ELECTROMAGNETIC BARRIER PENETRATION: MATLAB SIMULATION

A. Shalav, D.N. Pinder

ABSTRACT
A Matlab simulation was developed to help visualise and investigate electromagnetic tunnelling through particular non-dissipative and dissipative barriers within a waveguide. The theory behind the simulation is based on a transmission line model that accurately predicts previously published experimental results. The simulation (both development and use) demonstrates that physical concepts can be taught using various learning processes with an appreciation of the mathematics involved.

KEYWORDS
Electromagnetism, wave-guide, super luminal

INTRODUCTION

The four modes of learning as described by Fleming[1] consist of visual, aural, reading/writing and kinaesthetic experiences (VARK). Many classroom and lecture learning experiences are dominated by the aural and reading/writing learning modes[2]. A computer based simulation may provide useful learning experiences for the visual and kinaesthetic learner.

Students need to be motivated to study advanced undergraduate electromagnetism since the subject can appear to be abstruse and remote. A good way to motivate students is to encourage them to investigate a phenomenon that they find interesting and intriguing, such as super luminal propagation. Avi Shalav has developed a computer simulation package in partial fulfillment of the requirements for an MSc in physics. This simulation describes the propagation of a microwave pulse through a barrier within a rectangular metal wave-guide. The barrier can be dissipative or non-dissipative, the incident pulse shape and barrier length can be user defined, and the incident and transmitted pulses displayed for comparison purposes. The pulse suffers severe frequency dependent attenuation and distortion in passing through the barrier so pulse speed is not a trivial concept. Various definitions of pulse speed are presented and discussed and some speeds are also computed in the simulation. The simulation package confronts students with a host of important electromagnetic concepts, for example wave-guide propagation, transmission line analysis, wave speeds, distortion, filters, matching and Fourier techniques. The simulation was written to aid the visual and kinaesthetic learning modes of the user so that underlying physical concepts could be better understood. In writing the simulation, the programmer utilised all four learning modes. During the development of the program, the aural, reading/writing processes had dominated.

PULSE PROPAGATION THROUGH A WAVE-GUIDE

The propagation of electromagnetic waves through rectangular metal wave-guides is described in many standard text books[3,4]. Essentially dominant mode propagation can take place in an infinitely long rectangular metal wave-guide provided the frequency of excitation, f exceeds $c/2bn_1$, where c is the speed of light in a vacuum, n_1 the refractive index of the medium filling the wave guide and b is the width of the wave guide, see figure 1.

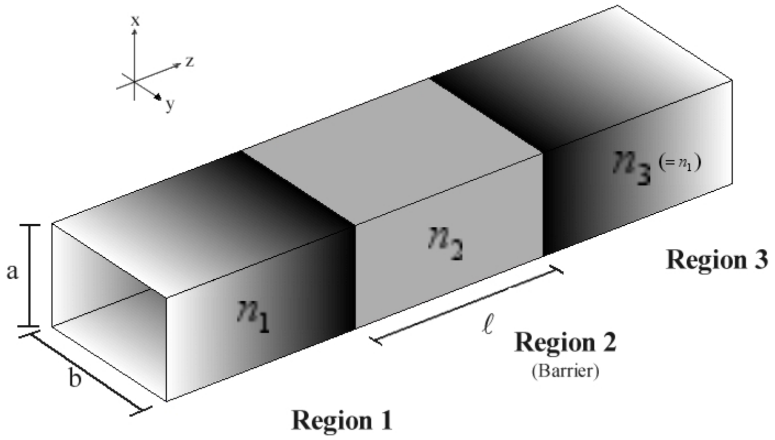


Figure 1. A Waveguide with a barrier of length ℓ. Regions 1 and 3 have the same refractive index. Region 2 can act as a barrier if $n_2 < n_1$

If the wave guide is a composite one with long input and output sections filled with a medium of refractive index n_1 and a middle region filled with a medium of refractive index n_2, then the composite guide can act as a barrier to electromagnetic waves whose frequencies satisfy the inequality $c/2bn_1 < f < c/2bn_2$.

Electromagnetic waves could not propagate through the middle region if it were infinitely long because the electric field would be $\pi/2$ out of phase with the transverse magnetic intensity. However, penetration can occur for finite barrier lengths. This can be readily appreciated by considering the reflections occurring at the interfaces and summing to obtain the resultant fields within the barrier. Physically the total electric field is not exactly $\pi/2$ out of phase with the total transverse magnetic intensity so the average Poynting vector is finite and energy flows through the finite length the barrier, even though no wavelength can be assigned to the wave within the barrier.

