) + \beta(t - T_{ij})h_{ij}(x_j) \end{aligned} \tag{6.1}$$

where $x_i \in \mathbb{R}^n$, $u_i \in \mathbb{R}$ is the state and control input of the i-th subsystem respectively, $f_i : \mathbb{R}^n \mapsto \mathbb{R}$ and $g_i : \mathbb{R}^n \mapsto \mathbb{R}$ are functions representing the local dynamics of the i-th subsystem. The term $\delta_{ij} : \mathbb{R}^n \mapsto \mathbb{R}$, $j = 1, \ldots, m,\ j \neq i$, is a continuous function representing the effect of the j-th subsystem onto the i-th subsystem dynamics. In the special case where a certain subsystem j does not affect the i-th subsystem, $\delta_{ij}(x_j) \equiv 0$ for all $x_j \in \mathbb{R}^n$. The term $h_{ij} : \mathbb{R}^n \mapsto \mathbb{R}$ is a continuous function that represents the unknown changes in the δ_{ij} interconnection due to a fault, while the $\beta(t - T_{ij}) : \mathbb{R}^+ \mapsto \mathbb{R}$ functions represent the corresponding time profile of the faults that occur at some unknown times T_{ij}. We consider abrupt faults (where the time profiles satisfy $\beta(t - T_{ij}) = 0$ for $t < T_{ij}$, and $\beta(t - T_{ij}) = 1$ for $t \geqslant T_{ij}$), as well as incipient faults (where $\beta(t - T_{ij}) = 0$ for $t < T_{ij}$ and $\beta(t - T_{ij})$ increases monotonically from 0 to 1 for $t \geqslant T_{ij}$).

Remark 6.1. Assuming known functions of the local state, f_i and g_i, simplifies the analysis and allows us to focus on the main results presented in this chapter. In addition, for notational convenience, we assume subsystems with equal dimensions $\mathbb{R}^n$, as well as single faults in the interconnections. The results presented in this chapter can be extended to the case of unknown f_i and g_i functions, subsystems with arbitrary different dimensions, as well as multiple faults that occur in each interconnection and in the local subsystems dynamics. □

The i-th subsystem described by (6.1) can be written in matrix form as

$$\dot{x}_i = Ax_i + B\left[f_i(x_i) + g_i(x_i)u_i + \sum_{j=1}^{m} \delta_{ij}(x_j) + \beta(t - T_{ij})h_{ij}(x_j) \right] \tag{6.2}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}.$$

We impose the following bounding assumption on the interconnection terms δ_{ij} and fault functions h_{ij}.

Assumption 6.1. *There exist constants $L_{\delta,ij} \in \mathbb{R}^+$ and $L_{h,ij} \in \mathbb{R}^+$ such that*

$$\left|\delta_{ij}(x_j) - \delta_{ij}(\bar{x}_j)\right| \leqslant L_{\delta,ij}\left|x_j - \bar{x}_j\right| \tag{6.3}$$

$$\left|h_{ij}(x_j) - h_{ij}(\bar{x}_j)\right| \leqslant L_{h,ij}\left|x_j - \bar{x}_j\right|, \tag{6.4}$$

for all $x_j, \bar{x}_j \in \mathcal{X}_j \subset \mathbb{R}^n$.

Define the function $\delta_{h,ij}(x_j, t)$ as

$$\delta_{h,ij}(x_j, t) = \delta_{ij}(x_j) + \beta(t - T_{ij})h_{ij}(x_j) \tag{6.5}$$

Using (6.3), (6.4) and the fact that $0 \leqslant \beta(t - T_{ij}) \leqslant 1$, there exist constants $L_{ij} \in \mathbb{R}^+$ such that

$$|\delta_{h,ij}(x_j, t) - \delta_{h,ij}(\bar{x}_j, t)| \leqslant L_{ij}\left|x_j - \bar{x}_j\right|, \tag{6.6}$$

for all $x_j, \bar{x}_j \in \mathcal{X}_j \subset \mathbb{R}^n$ and $t > 0$.

The control objective of this chapter is to develop a distributed fault tolerant control scheme for interconnected subsystems described by (6.1), such that each x_i follows a smooth reference trajectory vector $x_{d_i} = [x_{d_{i1}}, x_{d_{i2}}, \ldots, x_{d_{in}}]^\top$ in the presence of unknown interconnections and faults. Let $\tilde{x}_{ik} = x_{ik} - x_{d_{ik}}$ be the tracking error for the k-th state of the i-th subsystem. The tracking error vector of the i-th subsystem is defined by $\tilde{x}_i = [\tilde{x}_{i1}, \ldots, \tilde{x}_{in}]^\top$. Without loss of generality, it is assumed that $g_i(x_i) > 0$ for all $x_i \in \mathbb{R}^n$, in order to avoid stabilizability problems.

Given that the effect of the j-th subsystem on the i-th subsystem dynamics and the effect of a fault occurring in the interconnection are described in a closed-form and given by the functions δ_{ij} and h_{ij} respectively, then following the universal approximation results [19], given arbitrary $\bar{\mu}_{\delta,ij} > 0$ and $\bar{\mu}_{h,ij} > 0$, there exist sets of bounded constant parameters $\theta^*_{\delta,ij}$, $\theta^*_{h,ij}$ such that δ_{ij} and h_{ij} are represented within a compact set $\mathcal{X}_j \subset \mathbb{R}^n$ as

$$\delta_{ij}(x_j) = \theta^{*\top}_{\delta,ij}\phi_{\delta,ij}(x_j) + \mu_{\delta,ij}(x_j), \quad \bar{\mu}_{\delta,ij} = \sup_{\mathcal{X}_j}|\mu_{\delta,ij}(x_j)| \tag{6.7}$$

$$h_{ij}(x_j) = \theta^{*\top}_{h,ij}\phi_{h,ij}(x_j) + \mu_{h,ij}(x_j), \quad \bar{\mu}_{h,ij} = \sup_{\mathcal{X}_j}|\mu_{h,ij}(x_j)|, \tag{6.8}$$

where $\phi_{\delta,ij}$, $\phi_{h,ij}(x_j)$ are a set of basis functions (such as radial basis functions), $\theta^*_{\delta,ij}$, $\theta^*_{h,ij}$ are a set of unknown constant parameters, and $\mu_{\delta,ij}(x_j)$, $\mu_{h,ij}(x_j)$ are the residual approximation errors of δ_{ij} and h_{ij}, respectively. Consider the fault term $\beta(t - T_{ij})h_{ij}(x_j)$ which based on (6.8) satisfies

$$\beta(t - T_{ij})h_{ij}(x_j) = \theta^{*\top}_{h,ij}\phi_{h,ij}(x_j) + \mu_{\beta h,ij}(x_j, t), \tag{6.9}$$

where $\mu_{\beta h,ij}(x_j, t) = (\beta(t - T_{ij}) - 1)\theta^{*\top}_{h,ij}\phi_{h,ij}(x_j) + \beta(t - T_{ij})\mu_{h,ij}(x_j)$. The unknown uniform upper bound of $\mu_{\beta h,ij}(x_j, t)$, $\bar{\mu}_{\beta h,ij}$, satisfies

$$\bar{\mu}_{\beta h,ij} = \sup_{\mathcal{X}_j, t>0} \max\{\beta(t - T_{ij})\bar{\mu}_{h,ij}, |(1 - \beta(t - T_{ij}))\theta^{*\top}_{h,ij}|\} \tag{6.10}$$

Note that before the occurrence of a fault, the time profile $\beta(t - T_{ij})$ is equal to zero, and (6.9) is identically zero. After the occurrence of a fault the time profile satisfies $0 < \beta(t - T_{ij}) \leqslant 1$, which ensures that $\bar{\mu}_{\beta h,ij}$ defined by (6.10) always exists. Based on (6.7) and (6.9), the $\delta_{h,ij}(x_j, t)$ function is represented within a compact set $\mathcal{X}_j$ as

$$\delta_{h,ij}(x_j, t) = \theta^{*\top}_{ij}\phi_{ij}(x_j) + \mu_{ij}(x_j, t), \quad \bar{\mu}_{ij} = \sup_{\mathcal{X}_j, t>0}|\mu_{ij}(x_j, t)|, \tag{6.11}$$

where ϕ_{ij} is a set of basis functions, θ^*_{ij} is a set of unknown constant parameters, and $\mu_{ij}(x_j, t)$ is the residual approximation error, with an unknown uniform upper bound given by $\bar{\mu}_{ij} = \bar{\mu}_{\delta,ij} + \bar{\mu}_{\beta h,ij}$. The parameter estimates θ^*_{ij} are unknown and are estimated online with parameter estimates $\hat{\theta}_{ij}$. The unknown bounds $\bar{\mu}_{ij}$ are estimated online based on an adaptive bounding method. The adaptive laws for the parameter estimates and the bound estimates are presented later on.

In this chapter, we consider the case where each subsystem j broadcasts its state x_j to the other subsystems i according to a communication algorithm presented later

Figure 6.1: Block diagram of the broadcast of x_j to the i-th subsystem.

on. The communication algorithm $\mathcal{C}_{ij} : \mathbb{R} \to \mathbb{N}$ specifies at each $t > 0$ whether $x_j(t)$ is sampled and broadcasted to the i-th subsystem. Let the release time of the k-th broadcast of x_j be denoted with t_{ij}^k. The communication algorithm $\mathcal{C}_{ij}$ is a mapping of a continuously-defined signal $x_j(t)$, $t \in \mathbb{R}^+$, to a discretely-defined signal $\bar{x}_j^i[k]$, $k \in \mathbb{N}$ given by $\mathcal{C}_{ij}(x_j(t); \mathcal{E}_j) = \sum_{k\in\mathbb{N}} \boldsymbol{\delta}(t - t_{ij}^k) x_j(t)$, where $\boldsymbol{\delta}$ is the Dirac delta function, and $\mathcal{E}_j = \left[\mathcal{E}_j^1, \ldots, \mathcal{E}_j^v\right]^\top$ is a set of events related to the j-th subsystem, where $\mathcal{E}_j^k : \mathbb{R} \to \{0, 1\}$, $k = 1, \ldots, v$. The i-th subsystem receives the samples $\bar{x}_j^i[k]$, $k = 1, 2, \ldots$ and using a zero-order hold $\mathcal{Z}_{ij} : \mathbb{N} \to \mathbb{R}$ produces a continuous signal $\bar{x}_j^i(t)$. Therefore the value of $\bar{x}_j^i(t)$ is equal to the latest broadcasted sample of x_j, i.e., $\bar{x}_j^i(t) = \bar{x}_{ij}[m]$, $m = \underset{k}{\operatorname{argmin}}\left\{t - t_{ij}^k\right\}$. Fig. 6.1 illustrates the schematic diagram for the communication of x_j to the i-th subsystem.

Let the set of release times t_{ij}^k up to a certain $t > 0$ be denoted with $A_{ij}(t)$, i.e., $A_{ij}(t) = \left[t_{ij}^1, \ldots, t_{ij}^{m_{ij}}\right]^\top$, where $m_{ij} = \underset{k}{\operatorname{argmax}}\left\{t - t_{ij}^k \geqslant 0\right\}$. The cardinality of $A_{ij}(t)$, $|A_{ij}(t)|$, represents the number of broadcasts for a given time period. Define the cost function J as

$$
\begin{aligned}
J &= \sum_{i=1}^{m} J_{Q_i} + J_{W_i} + \sum_{j=1}^{m} J_{S_{ij}}, \\
J_{Q_i} &= \int_0^\infty \tilde{x}_i(\tau)^\top Q_i \tilde{x}_i(\tau) d\tau \\
J_{W_i} &= \int_0^\infty u_i(\tau) W_i u_i(\tau) d\tau \\
J_{S_{ij}} &= \int_0^\infty |A_{ij}(\tau)| S_{ij} |A_{ij}(\tau)| d\tau,
\end{aligned}
$$

where $Q_i : \mathbb{R}^n \mapsto \mathbb{R}$ is a positive definite matrix, and W_i, S_{ij} are positive constants. The terms J_{Q_i}, J_{W_i} and $J_{S_{ij}}$ represent the performance, control effort and communication cost respectively. Among all the possible sequences of release times $t_{ij}^1, t_{ij}^2, \ldots$, the optimal communication algorithm is realizing that one sequence that minimizes the cost function J. Now consider the case of time-driven communication algorithms, where x_j is broadcasted periodically with some period $T_{ij} > 0$. As the broadcasting period T_{ij} decreases, it is expected that the magnitude of either or both J_{Q_i} and J_{W_i} is decreased, leading to a smaller tracking error and control effort, respectively.

However at the same time, as the rate of communication increases, the magnitude of $J_{S_{ij}}$ increases. Therefore by increasing the communication rate it is not guaranteed that J becomes smaller, and it may actually increase. Therefore, it is important that the communication algorithm adapts to the requirements of the system, broadcasting information as needed.

Towards this direction, a promising approach is based on level-crossing sampling [4, 5, 34, 44]. According to level-crossing sampling, the communication decision is a signal-dependent algorithm: a broadcast is released only when some monitored signal exceeds a certain threshold. A common approach is to use the local state as a monitoring signal (state level-crossing communication). In this case, the j-th subsystem broadcasts its state x_j at time t, whenever $|x_j(t) - \bar{x}^i_j(t)| = d_{ij}$, where $d_{ij} > 0$ is a design constant representing the communication threshold. Note that level-crossing sampling exhibits hysteresis; the next broadcast depends not only on the current value of $x_j(t)$, but also on the most recent broadcasted value $\bar{x}^i_j(t)$. Level-crossing sampling ensures that neighboring broadcasting values of x_j are always an d_{ij} distance apart.

Remark 6.2. The benefit of level-crossing over time-driven communication is two-fold: (a) It reduces the required mean rate of broadcasts. In other words, in order to meet certain performance and stability properties, a level-crossing algorithm requires a smaller $|A_{ij}(t)|$ as compared to a time-driven communication algorithm [5]. Intuitively, less samples are required to extract the same amount of information from a slowly varying signal, compared to a fast-varying signal. Level-crossing communication broadcasts information only when the monitored signal has changed enough, therefore avoiding oversampling. (b) It improves the properties of the closed-loop system in the presence of disturbances or other unmodeled dynamics. In such cases, the level-crossing communication algorithm increases the broadcast rate to compensate for the greater uncertainty, while a time-driven communication algorithm may lead the system to performance degradation or even instability. □

Prior work in networked control systems has demonstrated the effectiveness of level-crossing sampling based schemes and their superiority over time-driven schemes (see, for example, [81] and [15]). In networked control systems the communication is taking place between sensors and actuators. The goal in such schemes is to design communication algorithms for reliably controlling a system remotely. Therefore, a communication algorithm is evaluated based on how well the networked control system performs, in terms of stability properties, control effort, tracking performance, etc. Indeed, this is also the case with dynamically interconnected systems; the criterion for evaluating a communication algorithm should be the performance of the interconnected system. But note that in the case of interconnected systems, this does not happen in a direct way. The performance of the interconnected system will improve, if the communication algorithm allows the subsystems to more efficiently compensate for the effects of the interconnections. Therefore, optimization of communication in interconnected systems, indirectly reduces either or both of the cost functions J_{Q_i} (tracking-error performance) and J_{W_i} (control effort).

Previous work in distributed control considers the case where the decision for communication is based on the local state. Typically in such schemes, each subsystem broadcasts its state according to a state level-crossing scheme for all $t > 0$, [48,82,86]. Although this approach works well for linear interconnections (or weak nonlinear interconnections), in the case of complex interconnections, with higher degrees of nonlinearities, it leads to suboptimal results. The following subsection illustrates this phenomenon through an example.

6.2.1 Motivating example

Consider scalar interconnected subsystems described by (6.1), known analytic interconnections δ_{ij}, $h_{ij} \equiv 0$, and for simplicity $g_i(x_i) = 1$. Consider the control law $u_i = u_{N_i} + u_{F_i}$, where u_{N_i} is the local control component that stabilizes the i-th isolated subsystem, and is given by

$$u_{N_i} = -K_i\tilde{x}_i - f_i(x_i) + x_{d_i}, \tag{6.12}$$

where $K_i > 0$, and u_{F_i} is the control component for addressing the presence of the interconnection effects, and given by

$$u_{F_i} = -\sum_{j=1}^{m} \delta_{ij}(\bar{x}_j^i) - d_{ij} \tag{6.13}$$

Consider the case where the j-th subsystem broadcasts its state whenever $|x_j - \bar{x}_j^i| > d_{ij}$. Let the Lyapunov function for the i-th subsystem be given by $V_i = \frac{1}{2}\tilde{x}_i^2$. Substituting u_{N_i} from (6.12), the time derivative of V_i satisfies

$$\dot{V}_i = -K_i\tilde{x}_i^2 + g_i(x_i)u_{\delta_i} + \sum_{j=1}^{m} \delta_{ij}(x_j)$$

Substituting u_{F_i} from (6.13) we obtain

$$\dot{V}_i = -K_i\tilde{x}_i^2 + \sum_{j=1}^{m} \delta_{ij}(x_j) - \delta_{ij}(\bar{x}_j^i) - d_{ij} \tag{6.14}$$

Since $\delta_{ij}(x_j)$ is analytic it can be represented by a Taylor series around $\bar{x}_j^i$. With a similar analysis as in Section 5.2, we obtain that the replacement error $\delta_{ij}(x_j)-\delta_{ij}(\bar{x}_j^i)$ satisfies

$$\delta_{ij}(x_j) - \delta_{ij}(\bar{x}_j^i) = (x_j - \bar{x}_j^i) + \lambda_{ij}, \tag{6.15}$$

where the term λ_{ij} characterizes the higher order terms of the Taylor series-based approximation. Based on (6.15), the time derivative of V_i satisfies

$$\dot{V}_i \leqslant -K_i\tilde{x}_i^2 + \sum_{j=1}^{m} |x_j - \bar{x}_j^i| + |\lambda_{ij}| - d_{ij}$$

Since $|x_j - \bar{x}_j^i| \leqslant d_{ij}$ we obtain that

$$\dot{V}_i \leqslant -K_i\tilde{x}_i^2 + \sum_{j=1}^{m} |\lambda_{ij}|,$$

which shows that $\dot{V}_i \leqslant 0$ in the set $\Lambda_i = \left\{x_i \middle| \, |\tilde{x}_i| \leqslant \sqrt{\frac{\sum_{j=1}^{m}|\lambda_{ij}|}{K_i}}\right\}$. In the case of linear interconnections, $\lambda_{ij} \triangleq 0$, and $\tilde{x}_i = 0$ is asymptotically stable. However, in the general case of nonlinear interconnections, the presence of nonlinear terms λ_{ij} with significantly large magnitude, the set Λ_i can become quite large. In the case where the nonlinear interconnection δ_{ij} satisfies a Lipschitz condition, $|\delta_{ij}(x_j)-\delta_{ij}(\bar{x}_j^i)| \leqslant L_{ij}|x_j-\bar{x}_j^i|$, the $|\delta_{ij}(x_j) - \delta_{ij}(\bar{x}_j^i)|$ term in (6.14) can be canceled with a term $\bar{L}_{ij}d_{ij}$, where $\bar{L}_{ij}$ is a design constant satisfying $\bar{L}_{ij} > L_{ij}$. Therefore in the case of Lipschitz interconnection functions, a state-based communication scheme can still guarantee the stability of the system in the presence of unknown nonlinear interconnections. However, analogously to time-driven communication schemes used in linearly interconnected systems [5], using state-based level-crossing communication schemes on nonlinearly interconnected subsystems results to suboptimal utilization of the available bandwidth. Fig. 6.2 shows an example of a nonlinear interconnection function δ_{ij}. Due to the nonlinearity of δ_{ij}, the distance between the samples at $x_j = 1$ and $x_j = 0$ is not representative of the magnitude of $\delta_{ij}(1) - \delta_{ij}(0)$. As we will see in later sections, by basing the communication decision on the interconnection function δ_{ij}, instead of the state x_j, we are able to optimize the performance benefits from communication. In the next section we investigate the problem of approximating a function based on the use of step functions. These results will be used for optimally representing the interconnection function δ_{ij}, based on the use of broadcasted samples of x_j.

