GRAPHICAL REPRESENTATION OF DATA: THE EFFECT OF THE USE OF DYNAMICAL STATISTICS TECHNOLOGICAL TOOL

Maria Meletiou-Mavrotheri, Despina A. Stylianou

ABSTRACT
In response to calls for reform an innovative course which has at its core element a dynamical statistical software package, Fathom®, was developed as an alternative approach to traditional introductory statistics in college. This study was designed to examine the course's effects on students' understanding of graphical representations of data. Because particular attention was paid to the processes students used when they were actually solving statistics problems with the help of Fathom, the study conveys more than general notions concerning whether or not technology might aid in the learning of statistics; the findings reveal some of the ways in which students used technology to perform specific tasks when solving problems and how technology helped their thinking. In particular, the study suggests that students using technology are more likely to engage in interpretation and translation of graphically presented information, while, in the absence of technology students only engage in interpretation. Additionally, while when approaching problems with paper-and-pencil, students rarely engage in extrapolation – the most cognitively demanding aspect of graphical comprehension – introduction of technology encourages students to make conjectures about observed trends in the data, and actively search for evidence to support their claims.

KEYWORDS
Data representation, graphs statistical reasoning, variation, dynamical statistics software, FATHOM®

INTRODUCTION

Statistics education is rapidly becoming the focus of reformers in mathematics education as a vital aspect of the education of citizens in democratic societies (National Council of Teachers of Mathematics [NCTM], 2000). Indeed, data literacy has become a fundamental skill for living in an information era where important decisions are made based on available data. Graphical or visual representation of quantitative data is one of the core aspects of statistics – pie and bar graphs especially are used broadly to present, disseminate and "explain" information in the media (Shaughnessy, Garfield, & Greer, 1996). Introductory statistics courses have also been traditionally using a multitude of graphical representations both as a tool for describing data and as a means to aid students. Hence, it might be expected that students are familiar with various types of data representation. Recent work in statistics education reveals, however, that students are likely to have beliefs about the features of graphs that are different from what is expected (e.g. Meletiou & Lee, 2002).

It is important to reinforce the understanding of graphing in the teaching of statistics. Advances of technology provide us with new tools and opportunities for the teaching of statistical concepts including the use of various graphical representations. These new technological tools are, in fact, designed explicitly to facilitate the visualization of statistical concepts providing an enormous potential for making statistical thinking accessible by all students. In response to calls for reform an innovative course which has at its core element a technological tool, the dynamical statistical software package

Fathom® (Finzer, 1999), was developed as an alternative approach to traditional introductory statistics in college.

This study was designed to examine the course's effects on students' understanding of graphical representations of data. In particular, this paper addresses the following two research questions in the context of a course on elementary statistics.

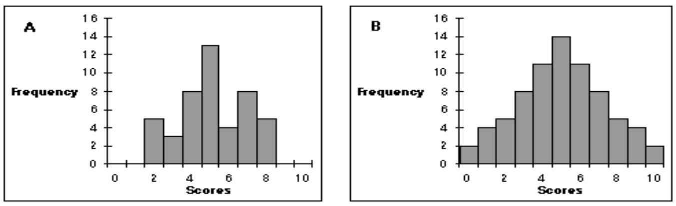
1. How does technology affect students' perceptions of data presented graphically?
2. How does technology affect students' approach to problems when involving a strong graphical element?

METHODOLOGY

Context and Participants

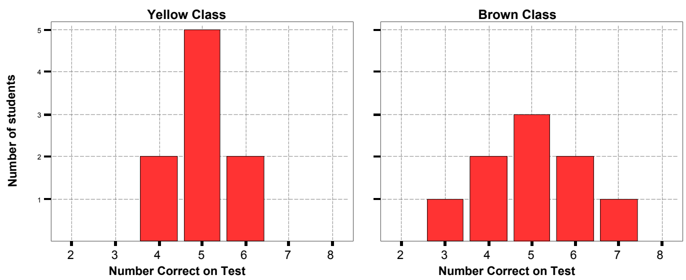
The site for the study was an introductory college statistics course. Class met two times a week, for two hours each time. There were thirty-five students in the class most of whom were majoring in a business related field of study. Only few students had studied mathematics at the pre-calculus level or higher. Instruments, Data Collection and Analysis Procedures: Students were given a pre-test on graph-understanding, to provide a baseline for the study. The three tasks that formed the pre-test are shown in figure 1. Each of these tasks was selected from previous studies in statistics education, mainly to provide a point of reference and comparison for our findings. Students' solutions were coded with respect to correctness, but also, with respect to their attention to numerical and graphical aspects of each problem.