The algebraic description is best pursued through a transmission line analysis as then multiple reflections are included with minimum algebraic trauma[5]. The transmitted electric field E_t is related to the incident electric field, E_i by

$$\frac{E_t}{E_i} = \frac{4Z_1Z_2}{(Z_1+Z_2)^2\exp(\alpha l) - (Z_1-Z_2)^2\exp(-\alpha l)} \qquad (1)$$

Where Z_1 is the wave impedance either side of the barrier, Z_2 is the wave impedance inside the barrier and α is the attenuation constant inside the barrier of length l. The barrier transfer function is shown in Appendix A where equation (1) is compared with the experimental results of Nimtz et al[6].

An incident pulse would be composed of many frequencies within the allowed frequency pass-band $c/2bn_2 - c/2bn_1$.

Any signal tunnelling through a barrier in a wave-guide is necessarily frequency band limited. Also each frequency component would suffer attenuation and a phase shift on traversing the barrier in accordance with equation (1). So an incident pulse would suffer severe frequency dependent attenuation and distortion on propagating through the barrier thus causing ambiguities in the measurement of the speed of the pulse.

WAVE SPEEDS

There are numerous definitions of wave speed[9,10] some are and some aren't useful in this context. We list some of these below with comments.

1. The *phase speed* is the most familiar wave speed concept. It refers to the propagation of a sinusoidal wave and is given by the product of the wavelength and frequency of the wave.
2. The *group speed* is the speed of a pulse composed of many sinusoids; it is given by the rate of change of frequency with reciprocal wavelength.

Neither the phase nor the group speed is a useful concept for describing non-dissipative barrier penetration since they are defined in terms of the wavelength and no wavelength can be assigned to a wave within the barrier.

3. The *Hartman phase and group speeds*[9] were defined to account for waves without wavelengths these speeds are defined in terms of the phase change occurring across the barrier, these speeds are calculated in the simulation.
4. The *energy speed* is the speed of propagation of the energy; it is defined as the power flow density of the wave divided by the energy density of the wave. The power flow density is readily calculated but the energy density of the propagation is not, also this is not a speed that can be easily measured in any circumstances.

The speed of a pulse penetrating a non-dissipative barrier is best described by the following speed definitions.

5. The *signal speed* has been discussed at length by Brillouin[10], who showed it is best applied to systems without attenuation. The speed is defined as the length of the barrier divided by the time of propagation, this time being the delay before the transmitted pulse reaches some specified fraction of the incident pulse peak height, Brillouin recommends the fraction be taken as 1/3. The calculated signal speed depends on the value of the fraction chosen if the pulse suffers distortion. The definition has to be modified for propagation with significant attenuation, and then the delay is taken to be the time taken for the transmitted pulse to achieve a specified fraction of its own peak height. If the fraction is taken as unity the *signal speed* is identical to the *peak speed*, i.e. the speed of propagation of the pulse peak. The signal speed is a useful experimental speed in that it can be unambiguously defined and readily measured. It is calculated in the simulation.
6. The *cross correlation speed* is the speed used in radar range finding, it is the most useful experimental definition of pulse speed. It is defined as the barrier length divided by the delay at which the cross correlation function between the incident and transmitted pulses reaches a maximum. This speed is particularly useful for systems suffering distortion because it takes an average over the whole of both pulses. This speed is calculated in the simulation.

There are other wave speeds but the above are the most significant ones for our purposes.

MATCHED FILTERS AND MATCHED SIGNALS

The tunnelling of an incident pulse is best studied using Fourier techniques. The discrete Fourier transform, DFT, provides a direct relationship between the time domain and the frequency domain of a given signal. In the simulation the DFT has been used to decompose the incident pulse into its harmonic components, then equation (1) has been applied to calculate the harmonic components of the transmitted pulse and then the inverse DFT has been used to describe the transmitted pulse in the time domain. The barrier acts as a high pass filter causing profound frequency dependent attenuation and distortion of the incident pulse, this in turn makes it very difficult to unambiguously calculate any of the wave speeds specified above. Seismologists and radar engineers have encountered similar difficulties when trying to record the arrival time of distorted pulses. They have made use of the "matched filter" concept[11]. The received pulse is passed through a filter that compensates for the known calculable distortion suffered by the pulse so restoring it to some semblance of its original form. Such a technique isn't available in this case since a matched filter would introduce delays of the same order as those under investigation. But the concept of matching signal and filter does suggest that one should choose an incident pulse shape that suffers the "least amount of tolerable distortion" on propagation through the barrier. Considering only the middle region of the pass-band for barriers longer than 2 cm the phase shift is almost insensitive to barrier length, see figure 2, and the attenuation in dBs varies linearly with frequency (-8.6 dB cm^{-1}). However, one needs more than just the transfer function to adequately appreciate the barrier's filter action. Figure 3 shows the effect on the amplitude spectrum of the transmitted pulse when an incident gaussian pulse tunnels through barriers of different length. The transmitted pulse remains pseudo gaussian but the transmitted amplitude spectrum migrates to higher frequencies as the barrier length increases. Without the aid of the simulation, this effect is not immediately obvious.