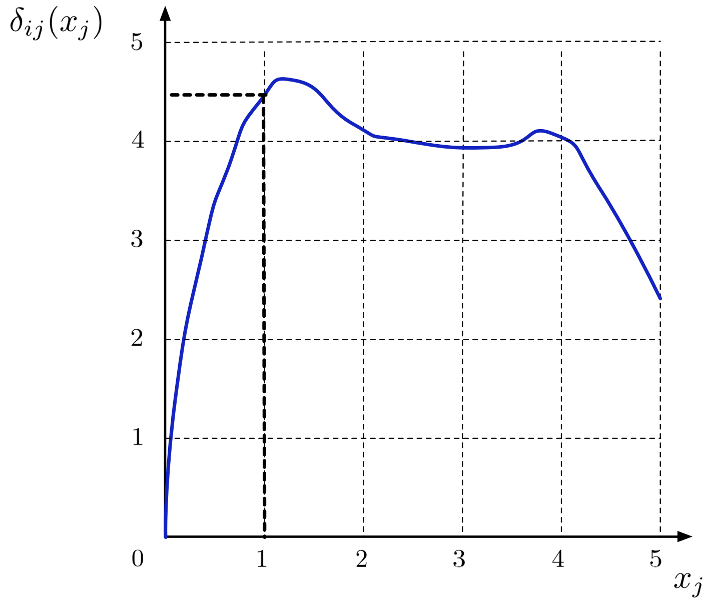


Figure 6.2: State level-crossing sampling of a nonlinear interconnection function.

6.3 Function Approximation with Step Functions

Consider the approximation of the unknown function $\delta_{h,ij}$ within a compact set $\mathcal{X}_j$, based on an approximation model $\delta^*_{h,ij}$ given by $\delta^*_{h,ij}(x_j;\theta_{ij}) = \theta^{*\top}_{ij}\phi_{ij}(x_j)$. The θ^*_{ij} parameters are in general unknown and estimated online using adaptive parameter estimates $\hat{\theta}_{ij}$. In order to guarantee that the parameter estimates $\hat{\theta}_{ij}$ converge to their optimal values θ^*_{ij}, the input training signal needs to satisfy a persistency of excitation condition [30]. However, even if $\hat{\theta}_{ij}$ converge to the optimal parameters θ^*_{ij}, and the approximation model $\theta^*_{ij}\phi_{ij}(x_j)$ is obtained, due to the fact that x_j is not available at all times, it is not possible for the i-th subsystem to realize $\theta^{*T}_{ij}\phi_{ij}(x_j)$. The only information available to the i-th subsystem about x_j between broadcasts is the latest broadcast of x_j. Therefore, the best that the i-th subsystem can do is approximate $\delta^*_{h,ij}$ with a constant value for $t \in [t^k_{ij}, t^{k+1}_{ij})$. Consider the sequence of broadcasts $t^1_{ij}, \ldots, t^k_{ij}$ for some $0 < t^k_{ij} < \infty$. Since $\delta_{h,ij}$ is continuous, it follows that $\delta_{ij}(x_j)$ is uniformly bounded for all $x_j \in \mathcal{X}_j$ and $t > 0$. In other words, there exists an $M_{ij} \in \mathbb{R}^+$ such that $|\delta_{h,ij}(x_j,t)| \leqslant M_{ij}$. Consider the case where $\delta^*_{h,ij}(x_j;\theta_{ij})$ is approximated with a constant value $b^l_{ij} \in \mathbb{R}$ for $t \in [t^l_{ij},\ t^{l+1}_{ij})$. By the fact that the number of approximations are finite and equal to k, and the fact that since $\delta^*_{h,ij}$ is bounded, the b^l_{ij} constant approximations are real numbers, it follows that the approximation of $\delta^*_{h,ij}$ by b^l_{ij} constants is a step function and given by

$$\bar{\delta}^*_{h,ij}(x_j) = \sum_{l=1}^{k} b^l_{ij}\chi_{t\in[t^k_{ij},t^{k+1}_{ij})}(|x_j(t)|),\ 0 < t < t^{k+1}_{ij}, \tag{6.16}$$

where $\chi_{\mathcal{S}} : \mathbb{R} \to \mathbb{R}$ is the characteristic function defined by

$$\chi_{\mathcal{S}}(x) = \begin{cases} 1 & \text{if } x \in \mathcal{S} \\ 0 & \text{if } x \notin \mathcal{S}. \end{cases}$$

Note however that as $t^k_{ij} \to \infty$, $\bar{\delta}^*_{h,ij}(x_j)$ can in general take infinitely many values, such that it is no longer a step function. Now we ask the question, under what conditions $\bar{\delta}^*_{h,ij}(x_j)$ can be represented with a step function for all $t > 0$. Define the partition of $\mathcal{X}_j$, $\mathcal{P}^{(\mathcal{X}_j,i)}_{\lambda}$ as the collection of sets $P^{(\mathcal{X}_j,i,\lambda)}_1, \ \ldots, \ P^{(\mathcal{X}_j,i,\lambda)}_{\lambda}$ that form an exact cover of $\mathcal{X}_j$, i.e., $\bigcup_{m=1}^{\lambda} P^{(\mathcal{X}_j,i,\lambda)}_m \triangleq \mathcal{X}_j$ and $P^{(\mathcal{X}_j,i,\lambda)}_{m_1} \bigcap P^{(\mathcal{X}_j,i,\lambda)}_{m_2} = 0$, for any $m_1, \ m_2 \in [1, \lambda]$. In others words, for any $x_j \in \mathcal{X}_j$ there is exactly one $m \in [1, \lambda]$ such that $x_j \in P^{(\mathcal{X}_j,i,\lambda)}_m$. If λ is finite then $\bar{\delta}^*_{h,ij}$ can be represented by a step function as follows

$$\bar{\delta}^*_{h,ij}(x_j) = \sum_{m=1}^{\lambda} a^m_{ij}\chi_{P^{(\mathcal{X}_j,i,\lambda)}_m}(|x_j|). \tag{6.17}$$

Therefore by limiting the number of distinct values that $\bar{\delta}^*_{ij}(x_j(t))$ can take for all $t > 0$ we are able to express $\bar{\delta}^*_{ij}(x_j)$ as a step function. Note that even if (6.17) holds, (6.16) is still not a step function for $t^k_{ij} \to \infty$, since it can take an infinite number of intervals. The (6.17) representation provides the basis for approximating the unknown interconnection function δ^*_{ij} at the i-th subsystem. The communication algorithm $\mathcal{C}_{ij}$ provides the information $x_j \in P^{(\mathcal{X}_j,i,\lambda)}_m$, for some $m = 1, \ldots, \lambda$, and the i-th subsystem approximates $\delta^*_{h,ij}(x_j)$ with an a^m_{ij} constant, until the next broadcast. By limiting the number of intervals λ, we ensure that the number of broadcasts does not become arbitrary large. The following theorem shows that any continuous function can be approximated with a step function to any desired accuracy within a compact set.

Theorem 6.1. *Consider a continuous function $f : \mathcal{X} \subset \mathbb{R} \to \mathbb{R}$, where $\mathcal{X} = [a, b]$. Given an $\epsilon > 0$, there exists a step function $s : \mathcal{X} \subset \mathbb{R} \to \mathbb{R}$, such that $\sup_{\mathcal{X}}|s - f| < \epsilon$ (I). Moreover, there exists a sequence of step function $s_{(n)}$, $n = 0, \ 1, \ldots$ such that $|s_{(n)} - f| \to 0$ as $n \to \infty$ (II).*

Proof. Since $f(x)$ is continuous, both the left and right limit of $f(x)$, $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$, exist for any $a \in \mathcal{X}$. Therefore $f(x)$ is a regulated function which shows that (I) and (II) are satisfied [13].

A different proof which can provide more insight is based on the uniform continuity of f. Since f is continuous on the compact set $\mathcal{X}$, then based on the Heine - Cantor theorem, [8], it is uniformly continuous on $\mathcal{X}$. Therefore given an $\epsilon > 0$ we can find a $\delta = \delta(\epsilon) > 0$ such that, for every $x,\ y \in \mathcal{X}$ with $|x - y| < \delta$ we have that $|f(x) - f(y)| < \epsilon$. Pick $\lambda \in \mathbb{N}$ such that $h = \frac{b-a}{\lambda} < \delta$, where $a < b,\ a, b \in \mathbb{R}$ are the endpoints of $\mathcal{X}$, and let $x_k = a + (k-1)h$, $k = 1,\ \ldots, \lambda$. Define the step function s as follows

$$s(x) = \sum_{m=1}^{k} f(x_m)\chi_{P_m^{(\mathcal{X},\lambda)}},$$

so that on each interval $P_m^{(\mathcal{X},\lambda)} = [x_m,\ x_{m+1})$, s is constantly equal to the value of f at the left endpoint of $P_m^{(\mathcal{X},\lambda)}$, and at $x = b$, $s(b) = f(b)$. For each $x \in \mathcal{X}$ we have that $x \in [x_j,\ x_{j+1})$ for some unique j, and therefore

$$|f(x) - s(x)| = |f(x) - f(x_j)| < \epsilon,$$

since $|x - x_j| < \delta$ which shows that (I) is satisfied. In order to prove (II), consider the sequence of partitions $P_j^{(\mathcal{X},2)},\ P_j^{(\mathcal{X},2^2)}, \ldots,\ P_j^{(\mathcal{X},2^n)}$. In other words, at stage n, $\mathcal{X}$ is divided into $\lambda = 2^n$ partitions. The step function for the n-th partition is given by

$$s_{(n)}(x) = \sum_{m=1}^{2^n} f(x_m)\chi_{P_j^{(\mathcal{X},2^n)}},$$

We want to show that $\lim_{n\to\infty} |f(x) - s_{(n)}(x)| = 0$. Let $x \in P_j^{(\mathcal{X},2^n)}$ for some $j = 1, \ldots, 2^n$, then for all $n > 0$ such that $\frac{b-a}{2^n} < \delta$,

$$\lim_{n\to\infty} |f(x) - s_{(n)}(x)| = \lim_{n\to\infty} |f(x) - f(x_j)|.$$

In addition, since $\lim_{n\to\infty} \frac{b-a}{2^n} = 0$ and $|x - x_j| < \frac{b-a}{2^n}$, it follows that $\lim_{n\to\infty} |x - x_j| = 0$. By the uniform continuity of f on X, it follows that $\lim_{n\to\infty} |f(x) - s_{(n)}(x)| = 0$, [21], which completes the proof. □

Theorem 6.1 guarantees the existence of a step function that can approximate any continuous function within a compact set, to an arbitrary accuracy. Additionally it shows that by increasing the number of intervals we can increase the approximation accuracy. Note however, that there exist infinite possible partitions of $\mathcal{X}$. Theorem 6.1 does not deal with the best approximation that we can achieve with a certain step function. Intuitively, the partitions should be such that f varies slowly within any interval, such that a constant value can approximate it well. Moreover, increasing the number of intervals of the partition, in general increases the frequency of broadcasts. This is due to the fact that information needs to be broadcasted whenever an interval boundary is crossed. As we want to exploit the available communication resources to their fullest, we are interested in designing step-function approximation models that approximate the unknown interconnection functions as closely as possible with the minimum number of intervals. We are therefore interested to answer the following questions:

1. Given a continuous function f and a $\lambda > 0$, such that $\mathcal{X}$ is partitioned to at most λ intervals, design a step function $s_\lambda(x)$ such that the $\mathcal{L}_\infty$ approximation error is minimized. What approximation error $\epsilon^* > 0$ can be obtained by the best step-function based approximator?

2. (*Inverse Problem*) Given an ϵ^* and a continuous function f defined on a compact set, what is the required number of partitions λ, such that the approximation of f with a step function s_λ satisfies $\sup\limits_{\mathcal{X}}|f(x) - s_\lambda| = \epsilon^*$?

According to the first question, we want to ensure that we use a step-function based approximator with the best approximation property [19]. Answering the second question will provide us with a constructive method for designing step functions that achieve a desired accuracy.

Theorem 6.2. *Given a $\lambda > 0$, the best step-function based approximation s^*_λ of a continuous function $f : \mathcal{X} = [a,\ b] \to \mathbb{R}$, in the $\mathcal{L}_\infty$ sense, is given by*

$$f(x) = s^*_\lambda(x) + \epsilon(x), \quad \epsilon^* = \sup_{\mathcal{X}}|\epsilon(x)| \tag{6.18}$$

where

$$s^*_\lambda(x) = \sum_{m=1}^{\lambda} a^*_m \chi_{P_m^{(\mathcal{X},\lambda)*}}, \tag{6.19}$$

$\epsilon(x)$ is the minimum functional approximation error, a^*_m are the optimal constant parameters defined by

$$a^*_m = \operatorname*{argmin}_{a_m \in \mathbb{R}} \left\{ \sup_{\mathcal{X}} \big| f(x) - s_\lambda(x) \big| \right\},$$

and given by

$$a^*_m = \frac{\inf\limits_{P_m^{(\mathcal{X},\lambda)*}} (f(x)) + \sup\limits_{P_m^{(\mathcal{X},\lambda)*}} (f(x))}{2}, \tag{6.20}$$

and $P_m^{(\mathcal{X},\lambda)*}$ *are the optimal partitions defined by*

$$P_m^{(\mathcal{X},\lambda)*} = \underset{P_m^{(\mathcal{X},\lambda)} \in \mathcal{P}_\lambda^{(\mathcal{X})}}{\operatorname{argmin}} \left\{ \sup_{\mathcal{X}} \left| f(x) - s_\lambda(x) \right| \right\},$$

and satisfying

$$\sup_{P_m^{(\mathcal{X},\lambda)*}} f(x) - \inf_{P_m^{(\mathcal{X},\lambda)*}} f(x) = c, \quad \forall m = 1, \ \ldots, \ \lambda, \tag{6.21}$$

where $c > 0$ *is a constant such that for* $x \in [a, b)$*, the equations*

$$\begin{aligned} f(x) &= f(a) + \left\lfloor \frac{\sup f(x) - f(a)}{c} \right\rfloor c \\ &\vdots \\ f(x) &= f(a) + c \\ f(x) &= f(a) \\ f(x) &= f(a) - c \\ &\vdots \\ f(x) &= f(a) - \left\lfloor \frac{|\inf f(x) - f(a)|}{c} \right\rfloor c \end{aligned} \tag{6.22}$$

have exactly $\lambda - 1$ *solutions. The approximation error is then given by* $\epsilon^* = \frac{c}{2}$.

In other words, the optimal constant value a_m^* is the midway value between the extremes of $f(x)$ in $P_m^{(\mathcal{X},\lambda)*}$, and the optimal partition of $\mathcal{X}$, as defined by the intervals $P_m^{(\mathcal{X},\lambda)*}$, are such that the range of f is divided into equally spaced regions.

Proof. Since $\epsilon^* = \sup_X |\epsilon(x)|$, we have that for all $k = 1, \ldots, \lambda$, $\sup_{P_k^{(\mathcal{X},\lambda)*}} |\epsilon(x)| \leqslant \epsilon^*$ and for at least one $m = 1, \ldots, \lambda$, $\sup_{P_m^{(\mathcal{X},\lambda)*}} |\epsilon(x)| = \epsilon^*$. Therefore, since for all $x \in P_k^{(\mathcal{X},\lambda)*}$, $s_\lambda(x) = a_m^*$ the optimal parameter a_m^* is defined by

$$a_m^* = \underset{a_m \in \mathbb{R}}{\operatorname{argmin}} \left\{ \sup_{P_m^{(\mathcal{X},\lambda)*}} \left| f(x) - a_m \right| \right\}.$$

First we note that the optimal parameter a_m^* satisfies $a_m^* \in [\inf f(x), \sup f(x)]$. Therefore a_m^* can be written as $a_m^* = \inf f(x) + k^*(\sup f(x) - \inf f(x))$, where $k^* \in [0, 1]$ is defined by

$$k^* = \underset{k \in [0,1]}{\operatorname{argmin}} \left\{ \sup_{P_m^{(\mathcal{X},\lambda)*}} \left| f(x) - \left[\inf f(x) + k^* \left(\sup f(x) - \inf f(x) \right) \right] \right| \right\}. \tag{6.23}$$

Observing that $\left| f(x) - \left[\inf f(x) + k^* \big(\sup f(x) - \inf f(x) \big) \right] \right|$ is maximized for either $\sup\limits_{P_m^{(\mathcal{X},\lambda)*}} f(x)$ or $\inf\limits_{P_m^{(\mathcal{X},\lambda)*}} f(x)$, (6.23) can be written as

$$
\begin{aligned}
k^* &= \underset{k \in [0,1]}{\operatorname{argmin}} \left\{ \underset{k \in [0,1]}{\operatorname{argmax}} \left\{ k^* \big(\sup f(x) - \inf f(x) \big), \big(1 - k^*\big) \big(\sup f(x) - \inf f(x) \big) \right\} \right\}. \\
&= \underset{k \in [0,1]}{\operatorname{argmin}} \left\{ \underset{k \in [0,1]}{\operatorname{argmax}} \left\{ k^*, \big(1 - k^*\big) \right\} \right\},
\end{aligned}
$$

which shows that $k^* = \frac{1}{2}$, and the optimal parameters a_m^* are given by (6.20).