Task 1: "Choosing distribution with more variability" (Garfield, delMas, & Chance, 1999)



A has more variability____ B has more variability____ Explain why.

Task 2: "Students' Scores" (Watson & Moritz, 1999)



Which class did better?

Are there the same number of raisins in each box? How can you tell?

Task 3: "Raisins in a box" (Friel & Bright,1996)

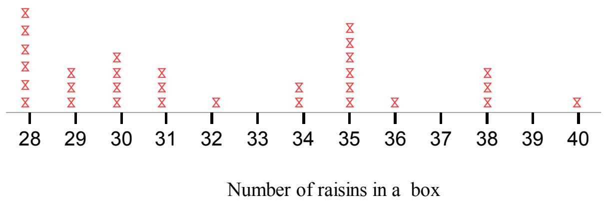


Figure 1. Pre-assessment tasks

During the course, students were first introduced to graphs using the traditional lecture method, and subsequently introduced to dynamical statistical software package Fathom®. In both cases students were given a set of tasks to solve. Visual representations of data were a core element of each task.

Students were observed and videotaped while solving these tasks, both in the presence and absence of technology. The method of analysis involved inductively deriving the descriptions and explanations of how the students interacted with the graphs and approached selected ideas of statistics through graphs. Using related behaviors and comments within each topic, we wrote descriptions of the students' strategies and actions. These descriptions formed the findings of the study that are described in the next section.

RESULTS

Pre-assessment

To provide a base-line for our study, students were given a pre-test on the first day of classes focusing on their understanding of bar graphs and histograms – two of the most important graphical tools used in the statistics classroom. The tasks were chosen from earlier studies in the field and they are shown in Figure 1. Students' performance in these tasks was indicative of a very limited understanding of graphical representations, confirming earlier findings in the research literature.

On the first task (adapted from Garfield, delMas, & Chance, 1999), only forty percent of the students recognized that distribution B has more variability than distribution A. Some students gave no response, while forty-five percent argued that distribution A has more variability. Student explanations for choosing distribution A indicate that instead of considering both the horizontal and the vertical axis of the graphs to compare the spread of the distributions they were looking at their vertical axes and comparing differences in the heights of the bars (i.e. differences in frequencies among the different categories). Chance, Garfield, and delMas (1999), have also found that students often confuse "bumpiness" of a histogram with "variability".

In the second task, students were asked to compare the performances of the two classes. The task was first reported by Watson and Moritz (1999), who observed three categories of strategies employed by students– visual, numerical, or a combination of the two. Using the same coding scheme, we found that the most popular type of strategy employed by students was the numerical one. Most of the students perceived the graphs as a way to obtain the actual scores in order to calculate a number that would summarize the data and allow comparisons between the two classes. Students who used visual strategies commented on aspects related to either the center or the spread of the graphs, while only two students used a combination of visual and numeric considerations.

In the third task students were given a line plot depicting the quantity of raisins in half-ounce boxes and were asked to determine whether all boxes have the same number of raisins. Data in line plots are ungrouped, making it easier for students to interpret such plots than bar graphs or a histograms, where there is data reduction in moving from raw data to frequencies of occurrence of different values or groups of values. Despite the seemingly easy nature of the task, only two students were able to reason using information both about the data values themselves (from the axis) and the frequencies of occurrence of these data values. The results in our study were less encouraging than in the Friel and Bright (1996) study, where the task first appeared. In that study, twenty-eight percent of sixth graders were able to consider both the range of data and the frequency of occurrence. The fact that thirty-five percent of the students in our study taking the pre-assessment either gave no response or stated that they did not know how to interpret the line plot, suggests lack of familiarity of Cypriot students with even this simple type of plot. Also, given that several other students responded in ways suggesting that they focused on the frequency or number of X's as the data values themselves, it seems that even when interpreting line plots, there might be confusion about the role of data values and frequencies.