This migration limits the length of barrier that can be investigated theoretically, since the amplitude spectrum of a pulse transmitted through long barriers approaches the upper cut-off frequency and these high frequency components of the pulse dominate the spectrum because they suffer far less attenuation than the lower frequencies. This effect is illustrated in figure 3, where it is clear barriers longer than 10 cm cannot be investigated. Distortion can be minimized if one ensures that the transmitted amplitude spectrum is narrow and confined to the mid region of the pass-band. Although the gaussian curve is often regarded as an ideal pulse shape its amplitude spectrum decreases too slowly as the frequency deviates from the mean for our purposes.

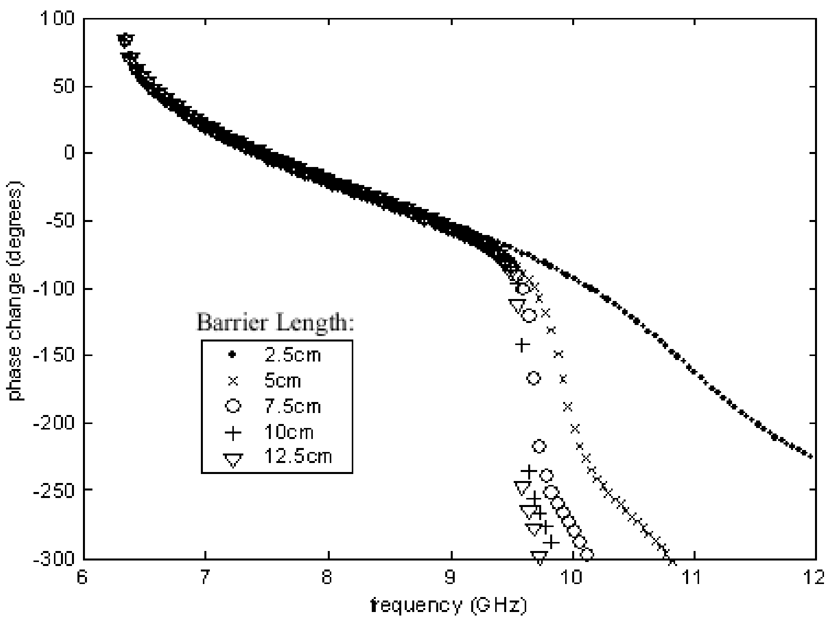


Figure 2. Phase against frequency for penetration through a long (>2cm) non-dissipative barrier

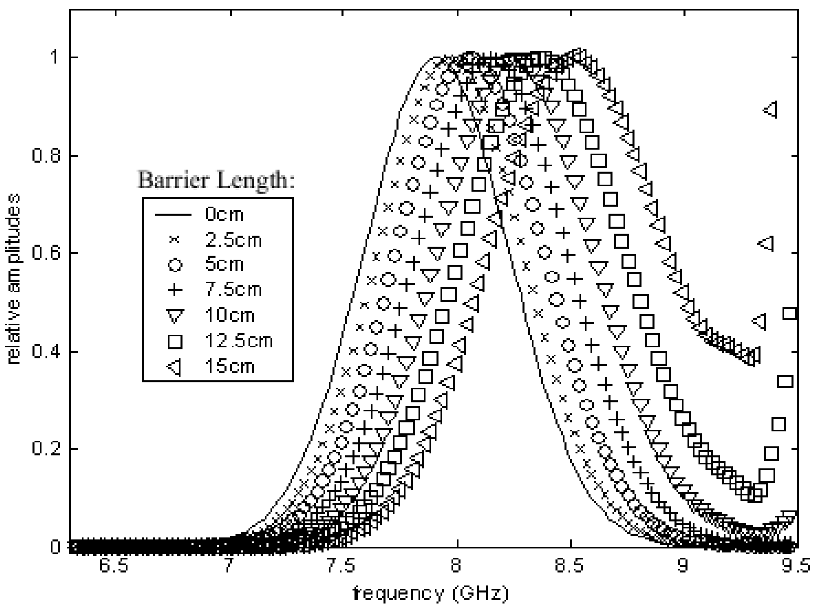


Figure 3. The transmitted normalised amplitude spectrum for an incident Gaussian signal traversing a non-dissipative barrier

The high frequency tail is attenuated far less than the mid frequency peak causing massive pulse distortion on propagation through long barriers. The Shalav curve has been defined to be a “more nearly matched” pulse shape for barrier tunnelling, The Shalav curve is almost identical to the gaussian curve except that it approaches zero far more quickly as the frequency deviates from the mean.

The amplitude spectrum of the transmitted pulse when an incident pulse with a Shalav profile tunnels through barriers of different lengths is shown in figure 4. Notice non-dissipative barriers as long as 600 cm can be investigated if the incident pulse has a Shalav profile. The amplitude spectrum still migrates to higher frequencies but this migration is arrested at the expense of pulse narrowing (kurtosis), this means that the pulse widens in the time domain.