We now want to test if there exists partitions that achieve an approximation error smaller than $\frac{c}{2}$. Consider the case where for some $k = 1, \ldots, \lambda$, $\sup\limits_{P_k^{(\mathcal{X},\lambda)**}} f(x) - \inf\limits_{P_k^{(\mathcal{X},\lambda)**}} f(x) = b < c$. In order for the $\mathcal{P}_\lambda^{(\mathcal{X})}$ to form an exact cover, then for some other $l = 1, \ldots, \lambda$, $l \neq k$, $\sup\limits_{P_l^{(\mathcal{X},\lambda)**}} f(x) - \inf\limits_{P_l^{(\mathcal{X},\lambda)**}} (f(x)) = d > c$, and therefore $\epsilon^{**} = \frac{d}{2} > \epsilon^*$, which shows that partitions satisfying (6.21) are optimal. Moreover a decreased number of partitions, $\mu < \lambda$ would lead to a larger approximation error. This is justified by the fact that (6.22) will need to have fewer solutions (μ), and therefore c needs to increase, which shows that $\epsilon^* = \frac{c}{2}$ will increase. Note that if (6.22) has more than λ solutions for all $0 < c < \frac{\sup f(x) - \inf f(x)}{2}$, then λ needs to increase. □

Theorem 6.3. *(Inverse Problem) Given an $\epsilon^* > 0$ and a continuous function $f : [a,\ b] \to \mathbb{R}$, the required $\lambda > 0$ such that $\sup |f(x) - s_\lambda| = \epsilon^*$, is given by the number of solutions to the equations*

$$
\begin{aligned}
f(x) &= f(a) + 2 \left\lfloor \frac{\sup f(x) - f(a)}{2\epsilon^*} \right\rfloor \epsilon^* \\
&\ \ \vdots \\
f(x) &= f(a) + 2\epsilon^* \\
f(x) &= f(a) \\
f(x) &= f(a) - 2\epsilon^* \\
&\ \ \vdots \\
f(x) &= f(a) - 2 \left\lfloor \frac{|\inf f(x) - f(a)|}{2\epsilon^*} \right\rfloor \epsilon^*,
\end{aligned}
\tag{6.24}
$$

for all $x \in [a, b)$.

Proof. Since $\epsilon^* = \frac{c}{2}$ and using (6.22), the number of solutions to (6.24) is equal to the required λ. Note that λ is bounded from below by $\frac{\sup f(x) - \inf f(x)}{2\epsilon^*}$, which occurs in the case of a monotonic function $f(x)$ for $x \in [a, b]$. □

In order to gain more insight in the design of the best step-function approximation, given by (6.19), consider the set

$$\mathcal{S} =\Big\{ f(a) - \left\lfloor \frac{|\inf f(x) - f(a)|}{c} \right\rfloor c, \ldots, f(a) - c, f(a), f(a) + c, \\ \ldots, f(a) + f(a) + \left\lfloor \frac{\sup f(x) - f(a)}{c} \right\rfloor c \Big\}.$$

Define the preimage of $\mathcal{S}$ as the set of all elements of $\mathcal{X}$ that map to $\mathcal{S}$

$$f^{-1}(\mathcal{S}) = \{x \in \mathcal{X} | f(x) \in \mathcal{S}\}$$

and define the ordered sequence $\check{x}_1, \ldots, \check{x}_{\lambda+1}$, $f(\check{x}_m) \in \mathcal{S}$, $m = 1, \ldots, \lambda + 1$. Then the optimal partition of $\mathcal{X}$ is defined by $P_m^{(\mathcal{X},\lambda)^*} = \{x | \check{x}_m \leqslant x < \check{x}_{m+1}\}$. Additionally we can define the optimal partition of $\mathcal{X}$ by considering partial inverses of f. Define a partition of $\mathcal{X}$ into intervals $[\mathcal{Q}_1^{\mathcal{X}}, \ldots, \mathcal{Q}_r^{\mathcal{X}}]$, such that $f_{\mathcal{Q}_l}^{-1}(f(x)) = x$ for all $l \in [1, r]$. In other words, we partition the domain of $f(x)$ such that the inverse of f, f^{-1}, is defined within each interval. The $P^{(\mathcal{X},\lambda)^*}$ intervals is constructed by finding all $x \in \mathcal{X}$ that satisfy $f_{\mathcal{Q}_l}^{-1}(kc) = x$, for some $k = 0, 1, \ldots$, within each $\mathcal{Q}_l$.

As the desired approximation error ϵ^* is reduced, the required number of intervals (λ) is increased. Since a greater number of intervals in general require more frequent broadcasts, the selection of the appropriate approximation accuracy, and thus the number of partition intervals, involves a tradeoff between communication cost and performance.

In Subsection 6.2.1 we have considered a simple system for illustrating that a state based communication scheme leads to suboptimal results in the case of nonlinear interconnections. Through Lyapunov analysis we have shown that the region of convergence increases as the nonlinearity of the interconnection becomes larger. In the next subsection, by considering a class of piecewise linear functions, we compare regular partitioning step function-based approximation (where the domain of the function is partitioned into equal length intervals), with the best, range partitioning, step function-based approximation given by Theorem 6.2.

6.3.1 Comparison of range partitioning with regular partitioning step approximation

Consider the regular partition of $\mathcal{X}$, $\mathcal{P}_c^{(\mathcal{X})}$ where the m-th interval is defined by $P_m^{(\mathcal{X},c)} = \Big\{ x \big| x \in [(m-1)c, mc] \Big\}$, and $c > 0$ is the length of the intervals. The regular-partitioning step approximation of a function $f : \mathbb{R} \to \mathbb{R}$ is given by

$$f(x) = s_c^d(x) + \epsilon(x), \quad |\epsilon(x)| \leqslant \epsilon^{d*} \tag{6.25}$$

where $s_c^d(x)$ is a step function given by

$$s_c^d(x) = \sum_{m=1}^{c} a_m^{d*} \chi_{P_m^{(\mathcal{X},c)}},$$

and the parameters a_m^{d*} are defined by

$$a_m^{d*} = \underset{a_m^d \in \mathbb{R}}{\operatorname{argmin}} \left\{ \sup_{P_m^{(\mathcal{X},c)}} \left| f(x) - s_c^d(x) \right| \right\} = \frac{\inf\limits_{P_m^{(\mathcal{X},c)}} f(x) + \sup\limits_{P_m^{(\mathcal{X},c)}} f(x)}{2}$$

In order to compare the $\mathcal{L}_\infty$ approximation of regular partitioning step approximation given by (6.25), with range partitioning step approximation given by (6.19), we consider the class of non-decreasing continuous functions $f_u : [0,1] \to [0,1]$ that satisfy $f_u(0) = 0$ and $f_u(1) = 1$. By the fact that the f_u functions are non-decreasing and that $f_u(0) = 0$ and $f_u(1) = 1$, both the range and regular partitioning step approximation leads to the same number of intervals. Therefore, by restricting our attention to functions f_u, we are able to perform a more reasonable comparison between the two approximation schemes.

Consider the class of h-piecewise linear functions $F^{(h)} : [0,1] \to [0,1]$, where a member function $f^{(h)}$ is defined by

$$\begin{aligned}
&f^{(h)}(0) = 0 && \\
&f^{(h)}(x) = k_1^{(h)} x, && 0 < x < \frac{1}{h} \\
&f^{(h)}(x) = k_2^{(h)}(x - \frac{1}{h}) + f^{(h)}(\frac{1}{h}), && \frac{1}{h} \leqslant x < \frac{2}{h} \\
&\vdots && \\
&f^{(h)}(x) = k_h^{(h)}(x - \frac{h-1}{h}) + f^{(h)}(\frac{h-1}{h}), && \frac{h-1}{h} \leqslant x < 1 \\
&f^{(h)}(1) = 1 &&
\end{aligned}$$

where $h \in \mathbb{N}^+$ and $k_j^{(h)} \in \{0, 1, \ldots, h-1, h\}$ for $j = 1, \ldots, h$. By definition, the functions $f^{(h)}$ are defined on a square lattice $[0,0] \times [1,1]$, with $(h+1) \times (h+1)$ distinct points, with the additional restriction that $f^{(h)}(0) = 0$ and $f^{(h)}(1) = 1$. Fig. 6.3 shows an example of a $f^{(5)}$ function.

Remark 6.3. It is well known that piecewise linear functions are approximators of continuous function within a compact set (see, for example, [17]). Given an f_u and an

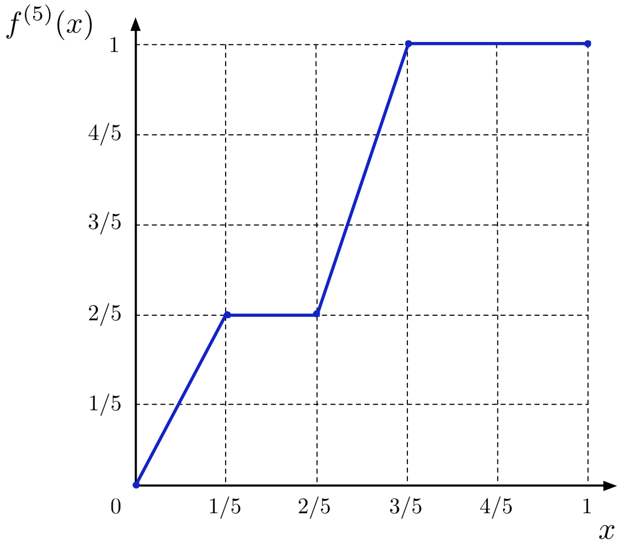


Figure 6.3: An $f^{(5)}(x)$ piecewise linear function.

$\epsilon^{(h)} > 0$, we can find an $f^{(h)}$ function with sufficiently large h such that $\sup |f^{(h)}(x) - f_u(x)| < \epsilon^{(h)}$. As a result, instead of considering f_u functions for comparing range and regular partitioning, we can equivalently consider $f^{(h)}$ functions. $\square$

Note that as h increases, the $f^{(h)}$ function can have larger slope. Therefore, $f^{(h)}$ function with large h can be used to approximate f_u functions with higher-order nonlinearities. The approximation of a certain $f^{(h)}$ with a regular-partitioning step function $s^d_{\frac{1}{h}}(x)$ partitions the domain of $f^{(h)}$ at $0, \frac{1}{h}, \ldots, \frac{h-1}{h}, 1$. In addition, since $k_j \in \{0, 1, \ldots, h-1, h\}$, the $\mathcal{L}_\infty$ approximation error $\epsilon^{d*} = \{ \sup_{P_m^{(\mathcal{X}, \frac{1}{h})}} \left| f(x) - s^d_{\frac{1}{h}}(x) \right| \}$, belongs to $\{0, \frac{1}{2h}, \ldots, \frac{h-1}{2h}, \frac{1}{2}\}$ and is given by

$$
\begin{aligned}
\epsilon^{d*} &= \frac{1}{2h}, \qquad \text{if } \max_{j=1,\ldots,h} k_j^{(h)} = 1 \\
\epsilon^{d*} &= \frac{2}{2h}, \qquad \text{if } \max_{j=1,\ldots,h} k_j^{(h)} = 2 \\
&\vdots \\
\epsilon^{d*} &= \frac{h-1}{2h}, \quad \text{if } \max_{j=1,\ldots,h} k_j^{(h)} = h-1 \\
\epsilon^{d*} &= \frac{1}{2}, \qquad \text{if } \max_{j=1,\ldots,h} k_j^{(h)} = h.
\end{aligned}
$$

Define the set of functions $\mathcal{F}^{(h,k_{\max})} = \left\{ f^{(h)} | \max_j k_j^{(h)} = k_{\max} \right\}$, where $k_{\max} \in \{1, \ldots, h-1, h\}$. Denote a member function of $\mathcal{F}^{(h,k_{\max})}$ with $f^{(h,k_{\max})}$. Then the regular-partitioning step approximation error of $f^{(h,k_{\max})}$ is given by $\epsilon^{d*} = \frac{k_{\max}}{2h}$. As expected, the approximation error increases as the maximum slope ($k_{\max}$) increases. Note that in the case of range-partitioning step approximation the approximation error is independent of $k_{\max}$ and is equal to $\epsilon^* = \frac{1}{2h}$. In order to compare range-partitioning with regular-partitioning step approximation, it suffices to calculate the number of possible functions $f^{(h,k_{\max})}$ for all $k_{\max} \in \{1, \ldots, h-1, h\}$. It is then straightforward to calculate the ratio of the $\mathcal{L}_\infty$ approximation error of the two approximation schemes, $\frac{\epsilon^{d*}}{\epsilon^*}$, as the weighted mean of $k_{\max}$ for all $k_{\max}$.

For any h, $|\mathcal{F}^{(h)}| < \infty$, or in other words, the possible $f^{(h)}$ functions are finite. This is due to the fact that $f^{(h)}$ are piecewise linear functions, and the possible break points are upper bounded by $h+1$. Since $|\mathcal{F}^{(h)}|$ is finite, $|\mathcal{F}^{(h,k_{\max})}|$ is finite. As it turns out the problem of finding the cardinality of the set $|\mathcal{F}^{(h,k_{\max})}|$ is equivalent to finding the number of arrangements of h indistinguishable balls in h boxes with the maximum number of balls in any box equal to $k_{\max} \in \{1, \ldots, h-1, h\}$, [1]. Denote with $N(h, k_{\max})$ the arrangement of h indistinguishable balls in h boxes with the maximum number of balls in any box equal to $k_{\max}$. Then $|\mathcal{F}^{(h,k_{\max})}| = N(h, k_{\max})$. For a certain h, the ratio $\frac{\epsilon^{d*}}{\epsilon^*}$ is calculated as

$$\begin{aligned}\frac{\epsilon^{d*}}{\epsilon^*} &= \sum_{k_{\max}=1}^{h} k_{\max} N(h, k_{\max}) \\ &= \big[N(h,1) + 2N(h,2) + \ldots + (h-1)N(h,h-1) + hN(h,h)\big] \qquad (6.26)\end{aligned}$$

As h increases we are able to consider a greater number of possible $f^{(h)}$ functions and therefore the calculation of $\frac{\epsilon^{d*}}{\epsilon^*}$ based on (6.26) becomes more accurate. In addition, a larger h includes $f^{(h)}$ functions with larger slopes, therefore it is expected that the $\frac{\epsilon^{d*}}{\epsilon^*}$ ratio will increase. To the best of our knowledge, a closed form expression for $N(h, k_{\max})$ (for any $h, k_{\max}$) does not exist. Therefore we rely on algorithmic computation. The values of $N(h,1), \ldots, N(h,h)$, for $h = 1, \ldots, 10$, are given in Table 6.1.

Fig. 6.4 shows the scatter plot of $\frac{\epsilon^{d*}}{\epsilon^*}$ for $h = 1, \ldots, 59$. The $\frac{\epsilon^{d*}}{\epsilon^*}$ ratio is strictly increasing with h, which confirms the fact that range-partitioning step approximation performs increasingly better as the functions $f^{(h)}$ can have larger slopes, or equivalently f_u functions can have larger nonlinearities. For instance, for $h = 59$, $\frac{\epsilon^{d*}}{\epsilon^*} = 6.23$. This shows that, if the range-partitioning approximation of a certain function $f^{(59)}$ results to λ intervals (for some $\lambda = 1, 2, \ldots$), then the regular partitioning approximation requires 6.23λ intervals to get the same mean approximation accuracy. From a communication perspective, a regular partitioning approximation must broadcast 6.23 times faster than range partitioning approximation to get the same mean approximation accuracy.

Table 6.1: Values of $N(h, k_{\max})$ for $h = 1, \ldots, 10$

	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8	k=9	k=10
h=1	1									
h=2	1	2								
h=3	1	6	3							
h=4	1	18	12	4						
h=5	1	50	50	20	5					
h=6	1	140	195	90	30	6				
h=7	1	392	735	392	147	42	7			
h=8	1	1106	2716	1652	672	224	56	8		
h=9	1	3138	9912	6804	2970	1080	324	72	9	
h=10	1	8952	35850	27600	12825	4950	1650	450	90	10

6.4 Communication Scheme Design

In the previous section, we have considered the approximation of a continuous function based on the use of steps functions. Theorems 6.2 and 6.3 provide the procedure for designing a step-function with the best $\mathcal{L}_\infty$ approximation property. Given that the functions $\delta_{h,ij}$ are known, the designer is able to design a communication algorithm that allows the i-th subsystem to best approximate $\delta_{h,ij}$ based on the use of broadcasted samples of x_j. In this work we consider the case of unknown interconnections δ_{ij} and fault functions h_{ij}. As a result, it is not possible to obtain an optimized communication scheme based on Theorems 6.2 and 6.3. In order to overcome this obstacle, we use adaptive approximation models for estimating the unknown $\delta_{h,ij}$ functions online, and then use these approximation models for deciding when to communicate. We now proceed to present the communication algorithm.

We consider the case where an estimate $\hat{x}_j^i$ of x_j may be available to the i-th subsystem. The $\hat{x}_j^i$ estimate is either based on the desired state x_{d_j}, or on distributed state estimation methods (see, for example, [86]). In addition, the j-th subsystem broadcasts a sample $\bar{x}_j^i[k]$ of its state x_j to the i-th subsystem according to a decision logic defined later on. If a sample $\bar{x}_j^i[k]$ of x_j is not available, the i-th subsystem utilizes the known $\hat{x}_j^i$ instead. In the case where $\hat{x}_j^i$ is a poor estimate of x_j or equivalently when $\hat{x}_j^i$ is not available at all, then the i-th subsystem utilizes only broadcasted samples of x_j. For all $t > 0$, the i-th subsystem utilizes either the estimate $\hat{x}_j^i(t)$, or

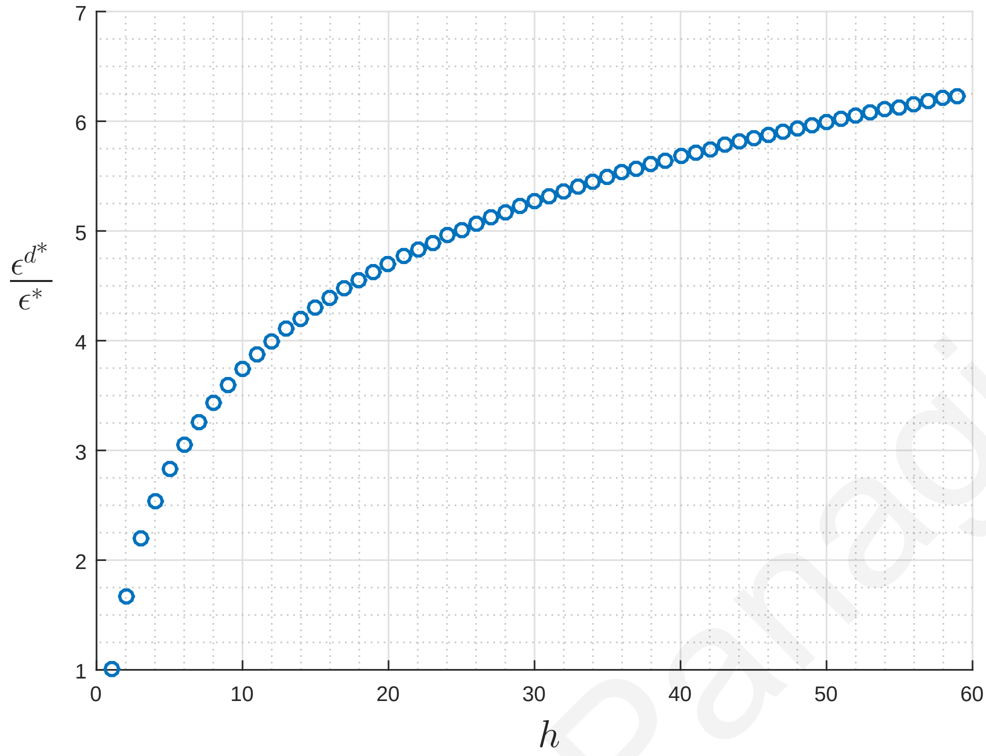


Figure 6.4: Mean ratio of regular and range partitioning step approximation errors of $f^{(h)}$ functions ($\frac{\epsilon^{d*}}{\epsilon^*}$) for $h = 1, \ldots, 59$

the latest broadcasted sample $\bar{x}_j^i[k]$. Define the indicator function U_{ij} as

$$U_{ij}(t) = \begin{cases} 1 & \text{if } \bar{x}_j^i[k] \text{ is used by the } i\text{−th subsystem} \\ 0 & \text{if } \hat{x}_j^i(t) \text{ is used by the } i\text{−th subsystem.} \end{cases}$$