Graphs are, along with numerical means, the main statistical tools used to assess the shape of a data distribution. However, as the analysis of student responses to the pre-assessment suggests, at the beginning of the course students had very poor understanding of graphical representations. Most were not able to correctly interpret histograms and bar graphs, as they did not seem aware of the data reduction involved. Student difficulties were not confined however to histograms and bar graphs; they also had difficulties in interpreting a simple type of plot – the line plot – which is a display of raw data. In a summary, the pre-assessment provided a baseline for our study – the assumption that students had a weak understanding of graphical representations. It is through this lens that we proceed to the analysis of the instructional experiment.

Instructional Experiment. In an effort to address the difficulties faced by students when working with graphical representations of data, an innovative statistics course was developed that focused specifically on graph comprehension. One of the core elements of the course was a dynamical statistics software package, Fathom®, and a set of problems was collected around the software to supplement it and provide a context for its use. Here, we describe how students gradually built their understanding of graphs, first using traditional paper and pencil means as investigation tools, and subsequently, the dynamic statistics package.

Our goal was to examine the role of technology (specifically, the role of the dynamic statistics tool) in shaping students' attitudes towards graphical representations and their approaches and strategies when solving statistics problems involving graphs. Thus, our data analysis paid particular attention to the processes students used when they were actually solving statistics problems with and without the help of Fathom®; we examined the ways in which students used technology to perform specific tasks when solving problems and how technology helped their thinking.

We framed our analysis around the three main behaviors/aspects of graph comprehension as these were identified by Friel, Curcio and Bright (2001), namely, reading and interpretation, translation, and extrapolation and interpretation (defined by Wood, 1968): Reading and Interpretation requires rearranging material and sorting the important from the less important factors. To interpret graphs, one can look for relationships among specifiers (e.g., the bars in graph) in a graph or between a specifier and a labeled axis.

Translation requires a change in the form of a communication. To translate between graphs and tables, one could describe the contents of a table of data in words or interpret the graph verbally.

Extrapolation and interpolation, extensions of interpretations, require stating not only the essence of the communication but also identifying some of the consequences. In working with graphs, one could extrapolate or interpolate by noting trends perceived in data or by specifying implications.

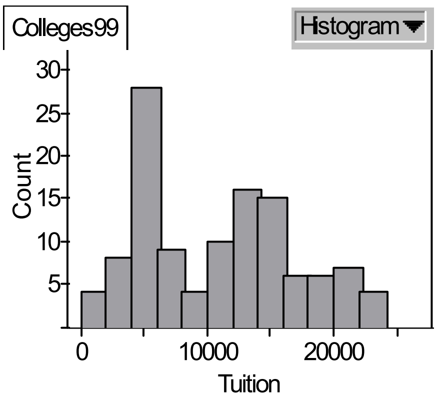
Here we present the analysis of one particular task, the "College Tuition" task (adapted from Rossman, Chance & Locke, 2001), shown in Figure 2. The aim was for students to understand the process of data reduction involved in a grouped data representation (the histogram) and realize that "the purpose of data reduction is to identify appropriate representations of the data which remove as much detail from the data as is possible while providing sufficient information to address the specific question at hand" (Friel, Bright, Fierson & Kader, 1997).

Students were asked to solve the task using traditional paper and pencil means. Subsequently, students were given the same task in a dynamic software environment. In each case we examined students' approaches with respect to the aforementioned three aspects of graph comprehension, and characterized and categorized students' behaviors based on this scheme. A summary of these characteristics, both in the paper-and-pencil and in the technological environment is presented in Table 1.

Paper and Pencil Stage

During the first stage of instruction, students approached tasks (such as the one shown in Figure 2) using traditional means of investigation – paper and pencil. Though several tasks were used in the course, here, we will focus on students' responses to the "College Tuition" task, which examines college tuition charges in a certain US state. The histogram in their activity sheet focuses on the distribution of tuition charges in this state's colleges during 1998-1999.