No matter which pulse shape is used the high pass filter action ensures that the amplitude spectrum of the transmitted pulse is shifted to higher frequencies. This means that the transmitted pulse must be sampled at a different rate than used for the input pulse to ensure that the true pulse envelope is obtained this effect is illustrated in Appendix D.

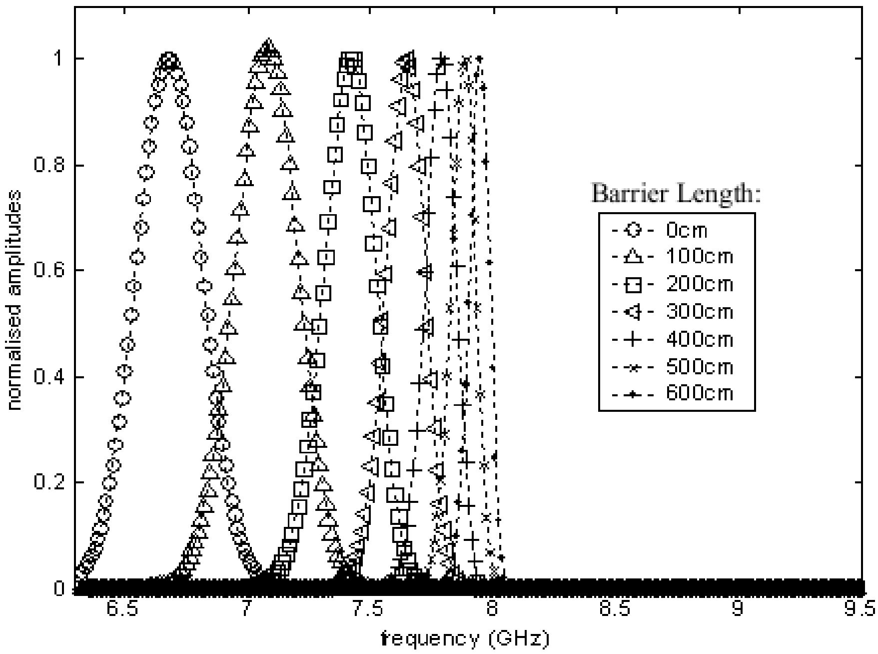


Figure 4. Transmitted amplitude spectra of a matched Shalav amplitude spectrum. The amplitude narrows with increasing barrier length

THE SIMULATION PACKAGE AND RESULTS

The main menu is shown in Appendix A, where the various available options are listed. Appendix B describes the penetration analysis. Appendix C displays some speeds and phase changes while Appendix D shows the transmitted pulse and illustrates the effects of inappropriate sampling. Like most computer simulations, the user interface was designed to appeal to the visual and kinaesthetic user. The effects on signal shape of altering the variables can be easily investigated. Physical concepts are therefore easily demonstrated without prior knowledge of complicated mathematics.

The cross-correlation, average Hartman group, and signal speeds (fraction 1/3 and unity) are shown in figure 5(a) for a gaussian incident pulse penetrating non-dissipative barriers with lengths up to 8 cm.

The signal speed for fraction 1/3 is sub luminal but the other speeds are super luminal. The average Hartman group and signal speed (for fraction unity) are shown in figure 5(b) for a Shalav profile incident pulse penetrating non-dissipative barriers of various lengths up to 300 cm.