Let $\hat{\bar{x}}_j^i(t)$ represent the best available information about x_j at time t, which based on $U_{ij}(t)$ is given by

$$\hat{\bar{x}}_j^i(t) = \begin{cases} \hat{x}_j^i(t) & \text{if } \; U_{ij}(t) = 0 \\ \bar{x}_j^i(t) & \text{if } \; U_{ij}(t) = 1. \end{cases} \tag{6.27}$$

The i-th subsystem receives the indicator function U_{ij} from the j-th subsystem whenever it transitions from 1 to 0, such that the i-th subsystem knows when it should use the state estimate $\hat{x}_j^i(t)$. It is easy to see that the transition $U_{ij} = 0 \to 1$ is implied by the fact that the i-th subsystem receives a sample of x_j. Define the indicator functions $X_{ij}(t)$, $D_{ij}(t)$ and $Q_i(t)$ as follows

$$X_{ij}(t) = \begin{cases} 0 & \text{if } |\hat{\bar{x}}_j^i(t) - x_j(t)|_2 \leqslant d_{ij} \\ 1 & \text{if } |\hat{\bar{x}}_j^i(t) - x_j(t)|_2 > d_{ij}, \end{cases}$$

$$D_{ij}(t) = \begin{cases} 0 & \text{if } |\hat{\theta}_{ij}\phi_{ij}(\hat{\bar{x}}_j^i(t)) - \hat{\theta}_{ij}\phi_{ij}(x_j(t))| \leqslant \bar{\delta}_{ij} \\ 1 & \text{if } |\hat{\theta}_{ij}\phi_{ij}(\hat{\bar{x}}_j^i(t)) - \hat{\theta}_{ij}\phi_{ij}(x_j(t))| > \bar{\delta}_{ij}, \end{cases}$$

$$Q_i(t) = \begin{cases} 0 & \text{if } |\tilde{x}_i(t)|_2 \leqslant \epsilon_i \\ 1 & \text{if } |\tilde{x}_i(t)|_2 > \epsilon_i, \end{cases}$$

where $\bar{\delta}_{ij}$ and $\epsilon_i > 0$ are design constants, and $|.|_2$ is the Euclidean norm. The communication algorithm $\mathcal{C}_{ij}(t)$ is given by

$$\mathcal{C}_{ij}(t) = \begin{cases} X_{ij}(t)x_j(t) & \text{if } Q_i(t) = 1 \quad (6.28) \\ D_{ij}(t)x_j(t) & \text{if } Q_i(t) = 0 \quad (6.29) \end{cases}$$

In other words, the communication algorithm switches between two cases:

(a) When the tracking error of the i-th subsystem exceeds a threshold ϵ_i ($|\tilde{x}_i(t)|_2 > \epsilon_i$), the communication algorithm is based on a state level-crossing communication scheme, i.e., a broadcast of x_j is released when $|\hat{\bar{x}}_j^i(t) - x_j(t)| > d_{ij}$.

(b) When the tracking error of the i-th subsystem is smaller than a threshold ϵ_i, ($|\tilde{x}_i(t)|_2 \leqslant \epsilon_i$), the communication algorithm is based on an approximation-model level-crossing communication scheme, i.e., a broadcast of x_j is released when $|\hat{\theta}_{ij}\phi_{ij}(\hat{\bar{x}}_j^i(t)) - \hat{\theta}_{ij}\phi_{ij}(x_j)| > \bar{\delta}_{ij}$.

The j-th subsystem receives the indicator function Q_i from the i-th subsystem only whenever it has changed. In addition, whenever Q_i transitions to zero, the i-th subsystem transmits the parameter estimates $\hat{\theta}_{ij}$ to the j-th subsystem. Fig. 6.5 illustrates the communication scheme, where the exchange of information according to (6.28) is shown at the top, and the exchange of information according to (6.29) is shown at the bottom.

Remark 6.4. In the proposed communication scheme, the tracking error $\tilde{x}_i$ is utilized for switching between the sampling algorithms (6.28) and (6.29). In order to explain the reasoning behind this approach, consider the realizable by the i-th subsystem approximation of $\delta_{h,ij}(x_j, t)$ given by $\hat{\theta}_{ij}\phi_{ij}(\hat{\bar{x}}_j^i)$. The difference $\delta_{h,ij}(x_j, t) - \hat{\theta}_{ij}\phi_{ij}(\hat{\bar{x}}_j^i)$ denotes the overall approximation error of the unknown function $\delta_{h,ij}(x_j, t)$. Based on the use of the triangular inequality, the overall approximation error satisfies

$$|\delta_{h,ij}(x_j, t) - \hat{\theta}_{ij}\phi_{ij}(\hat{\bar{x}}_j^i)| \leqslant |\delta_{h,ij}(x_j, t) - \delta_{h,ij}(\hat{\bar{x}}_j^i, t)| + |\tilde{\theta}_{ij}\phi_{ij}(\hat{\bar{x}}_j^i)| + \bar{\mu}_{ij}, \quad (6.30)$$

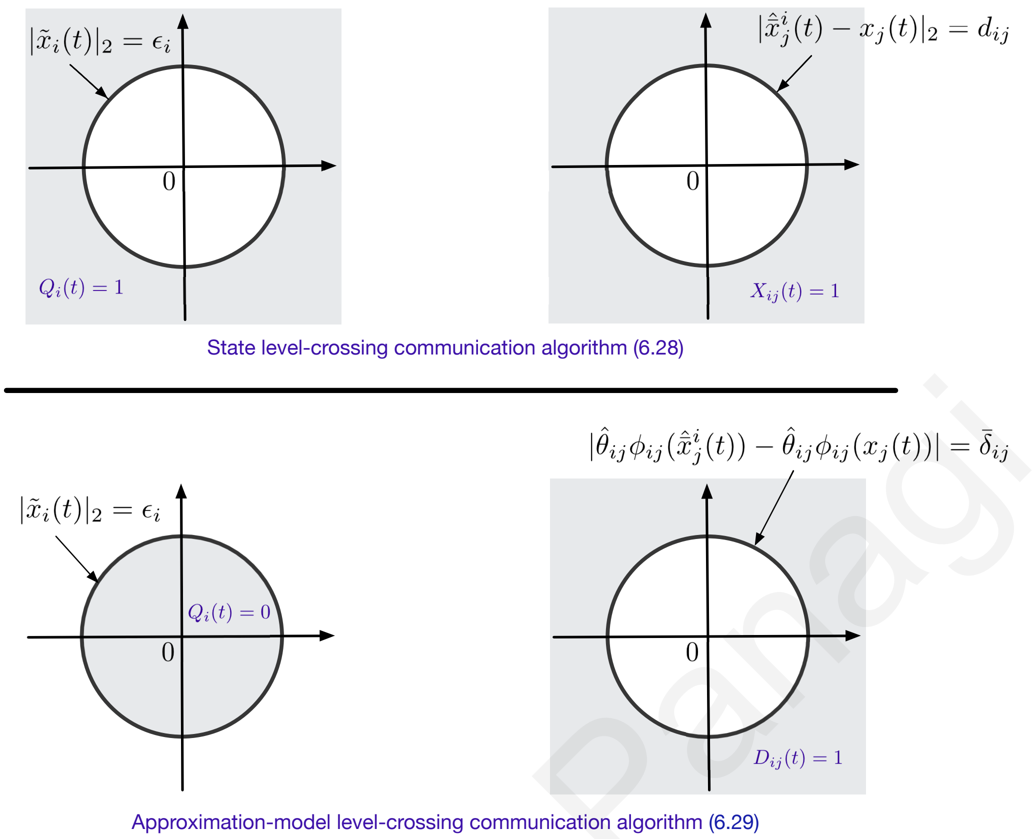


Figure 6.5: Decision logic of the communication scheme.

where $\tilde{\theta}_{ij} = \hat{\theta}_{ij} - \theta^*_{ij}$ are the parameter estimation errors. On the right hand side of (6.30), the term $\delta_{h,ij}(x_j, t) - \delta_{h,ij}(\hat{\tilde{x}}^i_j, t)$ represents the replacement error due to non-zero $\hat{\tilde{x}}^i_j - x_j$, the term $|\tilde{\theta}_{ij}\phi_{ij}(\hat{\tilde{x}}^i_j)|$ represents the parameters estimation error, and $\bar{\mu}_{ij}$ is an upper bound of the residual approximation error. The fact that the local tracking error $\tilde{x}_i$ is large ($Q_i(t) = 1$) is an indicator that the magnitude of $\delta_{h,ij}(x_j, t) - \hat{\theta}_{ij}\phi_{ij}(\hat{\tilde{x}}^i_j)$ is large. As $|\hat{\tilde{x}}^i_j - x_j| \to 0$, by the fact that $\delta_{h,ij}(x_j, t)$ satisfies a Lipschitz condition, $|\delta_{h,ij}(x_j, t) - \delta_{h,ij}(\hat{\tilde{x}}^i_j, t)| \to 0$. Therefore, while $Q_i(t) = 1$, it is required that the difference $\hat{\tilde{x}}^i_j - x_j$ becomes as small as possible. The state level-crossing communication scheme, (6.28), aims to minimize the replacement error $\delta_{h,ij}(x_j, t) - \delta_{h,ij}(\hat{\tilde{x}}^i_j, t)$, such that the overall approximation error is due mostly to the residual approximation error $\bar{\mu}_{ij}$ and the parameter estimation errors $\tilde{\theta}_{ij}$, which are due to the limited approximation capabilities of the approximator and the persistence of excitation property of the system, respectively. Using the triangular inequality once more, the overall approximation error satisfies

$$|\delta_{h,ij}(x_j, t) - \hat{\theta}_{ij}\phi_{ij}(\hat{\tilde{x}}^i_j)| \leqslant |\tilde{\theta}_{ij}\phi_{ij}(x_j)| + |\hat{\theta}_{ij}\big(\phi_{ij}(\hat{\tilde{x}}^i_j) - \phi_{ij}(x_j)\big)| + \bar{\mu}_{ij}. \tag{6.31}$$

The fact that the local tracking error $\tilde{x}_i$ is small ($Q_i(t) = 0$) is an indicator that the magnitude of the overall approximation error is small. Provided that the parameter estimation errors $\tilde{\theta}_{ij}$ are sufficiently small, the term $|\hat{\theta}_{ij}\big(\phi_{ij}(\hat{\bar{x}}^i_j) - \phi_{ij}(x_j)\big)|$ dominates the right hand side of (6.31), and therefore provides a good estimate of the replacement error $\delta_{h,ij}(x_j, t) - \delta_{h,ij}(\hat{\bar{x}}^i_j, t)$. In this case, the communication algorithm based on an approximation-model level-crossing scheme, (6.29), ensures that the magnitude of the replacement error is minimized. □

Remark 6.5. It is assumed that the available communication bandwidth for each subsystem j is finite, which means that data transmission cannot be continuous. In other words, it is assumed that $t^{k+1}_{ij} - t^k_{ij}$ are non-zero for all $k > 0$, such that within any finite time interval, x_j is broadcasted a finite number of times. The proposed communication scheme avoids explicit continuous broadcast of state information. However, without an ad-hoc restriction on the inter-broadcast times lower bound, it is possible for it to be arbitrarily close to zero or it may even result in the limit of the sequence t^k_{ij} to be a finite number (Zeno behavior). □

6.5 Distributed Fault Tolerant Control Design

Consider the tracking error dynamics of the i-th subsystem $\tilde{x}_i = x_i - x_{d_i}$ which based on (6.2) satisfy

$$\dot{\tilde{x}}_i = A\tilde{x}_i + B\left[f_i(x_i) + g_i(x_i)u_i + \sum_{j=1}^{m} \delta_{ij}(x_j) + \beta(t - T_{ij})h_{ij}(x_j) - \dot{x}_{d_{in}}\right]. \tag{6.32}$$

Let the distributed fault tolerant control law be given by

$$u_i = u_{N_i} + u_{F_i} \tag{6.33}$$

where u_{N_i} denotes the local nominal control law for stabilizing the i-th subsystem in the absence of interconnections and faults, and u_{F_i} is the augmented fault tolerant control law for addressing the unknown δ_{ij} interconnections and the change in dynamics due to the occurrence of faults in the δ_{ij} interconnections. The local nominal control law u_{N_i} is given by

$$u_{N_i} = \frac{1}{g_i(x_i)}\big(- K_i^T\tilde{x}_i - f_i(x_i) + \dot{x}_{d_{in}}\big), \tag{6.34}$$

where the vector $K_i = [k_{i1}, \ldots, k_{in}]^\top \in \mathbb{R}^n$ is chosen such that $A - BK_i^\top$ is a Hurwitz matrix. Since $A - BK_i^\top$ is Hurwitz, for any $Q_i > 0$ there exists $P_i > 0$ satisfying the

Lyapunov equation, $P_i(A - BK_i^\top) + (A - BK_i^\top)^\top P_i = -Q_i$. Based on P_i, the scalar tracking error e_i is defined as $e_i = B^\top P_i \tilde{x}_i$.

Let $\bar{\mu}_{ij}^\delta = \bar{\mu}_{ij} + L_{ij} d_{ij}$. The term $\bar{\mu}_i^\delta$ represents the combined inherent error due to the residual approximation error $\bar{\mu}_{ij}$ and the replacement error $\delta_{h,ij}(x_j, t) - \delta_{h,ij}(\hat{\tilde{x}}_j^i, t)$. Let $\bar{\mu}_i^\delta = \sum_{j=1}^m \bar{\mu}_{ij}^\delta$. The unknown $\bar{\mu}_i^\delta$ is estimated online by an adaptive estimate which is denoted by $\hat{\mu}_i^\delta$. The adaptive approximation based control law u_{F_i} is given by

$$u_{F_i} = -\frac{1}{g_i(x_i)} \left(u_{c_i} + \sum_{j=1}^m \hat{\theta}_{ij}^\top \phi_{ij}(\hat{\tilde{x}}_j^i) \right) \tag{6.35}$$

$$u_{c_i} = \begin{cases} \hat{\mu}_i^\delta sgn(e_i) & \text{if} \quad \tilde{x}_i^\top P_i \tilde{x}_i > \bar{\lambda}_{P_i} \epsilon_i^2 \\ 0 & \text{if} \quad \tilde{x}_i^\top P_i \tilde{x}_i \leqslant \bar{\lambda}_{P_i} \epsilon_i^2, \end{cases} \tag{6.36}$$

where $\bar{\lambda}_{P_i}$ is the maximum eigenvalue of P_i. The parameter estimates of the adaptive approximator $\hat{\theta}_{ij}$, and the adaptive bounding parameter $\hat{\mu}_i^\delta$ are updated according to

$$\dot{\hat{\theta}}_{ij} = \Gamma_{ij} \phi_{ij}(\hat{\tilde{x}}_j^i) q_i(e_i, \tilde{x}_i, \epsilon_i) \tag{6.37}$$

$$\dot{\hat{\mu}}_i^\delta = \gamma_i \Big| q_i(e_i, \tilde{x}_i, \epsilon_i) \Big| \tag{6.38}$$

where Γ_{ij} is a positive definite matrix and γ_i is a positive constant representing the adaptive gains, and $q_i(e_i, \tilde{x}_i, \epsilon_i)$ is a dead-zone, defined as

$$q_i(e_i, \tilde{x}_i, \epsilon_i) = \begin{cases} 0 & \tilde{x}_i^\top P_i \tilde{x}_i \leqslant \bar{\lambda}_{P_i} \epsilon_i^2 \\ e_i & \tilde{x}_i^\top P_i \tilde{x}_i > \bar{\lambda}_{P_i} \epsilon_i^2. \end{cases} \tag{6.39}$$

Note that the fact that $\tilde{x}_i^\top P_i \tilde{x}_i \leqslant \bar{\lambda}_{P_i} \epsilon_i^2$ ensures that $|\tilde{x}_i|_2 \leqslant \epsilon_i$.