COLLEGE TUITION
The following histogram displays the distribution of tuition charges (in thousands of US dollars) in 1998-99 for colleges in a certain US State.



How many of the colleges had tuition between $4000 and $6000?
How many colleges had tuition of at least $16,000? What proportion of the 117 colleges had tuition this large?
Can you determine how many colleges had tuition of at least $19,000? Why or why not?
What would happen if we changed the width and number of subintervals in the histogram? Comment on the different appearances of the distribution revealed as we would increase/decrease these values. How many rectangles do you think provide the most useful and informative display of the distribution of tuitions?

Figure 2. Instructional experiment task

Table 1. Characterizing students' behaviors with and without technology

Approaching Graphs on Paper	Approaching Graphs via Fathom®
(a) Reading and Interpretation - using a numeric strategy; perceiving graphs as a way to obtain the actual counts in order to do calculations.	*(a) Reading and Interpretation* - numeric strategies - constructing new types of graphs - changing graph attributes (e.g., scale,) and/or adding new attributes (e.g., "college type")
(b) Translation - no such behavior noted	*(b) Translation* - linking graphs to tables of data - linking various types of graphs of the same situation
(c) Extrapolation and Interpolation - no such behavior noted	*(c) Extrapolation and Interpolation* - looking for clusters - forming hypotheses on underlying reasons for clusters - creating appropriate representations to examine further the new hypotheses

Slightly less than half of the students answered correctly the first question, which was asking them to indicate the number of colleges with tuition between $4000 and $6000 by responding that the number of colleges in this tuition range is around 27-28. About one-fourth of the students added the counts of either the 2nd and 3rd bar or the 3rd and 4th bar and gave a number that ranged between 32-36 colleges. Another one-fourth of the students added the counts of the 2nd, 3rd and 4th bar and gave a number around 45.

The second question asked students to determine the number of colleges with tuition of at least $16000. The exact number of colleges in this range is 23. About forty-five percent of the students gave responses close to the exact number. Another twenty percent of the students included the count of colleges in the $14000-$16000 range in their estimates and gave numbers between 35-39. The remaining students either considered only the count of the $16000-$18000 bar or the count of the $14000-$16000 bar.

In the third question asking whether they could determine the number of colleges with tuition of at least $19,000, the majority of students (about two-thirds of the students) correctly pointed out that "*it is impossible from the given printed plot to determine exactly from the histogram since the colleges are grouped with a precision of $2000 tuition*", and thus *"graph shows only between $18000 and $20000"*. The remaining one-third of the students did not mention anything about bar width, but rather went ahead and gave an estimate for the number of colleges in the ≥$19000 range.

In the last question, students had to first comment on how the histogram would look like if were to change the width and number of subintervals in the histogram and then to define the number of rectangles that they thought would provide the most useful and informative display of tuition distribution. Students' typical approach to answering the first part of the question was to use the formula introduced in class for calculating the subinterval width, given the number of subintervals in a histogram. They calculated the class width for two or three different numbers of subintervals in order to show that *"if we increase the number of subintervals, the subinterval width decreases, and vice versa"*. The second part of the question was answered by only about half of the students, all of whom put down that *"5-15 classes"* provide the most useful display of the distribution of tuitions. The reason why all of these students gave the same number of classes was most likely because they had been told in class that, when drawing histograms, 5-15 classes are usually used.

Let us now turn our attention to the overall graph comprehension students exhibited during the traditional paper-and-pencil stage of instruction. We coded students' behaviors using the Friel et al. (2001) three main behaviors/aspects of graph comprehension, namely, reading and interpretation, translation, and extrapolation and interpretation, as explained earlier. Students appeared to show some gains in their ability to read and interpret graphs, compared to their earlier performance on the pre-assessment tasks. About half of the students were able to read graphs and respond correctly to the basic questions of the "College Tuition" task. Note, however, that students appear to focus primarily on numeric strategies associated with the graphs. That is, students, at the paper-and-pencil stage of instruction, perceived graphs as a way to obtain the actual counts in order to do calculations and respond to given questions. None of the students attempted to approach the task using a visual strategy. Hence, none of the students took the initiative to investigate the problem situation visually either by attempting to construct a different graph (that could potentially reveal different kinds of information or different aspects of the problem) or change any of the graph attributes (e.g., scale), to gain a better perspective in solving the problem.