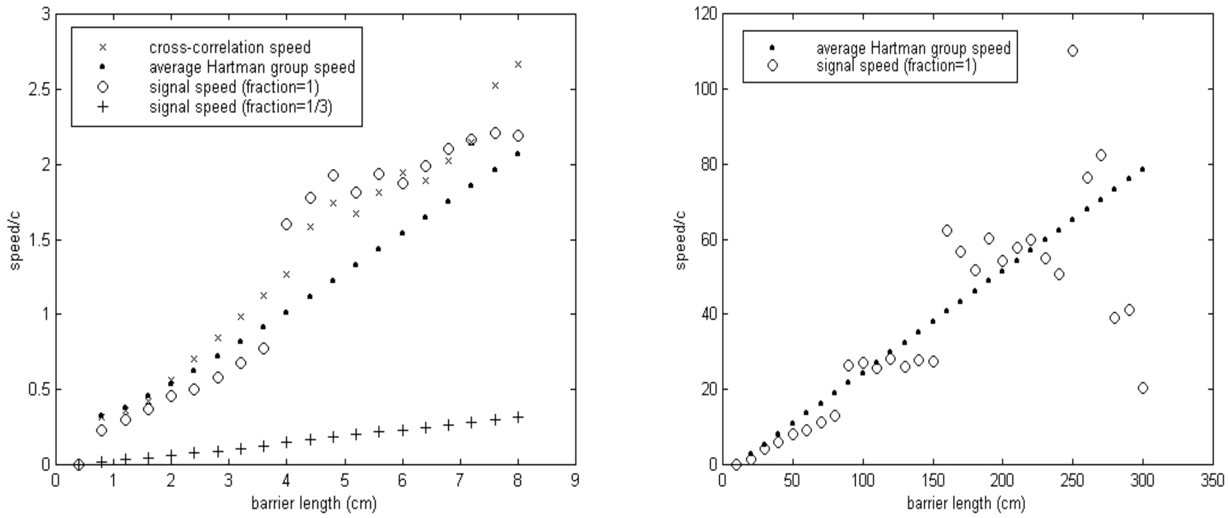


Figure 5. (a) Speed as a function of barrier length for a Gaussian shaped time signal penetrating a non-dissipative barrier. Similar results can be obtained from other investigated signals but not up to the barrier lengths shown above (due to significant signal distortion). (b) Average Hartman group speed and signal speed (fraction=1) for increasing barrier lengths for an improved matched signal. Average Hartman group and signal speeds of over 80 times the vacuum speed of light are predicted for barrier lengths over 300cm

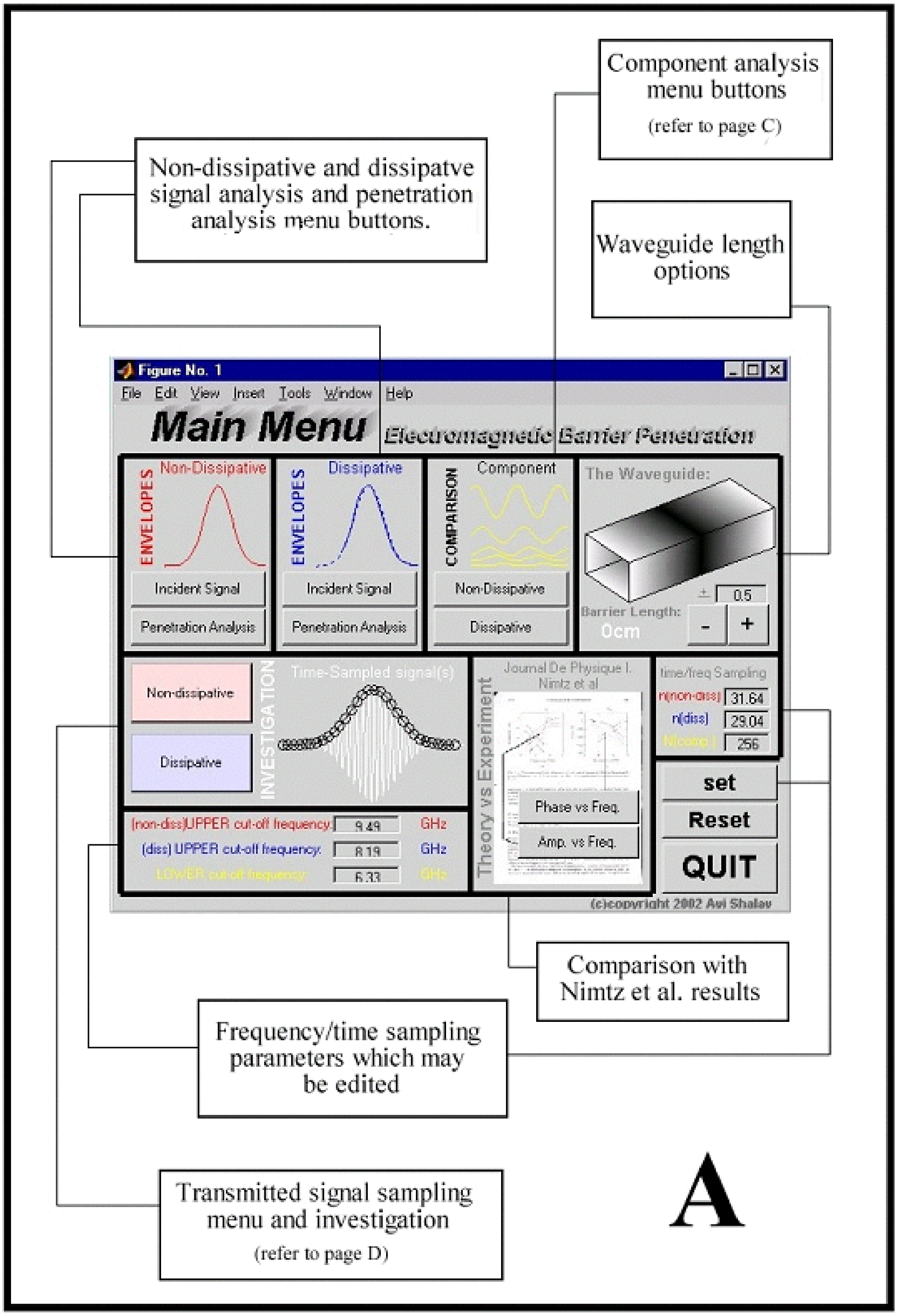
The signal speed values are affected by distortion. The speeds through dissipative barriers are not shown since these are sub luminal are hence “uninteresting”.