6.6 Stability Analysis

Theorem 6.4. *Given that the coverage set $\mathcal{X}_i$ is large enough such that $x_i(t) \in \mathcal{X}_i$ for all $t > 0$, the closed-loop system described by the interconnected system (6.1), the distributed fault tolerant control law (6.33), (6.34), (6.35) and (6.36), and the adaptation laws (6.37)-(6.38) guarantee that $|\tilde{x}_i(t)|$ is uniformly ultimately bounded by ϵ_i; i.e., the total time such that $\tilde{x}_i^\top P_i \tilde{x}_i > \bar{\lambda}_{P_i} \epsilon_i^2$ is finite.*

Proof. Let the Lyapunov function of the overall system be given by $V = \sum_{i=1}^m V_i$, where $V_i = V_{i1} + V_{i2}$ is the Lyapunov function of the i-th subsystem defined as

$$V_{i1} = \frac{1}{2}\tilde{x}_i^\top P_i \tilde{x}_i,$$
$$V_{i2} = \frac{1}{2}\sum_{j=1}^{m} \tilde{\theta}_{ij}^\top \Gamma_{ij}^{-1} \tilde{\theta}_{ij} + \frac{1}{2\gamma_i}(\hat{\mu}_i^\delta - \bar{\mu}_i^\delta)^2,$$

Substituting u_{N_i} in (6.32), we obtain the following expression for the tracking error dynamics

$$\dot{\tilde{x}}_i = \left(A - BK_i^\top\right)\tilde{x}_i + B\left(g_i(x_i)u_{F_i} + \sum_{j=1}^{m}\delta_{ij}(x_j) + \beta(t - T_{ij})h_{ij}(x_j)\right).$$

The time derivative of V_{i1} satisfies

$$\dot{V}_{i1} = -\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i + e_i\left(g_i(x_i)u_{F_i} + \sum_{j=1}^{m}\delta_{ij}(x_j) + \beta(t - T_{ij})h_{ij}(x_j)\right).$$

Using (6.5) and substituting u_{F_i} we obtain

$$\dot{V}_{i1} = -\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i - e_i u_{c_i} + e_i\left(\sum_{j=1}^{m}\delta_{h,ij}(x_j, t) - \hat{\theta}_{ij}^\top \phi_{ij}(\hat{\bar{x}}_j^i)\right).$$

Adding and subtracting $\delta_{h,ij}(\hat{\bar{x}}_j^i, t)$ we obtain

$$\begin{aligned}\dot{V}_{i1} = &-\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i - e_i u_{c_i} + e_i\left(\sum_{j=1}^{m}\delta_{h,ij}(x_j, t) - \delta_{h,ij}(\hat{\bar{x}}_j^i, t)\right.\\ &\left. + \delta_{h,ij}(\hat{\bar{x}}_j^i, t) - \hat{\theta}_{ij}^\top \phi_{ij}(\hat{\bar{x}}_j^i)\right).\end{aligned}$$

Based on (6.11),

$$\begin{aligned}\dot{V}_{i1} = &-\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i - e_i u_{c_i} + e_i\left(\sum_{j=1}^{m}\delta_{h,ij}(x_j, t) - \delta_{h,ij}(\hat{\bar{x}}_j^i, t)\right.\\ &\left. + \mu_{ij}(\hat{\bar{x}}_j^i, t) - \tilde{\theta}_{ij}^\top \phi_{ij}(x_j)\right).\end{aligned}$$

The time derivative of V_i satisfies

$$\begin{aligned}\dot{V}_i = &-\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i - e_i u_{c_i} + e_i\left(\sum_{j=1}^{m}\delta_{h,ij}(x_j, t) - \delta_{h,ij}(\hat{\bar{x}}_j^i, t)\right.\\ &\left. + \mu_{ij}(\bar{x}_j^i, t) - \tilde{\theta}_{ij}^\top \phi_{ij}(x_j)\right) + \sum_{j=1}^{m}\tilde{\theta}_{ij}^\top \Gamma_{ij}^{-1}\dot{\tilde{\theta}}_{ij} + \frac{1}{\gamma_i}(\hat{\mu}_i^\delta - \bar{\mu}_i^\delta)\dot{\hat{\mu}}_i^\delta.\end{aligned}$$

Substituting the adaptive law (6.37) for $\tilde{x}_i^\top P_i \tilde{x}_i > \bar{\lambda}_{P_i}\epsilon_i^2$ we obtain,

$$\begin{aligned}\dot{V}_i = &-\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i - e_i u_{c_i} + e_i\sum_{j=1}^{m}\delta_{h,ij}(x_j, t) - \delta_{h,ij}(\hat{\bar{x}}_j^i, t) + \mu_{ij}(\bar{x}_j^i, t)\\ &+ \frac{1}{\gamma_i}(\hat{\mu}_i^\delta - \bar{\mu}_i^\delta)\dot{\hat{\mu}}_i^\delta.\end{aligned}$$

Substituting u_{c_i} from (6.35) for $\tilde{x}_i^\top P_i \tilde{x}_i > \bar{\lambda}_{P_i} \epsilon_i^2$, we obtain,

$$\begin{aligned}\dot{V}_i = & -\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i + e_i \sum_{j=1}^{m} \delta_{h,ij}(x_j, t) - \delta_{h,ij}(\hat{\tilde{x}}_j^i, t) + \mu_{ij}(\hat{\tilde{x}}_j^i, t) \\ & - \frac{1}{\gamma_i}\bar{\mu}_i^\delta \dot{\hat{\mu}}_i^\delta + \frac{1}{\gamma_i}\hat{\mu}_i^\delta(\dot{\hat{\mu}}_i^\delta - \gamma_i |e_i|).\end{aligned}$$

Using (6.6) and the fact that $|\hat{\tilde{x}}_j^i - x_j| \leqslant d_{ij}$ for $\tilde{x}_i^\top P_i \tilde{x}_i > \bar{\lambda}_{P_i} \epsilon_i^2$ we have

$$\begin{aligned}\dot{V}_i \leqslant & -\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i + |e_i| \Big(\sum_{j=1}^{m} L_{ij} d_{ij} + \mu_{ij}(\hat{\tilde{x}}_j^i, t) \Big) - \frac{1}{\gamma_i}\bar{\mu}_i^\delta \dot{\hat{\mu}}_i^\delta + \frac{1}{\gamma_i}\hat{\mu}_i^\delta(\dot{\hat{\mu}}_i^\delta - \gamma_i |e_i|) \\ \leqslant & -\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i + |e_i| \bar{\mu}_i^\delta - \frac{1}{\gamma_i}\bar{\mu}_i^\delta \dot{\hat{\mu}}_i^\delta + \frac{1}{\gamma_i}\hat{\mu}_i^\delta(\dot{\hat{\mu}}_i^\delta - \gamma_i |e_i|)\end{aligned}$$

By substituting the adaptive law (6.38) for $\tilde{x}_i^\top P_i \tilde{x}_i > \bar{\lambda}_{P_i} \epsilon_i^2$ we obtain $\dot{V}_i \leqslant -\frac{1}{2}\tilde{x}_i^\top Q_i \tilde{x}_i$, which shows that $\tilde{x}_i$ converges into the set $\mathcal{W}_i = \left\{ \tilde{x}_i \middle| \tilde{x}_i^\top P_i \tilde{x}_i \leqslant \bar{\lambda}_{P_i} \epsilon_i^2 \right\}$. The fact that adaptation is stopped when $\tilde{x}_i \in \mathcal{W}_i$, $\tilde{x}_i, \hat{\theta}_{ij}, \hat{\mu}_i^\delta \in \mathcal{L}_\infty$ for all $t > 0$. Moreover, the fact that $\hat{\mu}_i^\delta$ is non-decreasing shows that $\tilde{x}_i$ enters $\mathcal{W}_i$ in finite time, i.e., there is some t_{0_i} such that $\tilde{x}_i \in \mathcal{W}_i$ for all $t > t_{0_i}$. □

Remark 6.6. Intuitively the distributed fault tolerant control scheme is divided into a *learning phase*, during which the unknown dynamics of the system are modeled, and an *operating phase*, in which the system utilizes the knowledge obtained at the learning phase to optimize the benefits from communication. As the i-th subsystem spends more time in the dead-zone it gradually transitions to the operating phase. The objective in the learning phase is to build as accurate approximation models of the unknown interconnections and fault functions as possible. The performance for the approximation of $\delta_{h,ij}(x_j, t)$ is in general improved as the state level-crossing communication threshold d_{ij} is reduced, and as the accuracy of the state estimator $\hat{x}_j^i$ is improved. It is noted that the stability of the system is decoupled from the choice of the communication threshold d_{ij} as well as the availability of a state estimator $\hat{x}_j^i$. During the operating phase, the i-th subsystem receives information from the other subsystems based on the approximation-model level-crossing communication algorithm given by (6.29), Based on this algorithm, the broadcasting samples of x_j are such that $\hat{\theta}_{ij}\phi_{ij}$ are optimally approximated with step functions (Theorem 6.2). Provided that a sufficiently accurate approximation model $\hat{\theta}_{ij}\phi_{ij}$ of the unknown $\delta_{h,ij}(x_j, t)$ function is obtained during the learning phase, the approximation-model level-crossing communication algorithm leads to a near optimal approximation of the unknown $\delta_{h,ij}(x_j, t)$ function with a step function, and therefore the uncertainty about the coupling dynamics is minimized. □

Remark 6.7. The proposed scheme does not require the availability of state estimators $\hat{x}_j^i$. In the case where an estimate of x_j is not available, the performance of the approximators depends only on the amount of state information exchanged between the subsystems. However, as we consider interconnections with higher degrees of nonlinearities, the required cost for communication to obtain sufficient approximation performance may become considerably high. Consider the replacement error $\delta_{h,ij}(x_j,t) - \delta_{h,ij}(\hat{\bar{x}}_j^i,t)$ which based on (6.6) satisfies

$$|\delta_{h,ij}(x_j,t) - \delta_{h,ij}(\hat{\bar{x}}_j^i,t)| \leqslant L_{ij}\left|x_j - \hat{\bar{x}}_j^i\right|. \tag{6.40}$$

In the case of higher order nonlinear functions $\delta_{h,ij}(x_j,t)$, the required constant L_{ij} to satisfy (6.40) can become quite large. The best achievable approximation is limited by the size of the replacement error. Therefore, while the i-th subsystem estimates $\delta_{h,ij}(x_j,t)$ it is important that the replacement error is minimized. As L_{ij} increases, the required communication threshold d_{ij} to minimize the inherent replacement error and improve the approximation performance goes to zero. Therefore, in practical applications it is best to incorporate as much communication-free knowledge about the remote states, by using distributed state estimators, or sharing the desired states *a priori*. Note that from a control and approximation perspective, the availability of state estimators is beneficial only during the learning phase of the unknown dynamics. After the subsystems enter the dead-zone, the availability of state estimators is only a matter of reducing the cost for communication. In fact, in the case where the reference signal x_{d_j} is utilized as an estimate for the state x_j, the j-th subsystem is free to change its reference signal without affecting the other subsystems, and thereafter, broadcast its state x_j to the other subsystems according to the approximation-model level-crossing communication algorithm. □

Remark 6.8. Theorem 6.6 assumes that the trajectory $x_i(t)$ of the i-th subsystem remains within the coverage region $\mathcal{X}_i$. This can be guaranteed by implementing a decentralized safety control scheme (such as the one presented in Section 3.5) for bringing the trajectory back within $\mathcal{X}_i$, in case that it leaves the coverage region. □

6.7 Simulation

To illustrate the design methodology for the distributed fault tolerant control, consider the following interconnected system:

$$\begin{aligned}\Sigma_1 : \dot{x}_1 &= x_1^2 + (1 + x_1^2)u_1 + \delta_{12}(x_2) + \beta(t - T_{12})h_{12}(x_2) \\ \Sigma_2 : \dot{x}_2 &= 5x_2^3 + (2 + 0.5x_2^2)u_2 + \delta_{21}(x_1) + \beta(t - T_{21})h_{12}(x_1),\end{aligned}$$

where $x_i \in \mathbb{R}$ is the state vector of the i-th subsystem ($i = 1, 2$). The feedback gains are chosen as $K_1 = K_2 = 1$. The desired trajectory vector $x_{d_i} = [x_{d_{i1}}, x_{d_{i2}}]^\top$ and the signal $\dot{x}_{d_{i2}}$ are generated using a third order filter with a bandwidth of 5 (rad/sec) and unity gain below this frequency. The filter input is chosen as a square wave of zero mean, 1.5 amplitude and a frequency of 0.4 Hz. The unknown interconnections are approximated with a lattice of equally spaced radial basis functions that cover the region $|x_i| \leqslant 10$, with the centers distance equal to 0.1. Outside this region, a decentralized safety control law is implemented based on the control design presented in Section 3.5. The state estimators $\hat{x}_2^1(t)$ and $\hat{x}_1^2(t)$ are based on the desired states $x_{d_2}(t)$ and $x_{d_1}(t)$ respectively. The state level-crossing communication thresholds d_1 and d_2 are chosen as $d_1 = d_2 = 0.5$, while the approximation models level-crossing communication thresholds are chosen as $\bar{\delta}_{12} = \bar{\delta}_{21} = 1$. The radii of the dead-zones are chosen as $\epsilon_1 = \epsilon_2 = 0.15$.

We consider the case in which abrupt faults occur in Σ_1 at $T_{12} = 30$ sec and in Σ_2 at $T_{21} = 70$ sec. For simulation purposes, the unknown interconnections δ_{12} and δ_{21} are chosen as

$$\begin{aligned}\delta_{12}(x_2) &= \frac{1}{25(x_2+7)^2+0.1} + \frac{1}{25(x_2-0.3)^2+0.1} + \frac{1}{(x_2-0.9)^2+0.4} \\ &\quad + \frac{1}{10(x_2-5)^2+0.1} + \frac{1}{25(x_2-6.3)^2+0.3} - \frac{16}{10} \\ \delta_{21}(x_1) &= 15\tanh(3x_1) + 4\sin(x_1),\end{aligned}$$

and the unknown fault functions h_{12} and h_{21} are chosen as

$$\begin{aligned}h_{12} &= -20\tanh\big(10(x_2+0.5)\big) + 20\tanh\big(10(x_2-0.5)\big) \\ h_{21} &= -30\tanh(5x_1).\end{aligned}$$

In Fig. 6.6 we plot the tracking error of each subsystem, $\tilde{x}_i$, $i = 1, 2$, indicating the time occurrence of the faults. As illustrated by the plot, the subsystems are able

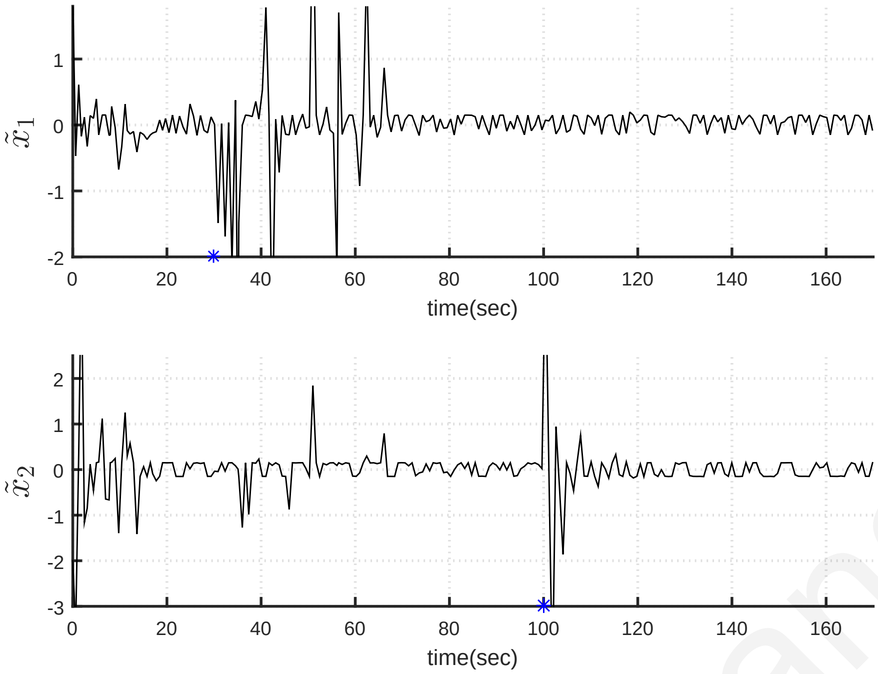


Figure 6.6: Time evolution of the tracking errors.

to track the reference trajectories in the presence of unknown interconnections and faults with significantly large magnitude and nonlinearities. In Fig. 6.7 we plot the parameter estimates, $\hat{\theta}_{12}$ and $\hat{\theta}_{21}$ of the approximation of the $\delta_{h,12}(x_2, t)$ and $\delta_{h,21}(x_1, t)$ functions, respectively. Before the occurrence of the faults, the parameter estimates are updated to approximate the unknown interconnections and drive the trajectories into the dead-zone (as shown by Fig. 6.6). As the subsystems spend more time in the dead-zone, the parameters estimates get closer to their final values. The sudden change in the dynamics due to the occurrence of faults in the interconnections drive the trajectories out of the dead-zone. However, the parameter estimates are adapted to accommodate the fault and steer the subsystems back into the dead-zone.

Fig. 6.8 shows the plot of the functions $\delta_{h,12}(x_2, t)$ and $\delta_{h,21}(x_1, t)$ and their respective approximations after the occurrence of the faults, for $t = 400$ sec. The fact that x_{d_i}, $i = 1, 2$ are available to the subsystems and the use of a small communication threshold, ensures sufficient approximation of the unknown interconnections and fault functions.

Fig. 6.9 compares the approximation error $\hat{\theta}_{12}\phi_{12}(\hat{\bar{x}}_2^1) - \delta_{h,12}(x_2, t)$ for $d_{12} = 0$, $d_{12} = 1$, and for the case where x_{d_2} is not available and $d_{12} = 1$. In the latter case $\hat{\bar{x}}_2^1$ is based only on the broadcasted samples of x_2 ($\bar{x}_2^1$). The plot shows the time

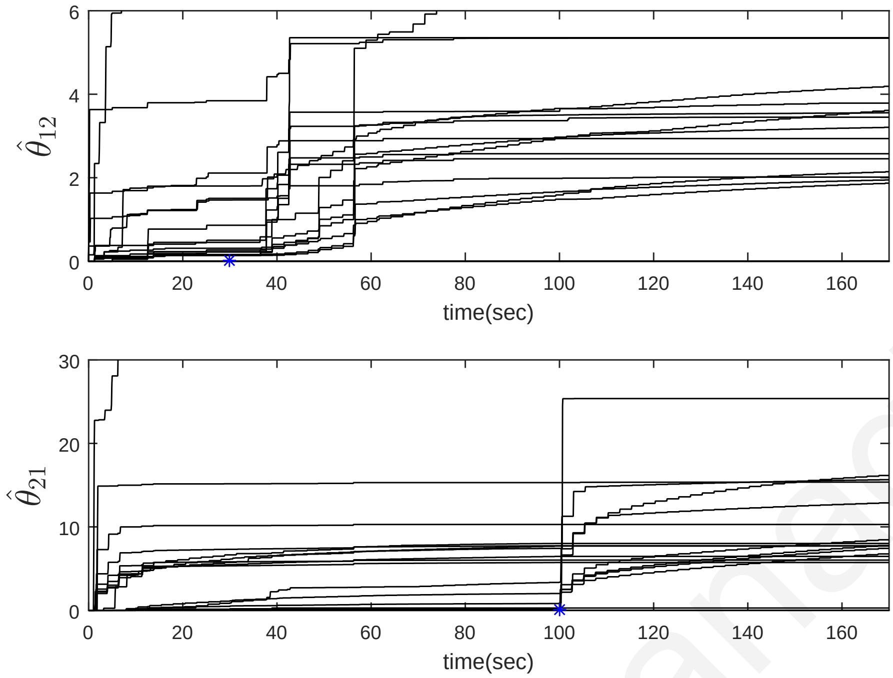


Figure 6.7: Time evolution of the adaptive parameter estimates.

evolution of the $\mathcal{L}_1$ norm of the approximation error, given by

$$\int_0^t \left|\hat{\theta}_{12}(\tau)\phi_{12}(\hat{\bar{x}}_2^1(\tau)) - \delta_{h,12}(x_2(\tau),\tau)\right|d\tau.$$

In the case of $d_{12} = 0$, x_2 is available to Σ_1 for all $t > 0$, such that the approximation error is due only to the residual approximation error and the parameter estimation errors. Therefore the case of $d_{12} = 0$ provides a baseline for the approximation we can expect from the distributed scheme, as any value of d_{12} greater than zero, produces an additional approximation error due to the replacement of x_2 with $\hat{\bar{x}}_2^1$. Comparing the case of $d_{12} = 1$ with and without knowledge of the desired state x_{d_2} shows that, although the replacement error is bounded by the same value in both cases, when x_{d_2} is available the approximation performance is improved. This can be explained by the fact that in the case where x_{d_2} is not available, the basis functions are not sufficiently excited and the parameter estimates converge to suboptimal values. The parameter estimation errors can not be further reduced unless a much smaller d_{12} is used, which will allow the approximator to explore more areas of the unknown function $\delta_{h,12}(x_2, t)$.