With respect to the second and third aspects of graph comprehension, students appeared to show little or no gains. None of the students actively attempted to link the graph to the data table and make translations between the two. While the instructor noted to the students that the table of data was available upon request, students did not sense the need to refer to the actual data as a way to compliment their understanding of the problem. Similarly, none of the students attempted to draw any further conclusions (i.e., extrapolation and interpretation).

Dynamic Statistics Stage

During the second stage of instruction, students were introduced to the dynamic statistics software Fathom® and used it as their means of investigation. Once again, we will focus on students' responses to the "College Tuition" task to allow for comparisons in students' behavior.

Students were given the same task, except that, this time, they were allowed to use the computer as an aid in responding to the given questions. The students' first action was to ask the computer for the data file. This was surprising, given that in their paper-and-pencil investigations they did not choose to use the data table (part of the data table as used in one student's investigation is shown in Figure 3 – left). It was expected that students would, once again, focus on the histogram. Students' second action was to get a "dot plot" of the distribution of tuition charges (Figure 3 – right).

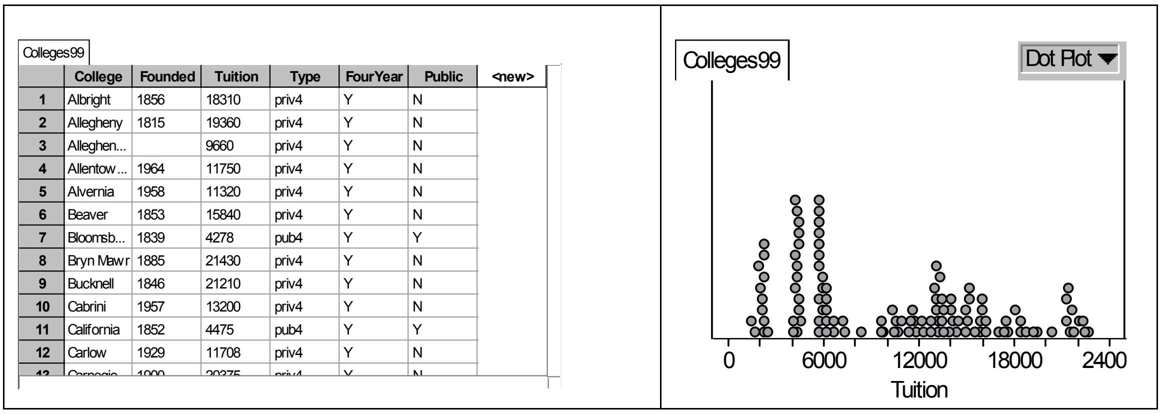


Figure 3. Data table and dot plot for the "College Tuition" task

Subsequently, students changed the dot plot to a histogram. They played a bit around with the histogram, trying to find a way to determine the number of colleges charging at least $19,000. Two-thirds of the students changed the bin width of the histogram's bars to $1,000 and, highlighting each bar in the ≥$19,000 range and using the toolbar at the bottom of the screen to find the number of colleges belonging to each of the categories, concluded that 13 institutions had tuition charges in this range. There were however some students (the same ones that in the third question of the first part of the activity did not mention anything about bin width) that did not bother to change the bin width, but rather highlighted each of the bars of the original plot (i.e., the one with bar width of $2,000) with tuition charges ≥$18,000 and, adding up the count of colleges corresponding to each bar, concluded that 17 colleges had tuition of at least $19,000.

At this point, one of the students made a remarkable observation:

Photis: We do have here some distinctive clusters here. This column here [he shows a bar], for example…we can see that most of the colleges have tuition between $5,000-$7,000. Of course, there are also these universities that…there are 10 universities which… let's say it's an outlier because we can see that most of the colleges don't have such high tuition.

Instructor: So, how many distinct clusters can you see?