CONCLUSION

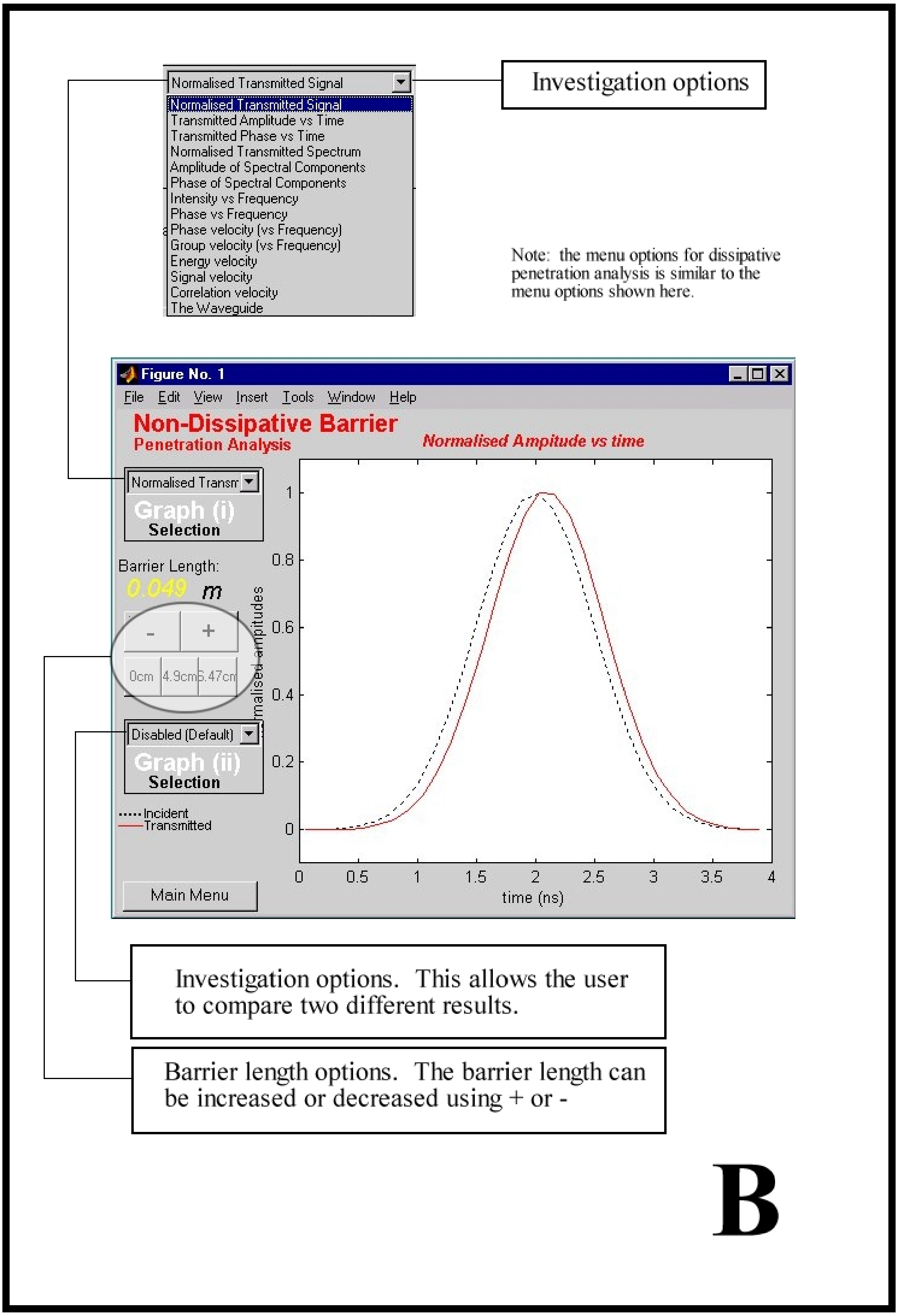
Without the aid of the simulation, it would have been difficult to compare results when applying different initial parameters. Effects due to signal distortion are not always obvious, but with the aid of the simulation these effects could be appreciated. As a consequence, the simulation could also be used to test and investigate other signal shapes in the hope of finding a ‘best’ matched signal.

The simulation effectively provides a visual interpretation of electromagnetic barrier penetration within a waveguide. This interpretation provides a visual link between physical and theoretical electromagnetic concepts. Such a link could be used to stimulate and motivate both the visual and kinaesthetic user.