Next, we compare the approximation-model level-crossing communication scheme with a state level-crossing scheme, where the j-th subsystem transmits its state whenever $|\hat{\bar{x}}_j^i(t) - x_j(t)|_2 > d_{ij}$ ($X_{ij}(t) = 1$), even when $|\tilde{x}_i(t)|_2 \leqslant \epsilon_i$ ($Q_i(t) = 0$). We consider

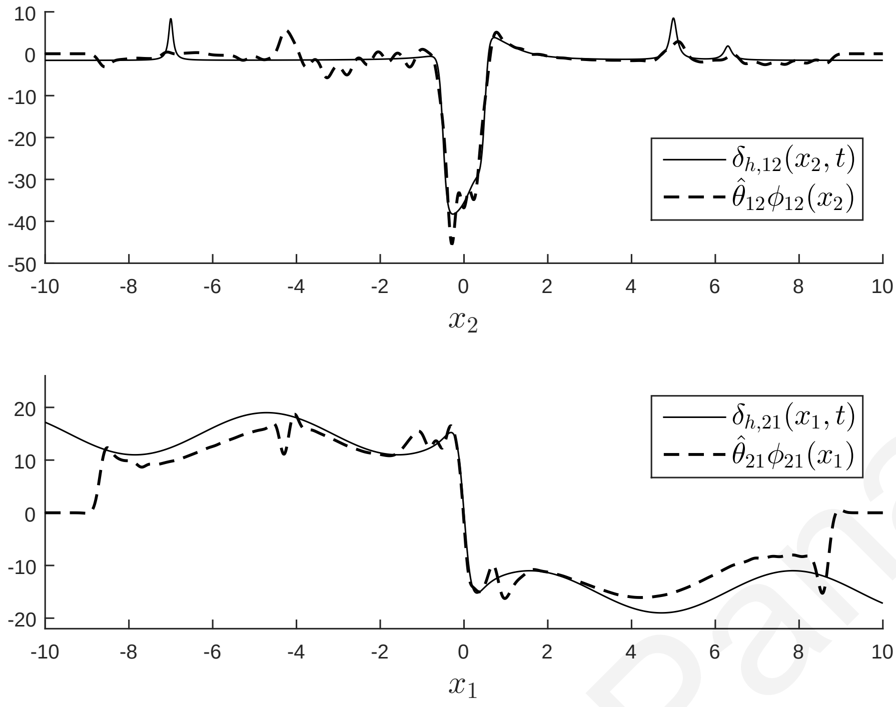


Figure 6.8: Approximation of the functions $\delta_{h,12}(x_2, t)$ and $\delta_{h,21}(x_1, t)$ for $t = 400$ sec.

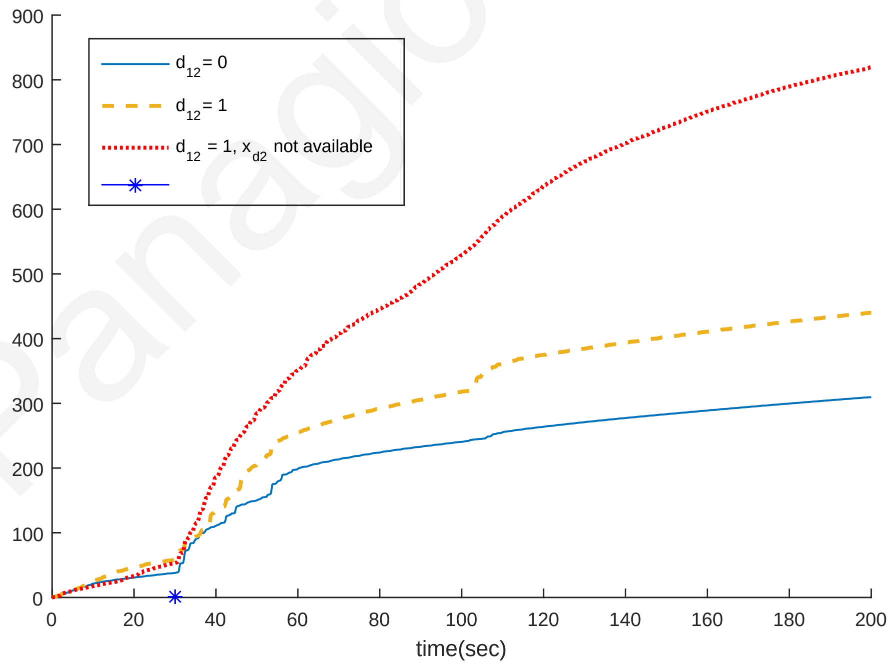


Figure 6.9: Time evolution of the $\mathcal{L}_1$ norm of the approximation error of $\delta_{h,12}(x_2, t)$, for (a) $d_{12} = 0$, (b) $d_{12} = 1$, and (c) $d_{12} = 1$ and x_{d_2} unknown.

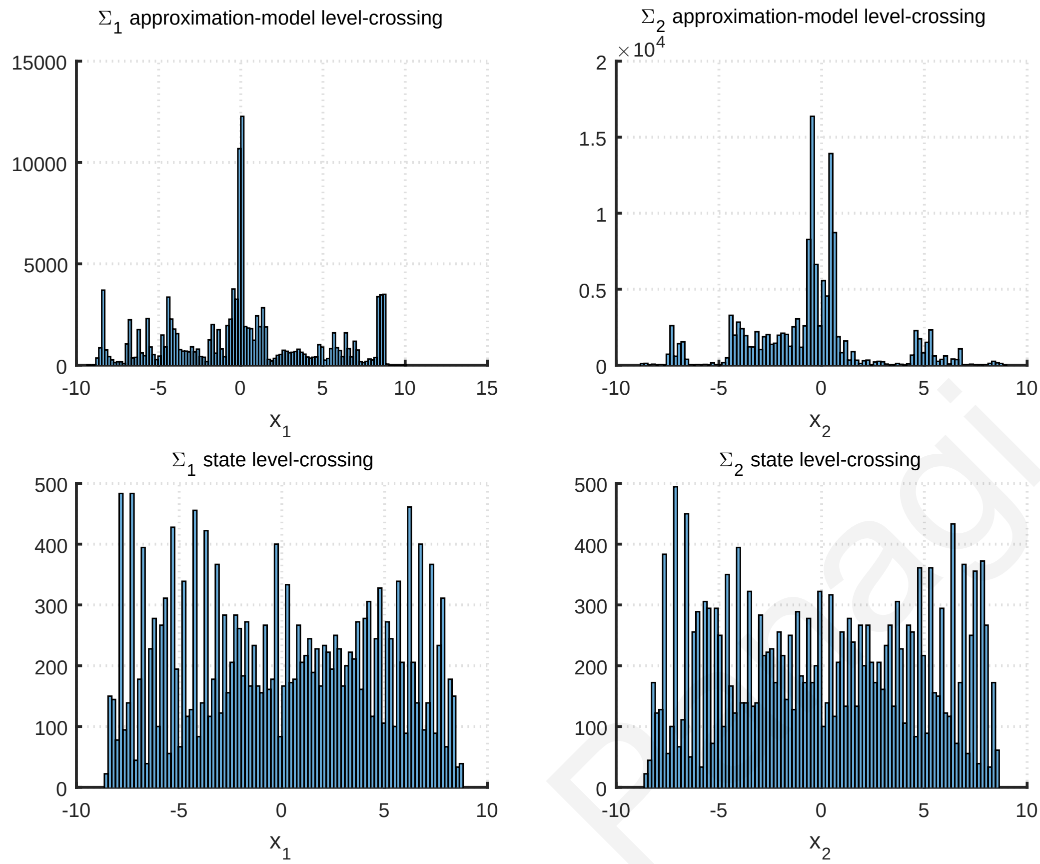


Figure 6.10: Broadcasted values of the states x_1 and x_2 for approximation-model and state level-crossing communication schemes.

the case where after $t = 200$ sec the desired state x_{d_j} is no longer available to the other subsystem, such that $\hat{\bar{x}}^i_j(t) = \bar{x}^i_j(t)$ for all $t > 200$ sec. In Fig. 6.10 we plot the histogram of broadcasted values of the state of each subsystem, for $t \in [0, 1000]$ sec. As the plot illustrates, when state level-crossing is used, the broadcasted values are spread out in the operating region. As a result, the subsystems communicate even when the change in the $\delta_{h,ij}(x_j, t)$ function is small, while not exchanging enough information when the $\delta_{h,ij}(x_j, t)$ function changes fast. In the case of approximation-model level-crossing communication, the broadcasted values are more frequent in areas where the $\delta_{h,ij}(x_j, t)$ function changes rapidly (such as the area around $x_1 = 0$), and minimal in areas where $\delta_{h,ij}(x_j, t)$ function is slowly varying or, in other words, when the replacement error is small.

Finally we compare the communication cost of the proposed communication scheme to the state level-crossing scheme, by considering the broadcast of x_2 to the subsystem Σ_1. We consider the case where the desired state x_{d_j} is not available to the i-th subsystem, $i = 1, 2$, for all $t > 0$. Moreover, the communication thresholds for the

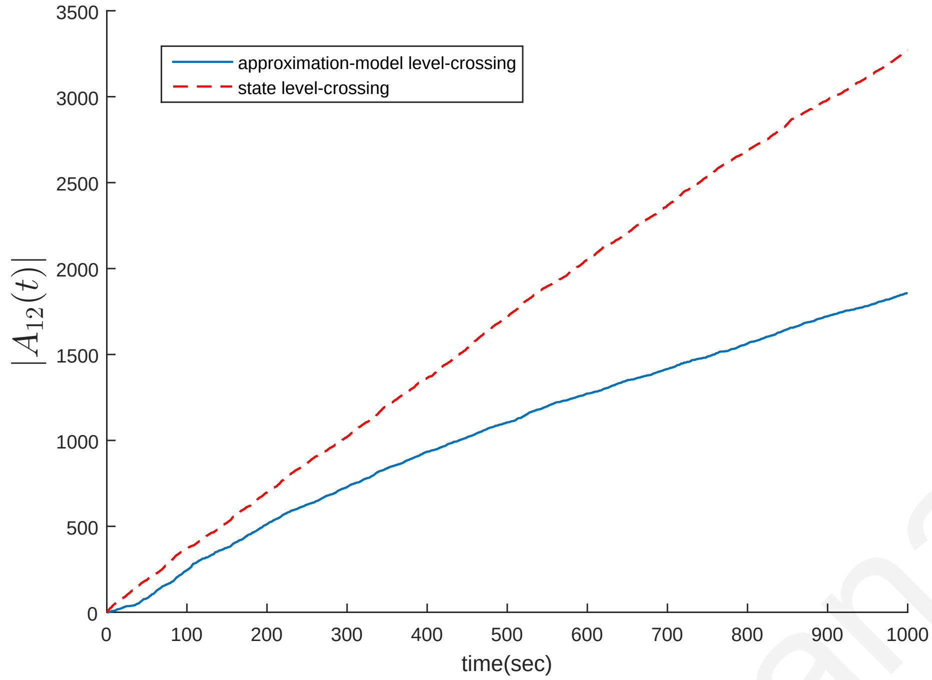


Figure 6.11: Comparison of communication cost for approximation-model and state level-crossing communication schemes.

two cases are chosen such that the sum of the performance and control-effort cost of Σ_1 $(J_{Q_1}(t) + J_{W_1}(t))$ is equalized at $t = 1000$ sec. The performance and control-effort cost is equalized with the selection of $d_{12} = 0.4$ and $\bar{\delta}_{12} = 4.6$ for the approximation-model level-crossing communication scheme, and $d_{12} = 0.8$ for the state level-crossing scheme. Fig. 6.11 shows the time evolution of the number of broadcasted samples, $|A_{12}(t)|$. The plot illustrates that the communication cost is substantially reduced when the approximation-model level-crossing algorithm is used inside the dead-zone. Intuitively, the subsystems communicate more frequently outside the dead-zone such that the approximation performance is improved, and then use this knowledge to optimize the exchange of information inside the dead-zone.

6.8 Conclusion

In this chapter we have presented a distributed fault tolerant control and communication scheme for a class of interconnected nonlinear uncertain systems. We considered the problem of designing optimized communication algorithms for optimizing the performance of the system, while minimizing the communication cost. The optimization of communication is formulated as a problem of obtaining the best approximation of the unknown coupling dynamics, based on the use of step functions. A rigorous analysis shows that step functions are universal approximators, and the step function with the best approximation property is derived. We considered a class of piecewise linear functions to show that range partitioning performs increasingly better than regular partitioning step-function approximation, as the nonlinearity of the function increases. The communication algorithm is based on the use of the local tracking errors, as well as adaptive approximation models for estimating the unknown interconnections and fault functions. By introducing adaptive approximation models into the communication decision algorithm, the subsystems are able to exchange information such that the replacement error is minimized. Robustness to the presence of residual approximation errors and replacement errors is ensured through the use of a dead-zone modification in the adaptive laws, combined with an adaptive bounding method. The use of a dead-zone prevents parameter drift in the presence of measurement noise and disturbances, as well as ensures that communication is optimized during normal operation of the system.

Chapter 7

Conclusion

7.1 Concluding Remarks and Contributions

This thesis investigated the problem of the control a class of nonlinear uncertain interconnected systems. A key challenge in interconnected systems is developing control and communication methods for dealing with uncertain interconnection dynamics. The difficulty arises from the fact that the interconnections are typically partially unknown, and at the same time, the states of other subsystems are completely or partially unknown. The situation becomes more challenging as we consider unpredictable failures that change the dynamics of the local subsystems and interconnections. In order to enhance the applicability of distributed schemes for the control of large-scale systems, it is important that the system is able to operate within certain performance margins even in the presence of faults.

A key contribution of this thesis is the development of a decentralized fault tolerant control scheme that guarantees the stability of the interconnected system, in the presence of unknown interconnections and multiple faults with significant magnitude, without the need for the exchange of state information between the subsystems (Chapter 3). We considered the class of interconnected systems where the interconnections and faults are bounded by unknown nonlinear functions of the local tracking error, γ_{ij}. Such bounding functions are satisfied in practice by several applications (e.g., inter-vehicle spacing regulation problem, [72]). Furthermore, we have shown the equivalence between bounding functions of the tracking error and bounding functions of the state. The decentralized control law is designed in an adaptive approximation framework for estimating the unknown upper bounding functions γ_{ij} of the interconnections and faults. Intuitively, the design of the decentralized fault tolerant control algorithm, is based on the *a priori* knowledge of the existence of bounding functions γ_{ij}. Each closed-loop isolated subsystem is made sufficiently stable, such that it is guaranteed it remains stable in the presence of unknown interconnections and fault functions.

The presence of even small approximation errors may cause instability issues to the feedback control scheme due to parameter drift. This issue was addressed with a dead-zone modification in the adaptive laws combined with an adaptive bounding method. The feedback control scheme guarantees the boundedness of the tracking errors to a small region around zero, inside the dead-zone. The stability analysis shows that each subsystem's tracking error enters the dead-zone in finite time. The size of the dead-zone can theoretically be designed as small as desired. However, as the dead-zone becomes smaller, the time interval to enter the dead-zone can become very large. This can drive the parameter estimates to large values and cause saturation of the control signal. Therefore the choice of the size of the dead-zone is a tradeoff between tracking performance and control effort.

A key characteristic of approximation models is that their ability to accurately represent a function is typically restricted within a compact set. Within this set, the approximation error can be as small as desired (for example, by increasing the number of basis functions). Outside this coverage set, the approximation error can become arbitrary large, to the point where no useful approximation of the function is obtained. From a control perspective, the inability of the approximator to restrict the size of the approximation error outside the coverage region can potentially destabilize the system. We address this issue with the development a decentralized safety control scheme based on sliding mode control with adaptive bounds. The design allows embedding any available knowledge about the bounds of the interconnections and faults into the decentralized safety control law, in order to reduce the control effort to bring the trajectory back into the coverage region. The stability analysis of the proposed scheme shows that the time spent outside the coverage region is finite.

There are some key limitations in the use of a completely decentralized architecture, which is usually effective only for weakly interconnected systems. In the case of strong interconnections, the local controller is typically forced to generate large control signals in order to compensate for the unknown interconnections, possibly leading to high-gain feedback. Prior research work has shown that the exchange of limited information between the subsystems can considerably reduce the control effort and improve the performance of the system. This motivates us to study distributed control schemes, where the subsystems exchange limited information online. We begin with the simple case where each subsystem continuously communicates its state whenever the local tracking error exceeds a certain design threshold. If the local tracking error is within this threshold, the other subsystems utilize the *a priori* available reference trajectory instead. Based on this communication scheme, we develop a distributed fault detection scheme based on the use of distributed nonlinear estimators (Chapter 4). The fault detection algorithm ensures that there are no false detection alarms. After a fault is detected, a fault accommodation algorithm based on adaptive approximation models is activated for estimating and accommodating the unknown fault function. A novelty of the proposed approach is that by approximating an upper bound of the unknown fault function instead of the fault function itself, robustness to the presence of residual approximation errors is ensured. The simulation study revealed a tight relationship between the choice of the communication threshold, and the ability of distributed estimators to detect faults. A smaller communication threshold allows the derivation of smaller fault detection thresholds and the fault accommodation algorithm is able to compensate for the presence of faults with less effort. At the same time, the simulation analysis revealed that for any sufficiently small communication threshold, the subsystems communicate when not needed. This phenomenon can be explained by the nonlinearity of the interconnections: a large local tracking error does not necessarily mean a large replacement error on its interconnected systems. As we consider higher-order interconnections, the local tracking error may misrepresent the magnitude of the impact on the other subsystems dynamics. As a consequence, the subsystems may communicate when not needed, while no exchange of information occurs when it could be beneficial.

Towards alleviating this problem we developed a coordinated communication scheme, in which two subsystems exchange information only when both of the tracking errors exceed a certain design threshold (Chapter 5). The basic idea is that while a subsystem performs well (i.e., the local tracking error is small), it is less likely that it needs information from other subsystems. As demonstrated by the simulation analysis, the proposed communication scheme substantially reduces the cost for communication, with no significant impact on the tracking performance. The tradeoff is that only boundedness of the tracking errors is guaranteed, while in the case of the self-triggering tracking-error based communication algorithm, asymptotic stability is ensured. However, the size of the convergence region can be made as small as desired by selecting smaller constants for certain communication thresholds. As these design constants approach zero, the coordinated communication scheme is reduced to the self-triggering tracking-error based communication scheme presented in Chapter 4.

The coordinated communication scheme presented in Chapter 5 reduces the cost for communication by avoiding to exchange information when not needed. However, it fails to address the issue where a small local tracking error can have a significant impact on the other subsystems. As the nonlinearities of the interconnection become larger, the magnitude of the replacement error can grow very large. In such cases, the replacement error can be reduced only by reducing the size of the communication threshold. Due to the high cost of communication, it is important that communication algorithms maximize the benefits from the available communication resources.

A key point in this thesis is that communication is optimized when the decision to communicate is based on the interconnection effects between the subsystems. More specifically, the exchange of information should be such that the subsystems are able to approximate, and therefore address, the interconnection functions as best as possible. In Chapter 6 we formulate the problem of communication optimization as a problem of obtaining the best approximation of the unknown coupling dynamics, based on the use of step functions. A rigorous analysis is presented which shows that a regular partition of the range of the function leads to the best $\mathcal{L}_\infty$ step function approximation. Following this analysis, a communication algorithm is presented based on the use of adaptive approximation models for estimating the unknown interconnections and fault functions. The decision to communicate is based on monitoring the value of the approximator rather than the value of the local state or tracking error. The proposed communication scheme ensures the boundedness of the replacement errors, without necessarily increasing the communication cost. Furthermore, the magnitude of the replacement errors is controllable through a communication threshold. The proposed scheme does not require the availability of estimates of remote states, as it was assumed in previous chapters. However, in practical applications it is best to incorporate as much communication-free knowledge about the other subsystems, in order to reduce the cost for communication. The simulation study shows a great reduction of the communication cost as compared to previous communication schemes, while keeping the tracking performance and control effort constant.