Photis: Three. Some have a very high tuition, some have an average tuition which is around $14,000-$15,000 and some are low cost colleges which have from $5,000-$7,000. I think that the explanation for this is that all of the colleges try to compete the other colleges in their own league. Let's say…you can't have high tuition colleges that are above $25,000. So, it will be… it wouldn't be competitive. If you have three major leagues of colleges – the low, medium, and high budget, they all try to meet the budget limits of their league, otherwise they will have too many students, which is not possible or, in other cases, very few students, which is also not desirable.

Photis proceeded to investigate the correctness of his hypothesis, by using the software's capability to re-arrange data using various attributes. In particular, he and his computer partner, Nicos, chose to group colleges by "type of college", that is, whether they are private or public institutions and whether they are 4-year or 2-year institutions. Photis and his partner proceeded to discuss the generalizability of their conjecture, and to examine features like variability based on the histogram features. Figure 4 shows the histograms used by Photis and Nicos in their investigations.

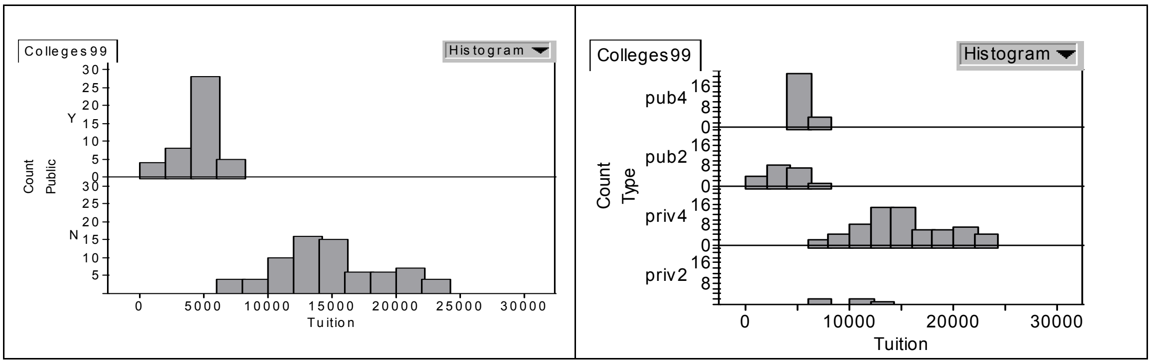


Figure 4. Photis and Nicos' histograms

Photis and Nicos observations led them to a new level of behavior with respect to graphs. Not only were the two students able to read and interpret the graph, but they extended these interpretations – the very essence of interpolation and extrapolation. The two students noted trends in the data and hypothesized the underlying rationale for these trends and potential implications.

The instructor brought the class' attention to Photis' observation, and asked them to examine this issue as well. Gradually almost all students in the class identified the three distinct clusters of the distribution, *"low cost, medium cost, and high cost"*. The ranges different students gave for each cluster varied from each other. Nonetheless, all students were able to recognize - with the aid of plots such as the one used by Photis and Nicos (Figure 4) that public institutions charge lower tuition than private institutions and thus tend to fall into the lower cluster:

Anna: Three distinct peaks may be seen, where first is observed with tuition between $1000 and $9000, second is from $9000 to $20000 and third from $2000 to $23000; since colleges falls into two major categories (private and public) we can notice that private colleges are more expensive (from $6000 to $24000) than public ($0 to $8000), which explains clustered shape of distribution histogram.
Elina*: There are three distinct categories. The $5000 range, $10000, $15000 and more. The lower costing colleges are the public ones, there are 35 colleges with fees ranging between $4000-$6000. The next cluster ranges from $7000-$13000. These cases fall into the private categories. It can be clearly noticed that public colleges are cheaper.*

The final task students had to do was to manipulate the width and number of subintervals in the histogram, comment on the different appearances of the distribution revealed when changing these values, and choose the number of bars that they thought would provide the most useful and informative display of the distribution of tuitions. After students had changed the bin width of the histogram several times, they made statements such as *"As the bin width gets bigger so do the bars but they become fewer in number"*; *"Different width give us different impressions"*. They ended up concluding that by having too many bars we get a graph that is *"difficult for the eye to see"*, whereas by having too few bars *"one loses valuable information"*. The interval width that students considered most desirable ranged between $1000-$2000. David's interactions with the software, as shown below, are indicative of the majority of the students' work on this last part of the activity.