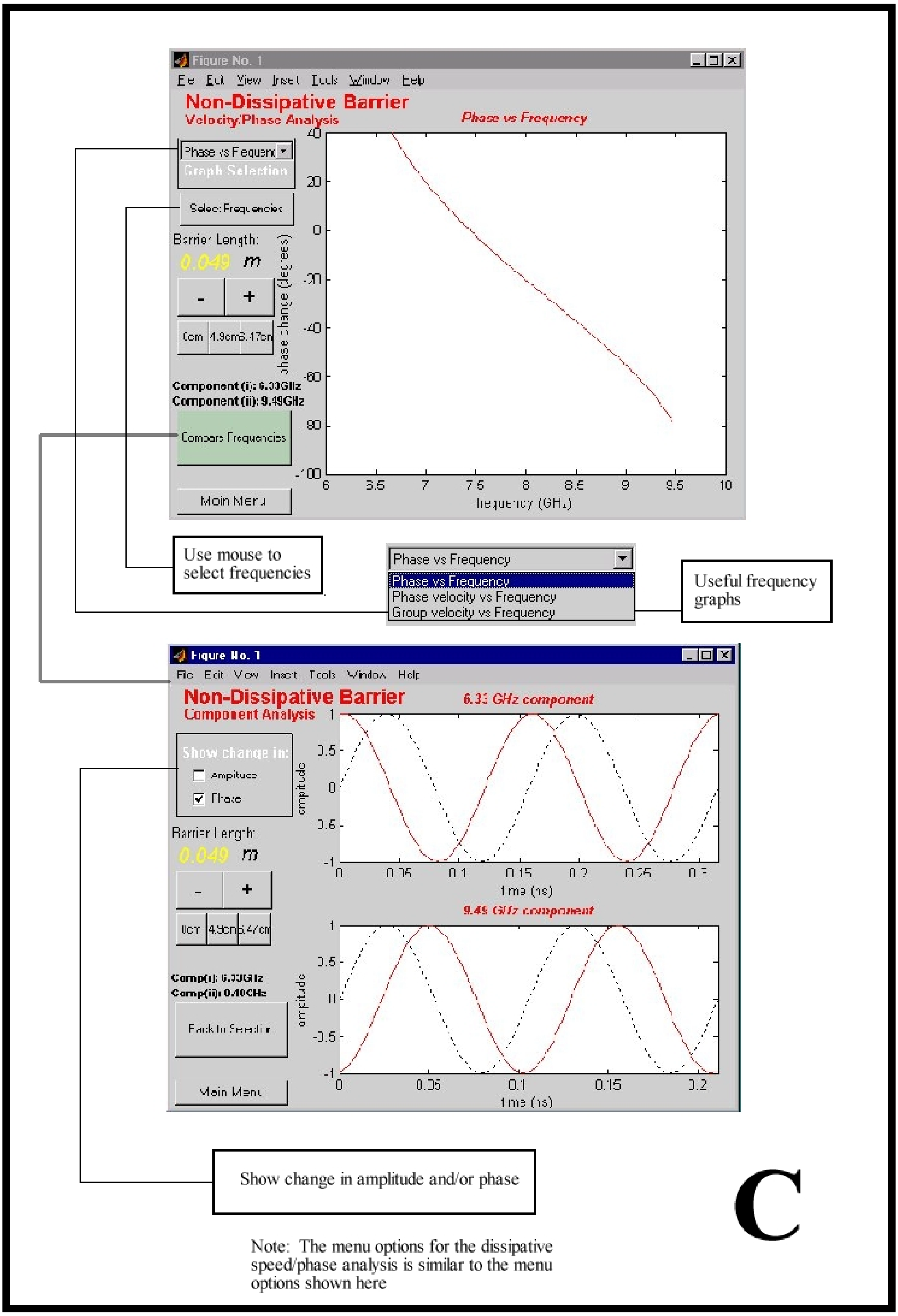
APPENDIX A
The Main Menu



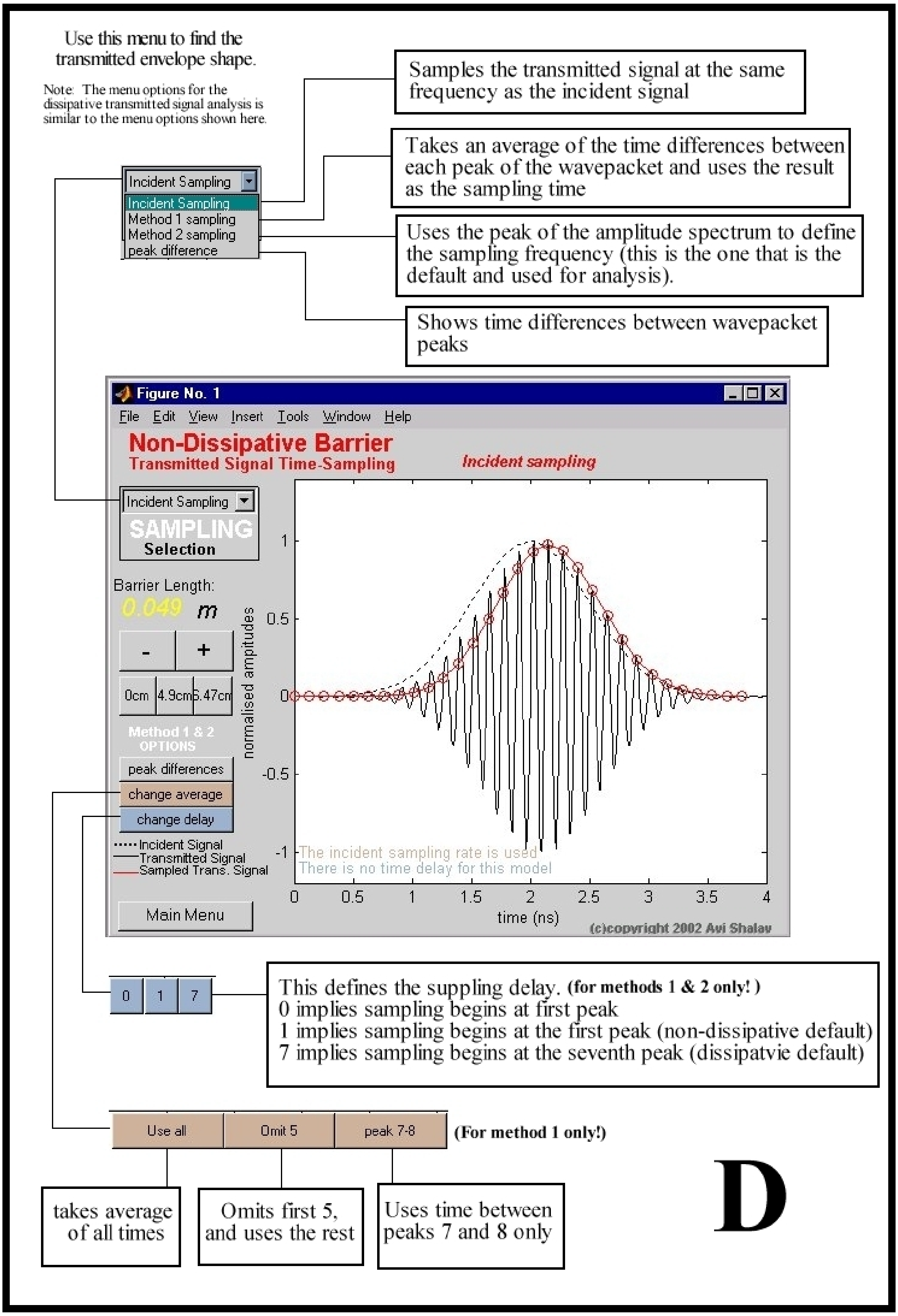
APPENDIX B
Penetration Analysis



APPENDIX C
Speed/Phase Analysis



APPENDIX D
Transmitted Signal Time-Sampling



REFERENCES

Fleming, D.N, I'm different; not dumb. Modes of presentation (VARK) in the tertiary classroom, in Zelmer, A.,(ed) Proceedings of the 1995 Annual Conference of the Higher Education and Research Development Society of Australasia, 1995, HERDSA, 18:p. 308-313.

Felder, R.M, Reaching the Second Tier: Learning and Teaching Styles in College Science Education. Journal of College Science Teaching, 1993. 23(5): p. 286-290.

Krauss, J., Electromagnetics. 4 ed. 1992: McGraw-Hill, Inc.

Lorrain, P., D.R. Corson, and F. Lorrain , Electromagnetic Fields and Waves. third edition 1988: W.H Freeman and Comapany.

Rulf, B., Transmission of microwaves through layered dielectrics-Theory, experiment, and application. American Journal of Physics, 1988. 56(1): p. 76-80.

Nimtz, G., H. Spieker, and H.M. Brodowsky, Tunneling with dissipation. Journal De Physique I, 1994. 4(10): p. 1379-1382.

Smith, R.L., The Velocities of Light. American Journal of Physics, 1970. 38(8): p. 978-984.

Bloch, S.C., Eighth velocity of light. American Association of Physics Teachers, 1977. 45(6): p. 538-549.

Hartman, T.E., Tunneling of a Wave Packet. Journal of Applied Physics, 1962. 33(12): p. 3427-3433.

Brillouin, L., Wave propagation and group velocity. 1960, New York: Academic Press Inc.

Skolnik, M.I., Introduction to Radar Systems. 1962, McGraw Hill: New York. chapters 9 and 10.

A.Shalav
Key Centre for Photovoltaic Engineering
University of New South Wales
Sydney
Australia
Email: A.Shalav@student.unsw.edu.au

D.N Pinder
Institute of Fundamental Sciences
Massey University
Palmerston North
New Zealand
Email: D.N.pinder@massey.ac.nz