7.1.1 Applicability of the approach in real systems and large-scale applications

The class of interconnected systems considered in this work is large enough so that it is not only of theoretical interest, but also of practical applicability. Moreover, although we have assumed identical subsystems, by the fact that the model assumes a significant uncertainty both in the local subsystems dynamics as well as in the interconnection effects, it can be used to model a wide range of real systems. The academic nature of the simulation examples considered in this thesis, allows for illustrating the effectiveness of the proposed approach and focus on certain aspects of the feedback control scheme. In general, the analysis and design of the distributed fault tolerant control and communication methodologies presented in this thesis provides the basis for the development of algorithms for improving the stability, performance, reliability of real-world large-scale and complex systems, as well as reduce the communication requirements in practical applications. In order to investigate the applicability of the approach in real-world applications, it is needed that more complex simulation setups are considered, including: (a) a large number of subsystems, and (b) communication delays and failures. In addition, in order to broaden the applicability of the proposed approach, a more general class of interconnected systems needs to be considered, such as, non-feedback linearizable subsystems and multiple-input multiple-output (MIMO) subsystems.

7.2 Future Research Directions

7.2.1 Extension to a more general class of interconnected systems

In this thesis we have considered interconnected uncertain nonlinear single-input single-output (SISO) subsystems that are exactly feedback linearizable. Real world systems are often dynamical systems with MIMO subsystems. Therefore, extending the results of this thesis to a MIMO framework would be beneficial from an applicability perspective. Towards this direction, a key challenge is to extend the optimized communication scheme presented in Chapter 6 to the case of interconnection effects represented by vector functions. In the case of highly complex vector interconnection functions, obtaining a good approximation of the coupling dynamics becomes more difficult, and a different approach might be needed to approximate the unknown coupling dynamics and design the communication decision algorithm.

The distributed fault tolerant control and communication schemes presented in Chapters 4, 5 and 6 consider the case of linear-growth and Lipschitz interconnections. In order to enhance the applicability of the proposed distributed design, we need to consider a more general class of interconnections. The key challenge in this direction is developing control and communication methods for ensuring the boundedness of the replacements errors. In the case of Lipschitz interconnections it is guaranteed that the replacement error is relatively bounded and typically small, provided that the communication threshold d_{ij} is small. However, in the more general case of non-Lipschitz interconnections, even a very small d_{ij} can produce a very large replacement error. The distributed fault tolerant control and communication scheme needs to be able to handle arbitrary large replacement errors, both from a stability and communication cost perspective. A potential approach is to combine the distributed fault tolerant control scheme with decentralized adaptive bounding methods for enhancing the stability of the system in the presence of large replacement errors.

7.2.2 Multiple unknown interconnections and fault functions

A key assumption of the optimized communication scheme presented in Chapter 6 is that the local tracking error is a good training signal for learning the unknown interconnections and fault functions. Note that, the optimality of the communication algorithm is as good as the accuracy of the obtained approximation models. However, by basing the approximation of a potentially large number of interconnection effects on a single training signal, it becomes increasingly difficult to obtain good approximation performance. In order to ensure sufficient approximation performance and therefore improve the efficiency of communication in the case of a large number of interconnection functions, it is required that we develop more sophisticated estimation methods for approximating the unknown coupling dynamics and design separate training signals for each interconnection and fault function. A potential approach towards this direction is the development of a bank of estimators (one for each interconnection function) for estimating the effect of each interconnection and fault function onto the local subsystem dynamics. Based on this approach, a residual error is generated that is used as an augmented training signal for approximating the corresponding unknown interconnection and fault function.

7.2.3 Sampling-based function approximation

A key limitation of the proposed approach, as well as prior research work in distributed control, is the assumption of shared knowledge of reference trajectories, or the availability of online estimators of remote subsystems states. In practice the reference trajectories might change at runtime, while it may be difficult to ensure the accuracy of the state estimators throughout the operation of the system. In Chapter 6 we have presented a distributed fault tolerant control scheme that combines state estimators with communication for approximating the uncertain dynamics in the system. The simulation analysis shows that the lack of state estimators can considerably increase the approximation error.

An interesting question to explore is what approximation performance can we expect when only discrete samples of the states of remote subsystems are available. In order to motivate this problem, consider the distributed input-output system given by $y_i = \delta(x_j)$, where $x_j \in \mathcal{X}_j \subset \mathbb{R}$ is the input, $y_i \in \mathbb{R}$ is the output, and $\delta : \mathbb{R} \to \mathbb{R}$ is an unknown continuous function. Assume that an observer has access to x_j for all $t > 0$ and transmits samples $\bar{x}_j^i(t)$ to an approximator $\hat{\delta}$, according to a level-crossing communication algorithm, with a certain communication threshold $d > 0$. The objective is the design of an adaptive approximation framework for approximating the unknown function δ, based only on the use of the communicated samples $\bar{x}_j^i(t)$ and the measurement of y, which is available for all $t > 0$. Intuitively, if the samples are close enough (i.e., if the communication threshold d is small), the approximation performance tends to the case where x_j is available for all $t > 0$. However, we need to determine the relationship between the best approximation performance and the properties of the function, as well as the choice of the communication threshold d. In addition, we need to explore alternative communication algorithms for enhancing the performance of the approximation. Finally, we need to investigate the use of other approximation structures (such as, piecewise linear functions and splines) that may be more suitable for function approximation, provided that only samples of the input are available.

Bibliography

[1] "The On-Line Encyclopedia of Integer Sequence," http://oeis.org/A180281.

[2] A. Anta and P. Tabuada, "To sample or not to sample: Self-triggered control for nonlinear systems," *Automatic Control, IEEE Transactions on*, vol. 55, no. 9, pp. 2030–2042, 2010.

[3] K.-E. Årzén, "A simple event-based pid controller," in *Proc. 14th IFAC World Congress*, vol. 18, 1999, pp. 423–428.

[4] K. J. Åström and B. Bernhardsson, "Comparison of periodic and event based sampling for first-order stochastic systems," in *Proc. of the 14th IFAC World congress*, vol. 11, 1999, pp. 301–306.

[5] K. Astrom and B. Bernhardsson, "Comparison of Riemann and Lebesgue sampling for first order stochastic systems," in *Proc. of the 41st Conference on Decision and Control*, vol. 2, 2002, pp. 2011–2016.

[6] M. Blanke, *Diagnosis and fault-tolerant control.* Springer Verlag, 2003.

[7] M. Bodson and J. Groszkiewicz, "Multivariable adaptive algorithms for reconfigurable flight control," *Control Systems Technology, IEEE Transactions on*, vol. 5, no. 2, pp. 217–229, 1997.

[8] N. Bourbaki, *Elements of Mathematics. General Topology.* Springer Verlag, 1989.

[9] W. Chen and J. Li, "Decentralized output-feedback neural control for systems with unknown interconnections," *Systems, Man, and Cybernetics, Part B: Cybernetics, IEEE Transactions on*, vol. 38, no. 1, pp. 258–266, 2008.

[10] J. Choi and J. Farrell, "Adaptive observer backstepping control using neural networks," *Neural Networks, IEEE Transactions on*, vol. 12, no. 5, pp. 1103–1112, 2001.

[11] M. Demetriou and M. M. Polycarpou, "On-line fault detection, diagnosis, isolation and accommodation of dynamical systems with actuator failures," *Godalming, United Kingdom, Springer-Verlag, London*, pp. 85–110, 2001.

[12] J. DePree and C. W. Swartz, *Introduction to Real Analysis.* John Wiley & Sons, New York, 1988.

[13] J. Dieudonne, *Foundations of modern analysis.* Academic Press, New York, 1960, vol. 286.

[14] D. V. Dimarogonas, E. Frazzoli, and K. H. Johansson, "Distributed event-triggered control for multi-agent systems," *Automatic Control, IEEE Transactions on*, vol. 57, no. 5, pp. 1291–1297, 2012.

[15] D. Dimarogonas and K. Johansson, "Event-triggered control for multi-agent systems," in *Proc. of the 48th Conference on Decision and Control*, 2010, pp. 7131–7136.

[16] X. Ding and P. Frank, "An adaptive observer-based fault detection scheme for nonlinear dynamic systems," *Proc. IFAC World Congress*, vol. 93, pp. 307–311, 1993.

[17] J. G. Dunham, "Optimum uniform piecewise linear approximation of planar curves," *Pattern Analysis and Machine Intelligence, IEEE Transactions on*, no. 1, pp. 67–75, 1986.

[18] J. Eterno, J. Weiss, D. Looze, A. Willsky, I. Alphatech, and M. Burlington, "Design issues for fault tolerant-restructurable aircraft control," *Proc. of the 24th IEEE Conference on Decision and Control*, vol. 24, 1985.

[19] J. Farrell and M. Polycarpou, *Adaptive Approximation Based Control.* John Wiley & Sons, Hoboken, NJ, USA, 2006.

[20] R. Ferrari, T. Parisini, and M. Polycarpou, "Distributed fault diagnosis with overlapping decompositions: an adaptive approximation approach," *Automatic Control, IEEE Transactions on*, vol. 54, no. 4, pp. 794–799, 2009.

[21] P. Fitzpatrick, *Advanced calculus.* American Mathematical Society, 2006, vol. 5.

[22] P. Frank and X. Ding, "Survey of robust residual generation and evaluation methods in observer-based fault detection systems," *Journal of process control*, vol. 7, no. 6, pp. 403–424, 1997.

[23] E. Garcia and P. J. Antsaklis, "Model-based event-triggered control for systems with quantization and time-varying network delays," *Automatic Control, IEEE Transactions on*, vol. 58, no. 2, pp. 422–434, 2013.

[24] D. Gavel and D. Siljak, "Decentralized adaptive control: structural conditions for stability," *Automatic Control, IEEE Transactions on*, vol. 34, no. 4, pp. 413–426, 1989.

[25] J. Gertler, "Analytical Redundancy Methods in Fault Detection and Isolation," *Fault Detection, Supervision and Safety for Technical Processes*, pp. 9–21, 1992.

[26] Y. Guo, D. Hill, and Y. Wang, "Nonlinear decentralized control of large-scale power systems," *Automatica*, vol. 36, no. 9, pp. 1275–1289, 2000.

[27] W. Heemels, J. Sandee, and P. Van Den Bosch, "Analysis of event-driven controllers for linear systems," *International Journal of Control*, vol. 81, no. 4, pp. 571–590, 2008.

[28] S. Huang, K. Tan, and T. Lee, "Decentralized control design for large-scale systems with strong interconnections using neural networks," *Automatic Control, IEEE Transactions on*, vol. 48, no. 5, pp. 805–810, 2003.

[29] P. Ioannou, "Decentralized adaptive control of interconnected systems," *Automatic Control, IEEE Transactions on*, vol. 31, pp. 291–298, 1986.

[30] P. Ioannou and J. Sun, *Robust Adaptive Control.* Prentice Hall, New Jersey, 1995.

[31] S. Jain and F. Khorrami, "Decentralized adaptive output feedback design for large-scale nonlinear systems," *Automatic Control, IEEE Transactions on*, vol. 42, no. 5, pp. 729–735, 1997.

[32] H. Jones, "Failure detection in linear systems." *Massachusetts Institute of Technology*, 1973.

[33] H. K. Khalil, *Nonlinear systems.* Prentice Hall, New Jersey, 2002.

[34] E. Kofman and J. H. Braslavsky, "Level crossing sampling in feedback stabilization under data-rate constraints," in *Proc. of the 45th Conference on Decision and Control*, 2006, pp. 4423–4428.

[35] M. Krstic, I. Kanellakopoulos, and P. Kokotovic, "Nonlinear design of adaptive controllers for linear systems," *IEEE Transactions on Automatic Control*, vol. 39, no. 4, pp. 738–752, 1994.

[36] F. L. Lewis, S. Jagannathan, and A. Yeildirek, *Neural Network Control of Robot Manipulators and Nonlinear Systems.* Taylor & Francis, London, 1999.

[37] J. Li, W. Chen, and J.-M. Li, "Adaptive NN output-feedback decentralized stabilization for a class of large-scale stochastic nonlinear strict-feedback systems," *International Journal of Robust and Nonlinear Control*, vol. 21, no. 4, pp. 452–472, 2011.

[38] W. Li, W. Gui, Y. Xie, and S. X. Ding, "Decentralized fault detection system design for large-scale interconnected systems," in *Fault Detection, Supervision and Safety of Technical Processes*, 2009, pp. 816–821.

[39] F. Lian, J. Yook, D. Tilbury, and J. Moyne, "Network architecture and communication modules for guaranteeing acceptable control and communication performance for networked multi-agent systems," *Industrial Informatics, IEEE Transactions on*, vol. 2, no. 1, pp. 12–24, 2006.

[40] M. Makoudi and L. Radouane, "A robust decentralized model reference adaptive control for non-minimum-phase interconnected systems," *Automatica*, vol. 35, no. 8, pp. 1499–1508, 1999.

[41] M. Massoumnia, G. Verghese, A. Willsky, and C. MIT, "Failure detection and identification," *Automatic Control, IEEE Transactions on*, vol. 34, no. 3, pp. 316–321, 1989.

[42] J. Meiry and R. Beard, "Failure accomodation in linear systems through self-reorganization." *Massachusetts Institute of Technology*, 1971.

[43] B. M. Mirkin, "Decentralized adaptive controller with zero residual tracking errors," in *Proc. of the 7th Mediterranean Conference on Control and Automation*, 1999, pp. 28–30.

[44] M. Miskowicz, "Send-on-delta concept: an event-based data reporting strategy," *Sensors*, vol. 6, no. 1, pp. 49–63, 2006.

[45] B. Y. N. Hovakimyan, E. Lavretsky and A. Calise, "Coordinated decentralized adaptive output feedback control of interconnected systems," *Neural Networks, IEEE Transactions on*, vol. 16, pp. 1045–9227, 2005.

[46] K. Narendra, N. Oleng, and S. Mukhopadhyay, "Decentralised adaptive control with partial communication," *IEE Proc. Control Theory and Applications*, vol. 153, no. 5, pp. 546–555, 2006.

[47] K. Narendra and N. Oleng, "Exact output tracking in decentralized adaptive control systems," *IEEE Transactions on Automatic Control*, vol. 47, no. 2, pp. 390–395, 2002.

[48] K. S. Narendra and S. Mukhopadhyay, "To communicate or not to communicate: A decision-theoretic approach to decentralized adaptive control," in *Proc. of the 2010 American Control Conference*, 2010, pp. 6369–6376.

[49] P. Panagi and M. M. Polycarpou, "Decentralized adaptive approximation based control of a class of large-scale systems," *Proc. of the 2008 American Control Conference*, pp. 4191–4196, 2008.

[50] P. Panagi and M. M. Polycarpou, "Decentralized adaptive approximation based control with safety scheme outside the approximation region," *Proc. of the 47th IEEE Conference on Decision and Control*, pp. 92–97, 2008.

[51] P. Panagi and M. M. Polycarpou, "Decentralized fault accommodation of a class of interconnected nonlinear systems using an adaptive approximation approach," *Proc. of the 17th Mediterranean Conference on Control and Automation*, pp. 546–551, 2009.

[52] P. Panagi and M. M. Polycarpou, "Distributed fault detection and accommodation with partial communication," *Proc. of the 4th International Symposium on Communications, Control and Signal Processing*, pp. 1–5, 2010.

[53] P. Panagi and M. M. Polycarpou, "Decentralized fault tolerant control of a class of interconnected nonlinear systems," *Automatic Control, IEEE Transactions on*, vol. 56, no. 1, pp. 178–184, 2011.

[54] P. Panagi and M. M. Polycarpou, "Distributed fault accommodation for a class of interconnected nonlinear systems with partial communication," *Automatic Control, IEEE Transactions on*, vol. 56, no. 12, pp. 2962–2967, 2011.

[55] P. Panagi and M. M. Polycarpou, "A coordinated communication scheme for distributed fault tolerant control," *Industrial Informatics, IEEE Transactions on*, vol. 9, no. 1, pp. 386–393, 2013.

[56] R. J. Patton, "Fault-tolerant control systems: The 1997 situation," in *IFAC symposium on fault detection supervision and safety for technical processes*, vol. 3, 1997, pp. 1033–1054.

[57] R. J. Patton and S. Klinkhieo, "A two-level approach to fault-tolerant control of distributed systems based on the sliding mode," in *Fault Detection, Supervision and Safety of Technical Processes*, 2009, pp. 1043–1048.

[58] R. J. Patton, C. Kambhampati, A. Casavola, P. Zhang, S. Ding, and D. Sauter, "A generic strategy for fault-tolerance in control systems distributed over a network," *European journal of control*, vol. 13, no. 2, pp. 280–296, 2007.

[59] J. Ploennigs, V. Vasyutynskyy, and K. Kabitzsch, "Comparative study of energy-efficient sampling approaches for wireless control networks," *Industrial Informatics, IEEE Transactions on*, vol. 6, no. 3, pp. 416–424, 2010.

[60] M. Polycarpou, "Stable Adaptive Neural Control Scheme for Nonlinear-Systems," *Automatic Control, IEEE Transactions on*, vol. 41, no. 3, pp. 447–451, 1996.

[61] M. Polycarpou and A. Helmicki, "Automated fault detection and accommodation: a learning systems approach," *Systems, Man and Cybernetics, IEEE Transactions on*, vol. 25, no. 11, pp. 1447–1458, 1995.

[62] M. Polycarpou and P. Ioannou, "A robust adaptive nonlinear control design," *Automatica*, vol. 32, no. 3, pp. 423–427, 1996.

[63] M. Polycarpou and A. Trunov, "Learning approach to nonlinear fault diagnosis: detectability analysis," *IEEE Transactions on Automatic Control*, vol. 45, no. 4, pp. 806–812, 2000.

[64] V. Raghunathan, C. Schurgers, S. Park, and M. Srivastava, "Energy-aware wireless microsensor networks," *Signal Processing Magazine, IEEE*, vol. 19, no. 2, pp. 40–50, 2002.

[65] N. Sandell Jr, P. Varaiya, M. Athans, and M. Safonov, "Survey of decentralized control methods for large scale systems," *Automatic Control, IEEE Transactions on*, vol. 23, no. 2, pp. 108–128, 1978.

[66] R. Sanner and J. Slotine, "Gaussian networks for direct adaptive control," *IEEE Transactions on Neural Networks*, vol. 3, no. 6, pp. 837–863, 1992.

[67] N. Sayiner, H. Sorensen, and T. Viswanathan, "A level-crossing sampling scheme for a/d conversion," *Circuits and Systems II: Analog and Digital Signal Processing, IEEE Transactions on*, vol. 43, no. 4, pp. 335–339, 1996.

[68] S. Seshagiri and H. Khalil, "Output feedback control of nonlinear systems using RBF neural networks," *Neural Networks, IEEE Transactions on*, vol. 11, no. 1, pp. 69–79, 2000.