David: OK. Now, I'll try say to put the width of 500 instead of 2000.This gives us [first histogram in Figure 5]…I don't like what I get because it's absolutely confusing. The three clusters you can still see…you can still see them… but, the number of rectangles is so big so you cannot really get an idea of the variation of those three main clusters. But on the other hand, if we changed the bin width to 5000

[he does it and gets the second graph in Figure 5]. The bin width is now so big that you lose information…by having this histogram here to 5000, we can only see 2 distinct clusters…one that is from let's say [highlights the first bar].. it ranges from $1000-$6000. With this bin width, you get a very general, sometimes misleading idea of the colleges.
Instructor: So, what bin width would you prefer?
David: 1000. Because…[changes the bin width and gets the third histogram in Figure 5]. On the other hand…this might also be confusing. I'm going to try 1500 [fourth histogram in Figure 5]. I think here it's more accurate…when you divide it by 2000 you only get 10-11 categories, but with something between $1000-$2000, you're being more specific without losing information.

Once again we need to turn our attention to the overall graph comprehension students exhibited during the dynamic statistics software assisted stage of instruction. Clearly, students had already exhibited a relatively good ability to read and interpret graphs. Note however, that the students' behavior now encompasses some new characteristics. Not only do students use numeric strategies associated with the graphs (that is, use graphs as a way to obtain the actual counts), but are now also interested in rearranging material and look at graph specifiers in a more visual manner. Students are changing some of the graph attributes, such as scale, and adding new attributes, such as the new cluster categories, to better understand the situation at hand. Graphs are not static figures to be read, but, more dynamic tools that can be manipulated to reveal hidden information. This behavior is, in fact, more consistent with the definition of graph reading and interpretation provided by Friel et al. (2001).

With respect to the second aspect of graph comprehension, translation, students appeared to show evidence of some gains. The majority of the students took the initiative to bring into their solution process the table of data. While there is little evidence that students actually used the table of data in a significant way, it is important for them to make the first step of noticing the relevance, or connection of the data table, as another possible representation of the problem situation, or as a potentially helpful tool. Further, students showed evidence of interest in translations between various types of graphs. When examining the clusters of data, students opted to create a histogram that also exhibited the new attribute "type of college".

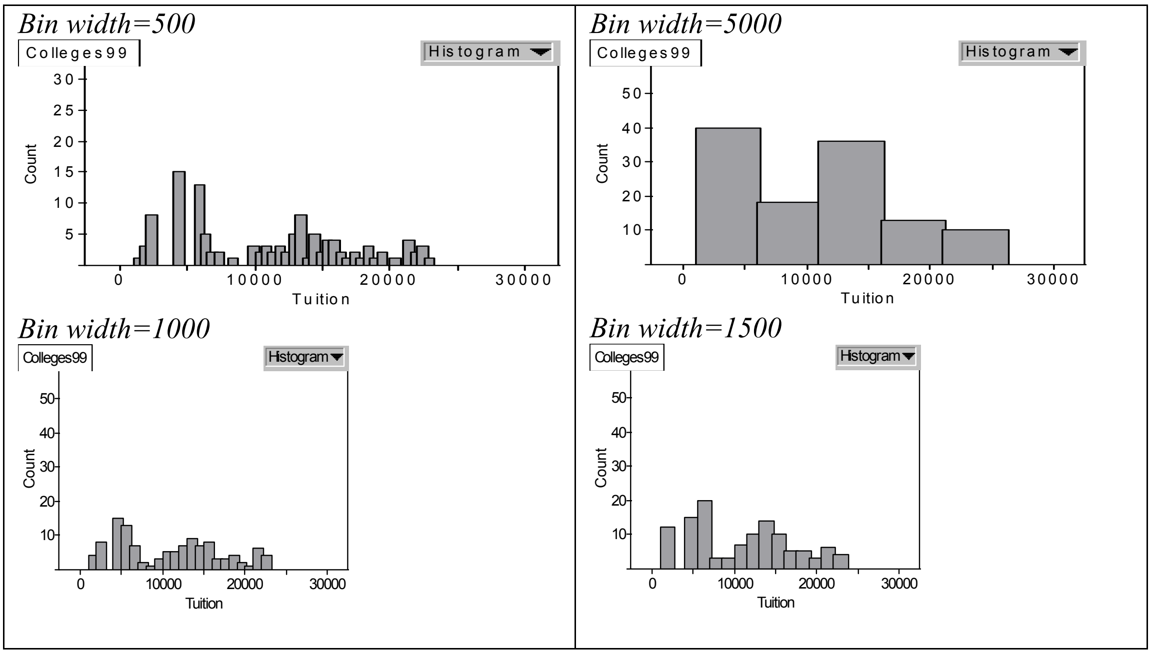


Figure 5. David's histograms

Finally, the most notable change in students' behavior was with respect to the third (and most cognitively demanding) aspect of graph comprehension, interpolation and extrapolation. Students gradually gained the confidence (along with interest and willingness) to further investigate the problem situation, beyond the given instructions. Students, to different degrees, of course, came to the point to not only notice trends in the graphical data but to also state hypotheses and conjectures for further implications. Moreover, students actively searched for evidence to support their claims, via graph manipulation and careful examination of the role each of the graph specifiers.

CONCLUSIONS

This study was designed to investigate the effects of a technology-based course on students' understanding of graphical representations of data. In particular, we examined the ways in which technology affected students' perceptions of data presented graphically and their approaches to problems involving a strong graphical element. The findings presented here are preliminary and further, more in-depth investigation is necessary. Nonetheless, findings do suggest that integration of technology in the classroom brought about important changes in students' ways of learning statistics. While we did not measure student "attitudes", we have evidence that the presence of the dynamic statistics software Fathom increased students' interest in *actively* pursuing problems involving a strong graphical element. We also have evidence for higher cognitive involvement, for improved overall graph comprehension.

During the traditional paper-and-pencil stage of instruction, we saw the students showing some gains in their ability to read and interpret graphs compared to their earlier performance on the pre-assessment tasks, but focusing primarily on numeric strategies associated with the graphs. Additionally, students showed little or no gains with respect to the second and third aspects of graph comprehension (i.e. translation, and extrapolation and interpretation). By contrast, students exhibited much better overall graph comprehension during the dynamic statistics software assisted stage of instruction. In addition to a relatively good ability to read and interpret graphs, students were now approaching problem situations using a combination of visual and numerical strategies. There was also some change in students' behavior with respect to the second aspect of graph comprehension, translation but, most importantly, with respect to the most cognitively demanding aspect of graph comprehension, interpolation and extrapolation. Students made conjectures about observed trends in the data, and actively searched for evidence to support their claims by creating, transforming, and interpreting graphical data representations.

Graph manipulation is a tedious and complex task when done by hand and this might have been the main reason students in this study did not employ the table of data when approaching the task in the absence of technology. Use of technology facilitates students' interest in exploration and provides the means for them to focus on statistical conceptual understanding and problem-solving and not on recipes and formal derivations, which become secondary in importance. Attributes of technological tools such as Fathom, which were especially designed for statistics learning, like the ability to operate quickly and accurately, to dynamically link multiple representations, to provide immediate feedback, and to transform a whole representation into a manipulable object seem to enhance students' flexibility in using representations and to facilitate the use of advanced cognitive levels of statistical problem solving. For example, the computer makes it possible to shift students' attention to issues of scaling and designing data graphs to support hypotheses and conjectures. Use of such software in the statistics classroom, supports active knowledge construction by "doing" and "seeing" statistics in a powerful and flexible learning environment (Ben-Zvi, 2000), and provides opportunities for students to reflect upon observed phenomena, and develop their metacognitive capabilities.

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Maria Meletiou-Mavrotheris
Ministry of Education
Nicosia, Cyprus
Email: meletiu@spidernet.com.cy

Despina A. Stylianou
University of Massachusetts, Dartmouth
Mathematics Department
285 Old Westport Road
N. Dartmouth, MA 02747, USA
Email: dstylianou@umassd.edu