[69] L. Shi and S. Singh, "Decentralized adaptive controller design for large-scale systems with higher order interconnections," *Automatic Control, IEEE Transactions on*, vol. 37, no. 8, pp. 1106–1118, 1992.

[70] D. D. Siljak, *Decentralized control of complex systems.* Courier Corporation, 2011.

[71] J. Spooner and K.M.Passino, "Decentralized adaptive control of nonlinear systems using radial basis neural networks," *Automatic Control, IEEE Transactions on*, vol. 44, no. 11, pp. 2050–2057, 1999.

[72] J. Spooner and K. Passino, "Adaptive control of a class of decentralized nonlinear systems," *Automatic Control, IEEE Transactions on*, vol. 41, no. 2, pp. 280–284, 1996.

[73] J. Spooner and K. Passino, "Stable adaptive control using fuzzy systems and neural networks," *Fuzzy Systems, IEEE Transactions on*, vol. 4, no. 3, pp. 339–359, 1996.

[74] X. Tang, G. Tao, and S. Joshi, "Adaptive actuator failure compensation for nonlinear MIMO systems with an aircraft control application," *Automatica*, vol. 43, no. 11, pp. 1869–1883, 2007.

[75] Y. Tang, M. Tomizuka, G. Guerrero, and G. Montemayor, "Decentralized Robust Control of Mechanical Systems," *Automatic Control, IEEE Transactions on*, vol. 45, no. 4, p. 771, 2000.

[76] G. Tao, S. Joshi, and X. Ma, "Adaptive state feedback and tracking control of systems with actuator failures," *Automatic Control, IEEE Transactions on*, vol. 46, no. 1, pp. 78–95, 2001.

[77] S. Tong, H. Li, and G. Chen, "Adaptive fuzzy decentralized control fora class of large-scale nonlinear systems," *Systems, Man and Cybernetics, Part B, IEEE Transactions on*, vol. 34, no. 1, pp. 770–775, 2004.

[78] S. Tong and Y. Li, "Adaptive fuzzy output feedback tracking backstepping control of strict-feedback nonlinear systems with unknown dead zones," *Fuzzy Systems, IEEE Transactions on*, vol. 20, no. 1, pp. 168–180, 2012.

[79] A. Vemuri and M. Polycarpou, "Neural-network-based robust fault diagnosis in robotic systems," *IEEE transactions on neural networks*, vol. 8, no. 6, pp. 1410–1420, 1997.

[80] A. Vemuri and M. Polycarpou, "Robust nonlinear fault diagnosis in input-output systems," *International Journal of Control*, vol. 68, no. 2, pp. 343–360, 1997.

[81] X. Wang and M. Lemmon, "Event-triggering in distributed networked control systems," *Automatic Control, IEEE Transactions on*, no. 99, pp. 1–1, 2011.

[82] X. Wang and N. Hovakimyan, "Distributed control of uncertain networked systems: A decoupled design," *Automatic Control, IEEE Transactions on*, vol. 58, no. 10, pp. 2536–2549, 2013.

[83] Y. Wang, H. Ye, S. Ding, Y. Cheng, P. Zhang, and G. Wang, "Fault detection of networked control systems with limited communication," *International Journal of Control*, vol. 82, no. 7, pp. 1344–1356, 2009.

[84] G. Yang, S. Zhang, J. Lam, and J. Wang, "Reliable control using redundant controllers," *Automatic Control, IEEE Transactions on*, vol. 43, no. 11, pp. 1588–1593, 1998.

[85] Y. Yang, G. Feng, and J. Ren, "A combined backstepping and small-gain approach to robust adaptive fuzzy control for strict-feedback nonlinear systems," *IEEE Transactions on Systems, Man and Cybernetics, Part A*, vol. 34, no. 3, pp. 406–420, 2004.

[86] J. K. Yook, D. M. Tilbury, and N. R. Soparkar, "Trading computation for bandwidth: Reducing communication in distributed control systems using state estimators," *Control Systems Technology, IEEE Transactions on*, vol. 10, no. 4, pp. 503–518, 2002.

[87] T. Zhang, S. S. Ge, and C. C. Hang, "Adaptive neural network control for strict-feedback nonlinear systems using backstepping design," *Automatica*, vol. 36, no. 12, pp. 1835–1846, 2000.

[88] X. Zhang, T. Parisini, and M. Polycarpou, "Adaptive fault-tolerant control of nonlinear uncertain systems: an information-based diagnostic approach," *Automatic Control, IEEE Transactions on*, vol. 49, no. 8, pp. 1259–1274, 2004.

[89] X. Zhang, T. Parisini, and M. Polycarpou, "Adaptive fault-tolerant control of nonlinear uncertain systems: an information-based diagnostic approach," *Automatic Control, IEEE Transactions on*, vol. 49, no. 8, pp. 1259–1274, 2004.

[90] X. Zhang, M. Polycarpou, and T. Parisini, "Decentralized fault detection in a class of large-scale nonlinear uncertain systems," *Proc. of the 48th IEEE Conference on Decision and Control*, pp. 5650–5655, 2009.

[91] Y. Zhang and J. Jiang, "Bibliographical review on reconfigurable fault-tolerant control systems," *Annual Reviews in Control*, vol. 32, no. 2, pp. 229–252, 2008.

[92] Y. Zhang and J. Jiang, "Active fault-tolerant control system against partial actuator failures," *IEE Proc. Control Theory and Applications*, vol. 149, no. 1, pp. 95–104, 2002.

[93] M. Zhong and C. Cassandras, "Asynchronous distributed optimization with event-driven communication," *Automatic Control, IEEE Transactions on*, no. 99, p. 1, 2010.

[94] M. Zhong, S. Ding, J. Lam, and H. Wang, "An LMI approach to design robust fault detection filter for uncertain LTI systems* 1," *Automatica*, vol. 39, no. 3, pp. 543–550, 2003.

[95] Q. Zhou, P. Shi, H. Liu, and S. Xu, "Neural-network-based decentralized adaptive output-feedback control for large-scale stochastic nonlinear systems," *Systems, Man, and Cybernetics, Part B: Cybernetics, IEEE Transactions on*, vol. 42, no. 6, pp. 1608–1619, 2012.

Appendices

Proof of Lemma 1.1

Let the Lyapunov function be given by $V = \frac{1}{2}\sum_{i=1}^{m} \tilde{x}_i^2$. Substituting u_i from (1.6) into the tracking error dynamics (1.5), the time derivative of V satisfies

$$\dot{V} = \sum_{i=1}^{m} -k_i\tilde{x}_i^2 + \sum_{j=1}^{m} \tilde{x}_i\theta_{ij}\phi_{ij}(x_j).$$

From (1.4) we have

$$\dot{V} \leqslant \sum_{i=1}^{m} -k_i\tilde{x}_i^2 + \sum_{j=1}^{m} |\tilde{x}_i|\theta_{ij}L_{ij}|\tilde{x}_j| + |\tilde{x}_i|\theta_{ij}\bar{\sigma}_{ij}.$$

Using the inequality $2\alpha\beta \leqslant \alpha^2 + \beta^2$ for $\alpha, \beta \in \mathbb{R}$,

$$\begin{aligned}\dot{V} &\leqslant \sum_{i=1}^{m} -k_i\tilde{x}_i^2 + \sum_{j=1}^{m} \frac{1}{2}\theta_{ij}^2 L_{ij}^2\tilde{x}_i^2 + \frac{1}{2}\tilde{x}_j^2 + |\tilde{x}_i|\theta_{ij}\bar{\sigma}_{ij} \\ &\leqslant -\sum_{i=1}^{m}\sum_{j=1}^{m}\Big[\big(k_i - \frac{1}{2} - \frac{1}{2}\theta_{ij}^2 L_{ij}^2\big)|\tilde{x}_i| - \theta_{ij}\bar{\sigma}_{ij}\Big]|\tilde{x}_i|\end{aligned}$$

Which shows that for $k_i = \lambda_i + \frac{1}{2}\sum_{j=1, j\neq i}^{m} 1 + \theta_{ij}^2 L_{ij}^2$, for some $\lambda_i > 0$, $\tilde{x}_i$ is ultimately bounded with the ultimate bound $|\tilde{x}_i| < \frac{\theta_{ij}\bar{\sigma}_{ij}}{\lambda_i}$.

Proof of Lemma 1.2

Let the Lyapunov function be given by $V = \sum_{i=1}^{m} \frac{1}{2}\tilde{x}_i^2 + \sum_{j=1}^{m} \frac{1}{2\gamma_{ij}^a}\tilde{\theta}_{ij}^{a^2} + \frac{1}{2\gamma_{ij}^b}\tilde{\theta}_{ij}^{b^2}$, where $\tilde{\theta}_{ij}^a = \hat{\theta}_{ij}^a - \theta_{ij}^a$ and $\tilde{\theta}_{ij}^b = \hat{\theta}_{ij}^b - \theta_{ij}^b$ are the parameter estimation errors. Substituting u_i from (1.11) into the tracking error dynamics (1.5), the time derivative of V satisfies

$$\dot{V} \leqslant \sum_{i=1}^{m} -k_i\tilde{x}_i^2 - \tilde{x}_i u_{s_i} + \sum_{j=1}^{m} \tilde{x}_i\theta_{ij}\phi_{ij}(x_j) + \frac{1}{\gamma_{ij}^a}\tilde{\theta}_{ij}^a\dot{\tilde{\theta}}_{ij}^a + \frac{1}{\gamma_{ij}^b}\tilde{\theta}_{ij}^b\dot{\tilde{\theta}}_{ij}^b.$$

Based on (1.8) we obtain

$$\begin{aligned}\dot{V} \leqslant \sum_{i=1}^{m} -k_i\tilde{x}_i^2 - \tilde{x}_i u_{s_i} &+ \sum_{j=1}^{m} \left(\frac{1}{2} + \frac{1}{2}\theta_{ij}^2 L_{ij}^2\right)\tilde{x}_i^2 + |\tilde{x}_i|\theta_{ij}\bar{\sigma}_{ij} \\ &+ \sum_{j=1}^{m} \frac{1}{\gamma_{ij}^a}\tilde{\theta}_{ij}^a\dot{\tilde{\theta}}_{ij}^a + \frac{1}{\gamma_{ij}^b}\tilde{\theta}_{ij}^b\dot{\tilde{\theta}}_{ij}^b.\end{aligned}$$

Using (1.9) and (1.10), and substituting u_{s_i} from (1.12) we obtain

$$\dot{V} \leqslant \sum_{i=1}^{m} -k_i\tilde{x}_i^2 + \sum_{j=1}^{m} -\tilde{\theta}_{ij}^a\tilde{x}_i^2 - \tilde{\theta}_{ij}^b|\tilde{x}_i| + \frac{1}{\gamma_{ij}^a}\tilde{\theta}_{ij}^a\dot{\tilde{\theta}}_{ij}^a + \frac{1}{\gamma_{ij}^b}\tilde{\theta}_{ij}^b\dot{\tilde{\theta}}_{ij}^b.$$

Substituting the adaptive laws (1.13) and (1.14) shows that $\dot{V} \leqslant -\sum_{i=1}^{m} k_i\tilde{x}_i^2$, which completes the proof.

Proof of Lemma 1.3

Let the Lyapunov function be given by $V = \sum_{i=1}^{m} \frac{1}{2}\tilde{x}_i^2 + \sum_{j=1}^{m} \frac{1}{2\gamma_{ij}^a}\tilde{\theta}_{ij}^{a^2} + \frac{1}{2\gamma_{ij}}\tilde{\theta}_{ij}^2$, where $\tilde{\theta}_{ij}^a = \hat{\theta}_{ij}^a - \theta_{ij}^a$ and $\tilde{\theta}_{ij} = \hat{\theta}_{ij} - \theta_{ij}$ are the parameter estimation errors. Substituting u_i from (1.15) into the tracking error dynamics (1.5), the time derivative of V satisfies

$$\dot{V} \leqslant \sum_{i=1}^{m} -k_i\tilde{x}_i^2 - \tilde{x}_i u_{s_i} + \sum_{j=1}^{m} \tilde{x}_i\theta_{ij}\phi_{ij}(x_j) + \frac{1}{\gamma_{ij}^a}\tilde{\theta}_{ij}^a\dot{\tilde{\theta}}_{ij}^a + \frac{1}{\gamma_{ij}}\tilde{\theta}_{ij}\dot{\tilde{\theta}}_{ij}.$$

Adding and subtracting $\theta_{ij}\phi_{ij}(x_{d_j})$ and substituting u_{s_i} from (1.16) we obtain

$$\dot{V} \leqslant \sum_{i=1}^{m} -k_i\tilde{x}_i^2 + \sum_{j=1}^{m} \tilde{x}_i\big(\theta_{ij}\phi_{ij}(x_j) - \theta_{ij}\phi_{ij}(x_{d_j})\big) + \tilde{x}_i\tilde{\theta}_{ij}\phi_{ij}(x_{d_j}) - \hat{\theta}_{ij}^a\tilde{x}_i^2 \\ + \frac{1}{\gamma_{ij}^a}\tilde{\theta}_{ij}^a\dot{\tilde{\theta}}_{ij}^a + \frac{1}{\gamma_{ij}}\tilde{\theta}_{ij}\dot{\tilde{\theta}}_{ij}$$

Substituting the adaptive law (1.17) we obtain

$$\dot{V} \leqslant \sum_{i=1}^{m} -k_i\tilde{x}_i^2 + \sum_{j=1}^{m} \tilde{x}_i\big(\theta_{ij}\phi_{ij}(x_j) - \theta_{ij}\phi_{ij}(x_{d_j})\big) - \hat{\theta}_{ij}^a\tilde{x}_i^2 + \frac{1}{\gamma_{ij}^a}\tilde{\theta}_{ij}^a\dot{\tilde{\theta}}_{ij}^a$$

From (1.2) we have

$$\dot{V} \leqslant \sum_{i=1}^{m} -k_i\tilde{x}_i^2 + \sum_{j=1}^{m} |\tilde{x}_i||\theta_{ij}|L_{ij}|\tilde{x}_j| - \hat{\theta}_{ij}^a\tilde{x}_i^2 + \frac{1}{\gamma_{ij}^a}\tilde{\theta}_{ij}^a\dot{\tilde{\theta}}_{ij}^a$$

Using the inequality $2\alpha\beta \leqslant \alpha^2 + \beta^2$ for $\alpha, \beta \in \mathbb{R}$,

$$\dot{V} \leqslant \sum_{i=1}^{m} -k_i\tilde{x}_i^2 + \sum_{j=1}^{m} \frac{1}{2}\tilde{x}_i^2\theta_{ij}^2L_{ij}^2 + \frac{1}{2}\tilde{x}_j^2 - \hat{\theta}_{ij}^a\tilde{x}_i^2 + \frac{1}{\gamma_{ij}^a}\tilde{\theta}_{ij}^a\dot{\tilde{\theta}}_{ij}^a$$

Using (1.9) we obtain

$$\dot{V} \leqslant \sum_{i=1}^{m} -k_i\tilde{x}_i^2 - \sum_{j=1}^{m} \tilde{x}_i^2\tilde{\theta}_{ij}^a + \frac{1}{\gamma_{ij}^a}\tilde{\theta}_{ij}^a\dot{\tilde{\theta}}_{ij}^a$$

Substituting the adaptive law (1.13) we obtain that $\dot{V} \leqslant -\sum_{i=1}^{m} k_i\tilde{x}_i^2$ which completes the proof.

Proof of Lemma 1.4

Let the Lyapunov function be given by $V = \sum_{i=1}^{m} \frac{1}{2}\tilde{x}_i^2 + \sum_{j=1}^{m} \frac{1}{2\gamma_{ij}^c}\tilde{\theta}_{ij}^{c^2} + \frac{1}{2\gamma_{ij}}\tilde{\theta}_{ij}^2$, where $\tilde{\theta}_{ij}^c = \hat{\theta}_{ij}^c - \theta_{ij}^c$ and $\tilde{\theta}_{ij} = \hat{\theta}_{ij} - \theta_{ij}$ are the parameter estimation errors. Substituting u_i from (1.21) into the tracking error dynamics (1.5), the time derivative of V satisfies

$$\dot{V} \leqslant \sum_{i=1}^{m} -k_i\tilde{x}_i^2 - \tilde{x}_i u_{s_i} + \sum_{j=1}^{m} \tilde{x}_i\theta_{ij}\phi_{ij}(x_j) + \frac{1}{\gamma_{ij}^c}\tilde{\theta}_{ij}^c\dot{\tilde{\theta}}_{ij}^c + \frac{1}{\gamma_{ij}}\tilde{\theta}_{ij}\dot{\tilde{\theta}}_{ij}.$$

Adding and subtracting $\theta_{ij}\phi_{ij}(\hat{x}_j^i)$ and substituting u_{s_i} from (1.22) we obtain

$$\begin{aligned}\dot{V} \leqslant \sum_{i=1}^{m} -k_i\tilde{x}_i^2 &+ \sum_{j=1}^{m} \tilde{x}_i\big(\theta_{ij}\phi_{ij}(x_j) - \theta_{ij}\phi_{ij}(\hat{x}_j^i)\big) + \tilde{x}_i\tilde{\theta}_{ij}\phi_{ij}(\hat{x}_j^i) - \hat{\theta}_{ij}^c|\tilde{x}_i| \\ &+ \frac{1}{\gamma_{ij}^c}\tilde{\theta}_{ij}^c\dot{\tilde{\theta}}_{ij}^c + \frac{1}{\gamma_{ij}}\tilde{\theta}_{ij}\dot{\tilde{\theta}}_{ij}\end{aligned}$$

Substituting the adaptive law (1.24) we obtain

$$\dot{V} \leqslant \sum_{i=1}^{m} -k_i\tilde{x}_i^2 + \sum_{j=1}^{m} \tilde{x}_i\big(\theta_{ij}\phi_{ij}(x_j) - \theta_{ij}\phi_{ij}(\hat{x}_j^i)\big) - \hat{\theta}_{ij}^c|\tilde{x}_i| + \frac{1}{\gamma_{ij}^c}\tilde{\theta}_{ij}^c\dot{\tilde{\theta}}_{ij}^c$$

From (1.2) we have

$$\dot{V} \leqslant \sum_{i=1}^{m} -k_i\tilde{x}_i^2 + \sum_{j=1}^{m} |\tilde{x}_i||\theta_{ij}|L_{ij}|x_j - \hat{x}_j^i| - \hat{\theta}_{ij}^c|\tilde{x}_i| + \frac{1}{\gamma_{ij}^c}\tilde{\theta}_{ij}^c\dot{\tilde{\theta}}_{ij}^c$$

From (1.19) and (1.20) we obtain

$$\dot{V} \leqslant \sum_{i=1}^{m} -k_i\tilde{x}_i^2 - \sum_{j=1}^{m} \tilde{\theta}_{ij}^c|\tilde{x}_i| + \frac{1}{\gamma_{ij}^c}\tilde{\theta}_{ij}^c\dot{\tilde{\theta}}_{ij}^c$$

Substituting the adaptive law (1.23) shows that $\dot{V} \leqslant -\sum_{i=1}^{m} k_i\tilde{x}_i^2$ which completes the proof.