LEARNING OF GEOMETRY SUPPORTED BY THE PROGRAM CABRI

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ABSTRACT
Programs for computers have been developed many years with the aim to help teacher of mathematics to make education of geometry more efficient. If we criticize software according to quantity of their use in schools the Sketchpad and Cabri Geometrie have the best position among software in the Europe and the USA. This contribution deals with the software Cabri and its use in education of geometry. The authors focused on dynamic features of Cabri Geometrie II Plus mainly various curves being a locus of special points in the plane. Cabri provides enough possibilities also for the users who concentrate to drawing the solutions of more difficult and more attractive problems, but which are presented in a finished static form. However we assume that the real help to teachers of mathematics is mainly in making full use of the dynamic features of the software as we try to show in this contribution.

KEYWORDS
Hypotheses about loci of points**,** Cabri Géomètre

INTRODUCTION

Cabri belongs to the most spread softwares which are used to support the education of geometry. The common characteristic of these programs is that they replace the traditional geometrical way of constructions by pencil, ruler, compass and protractor. This way of constructions enables students to make conjectures and draw their own conclusions. The context in which Cabri is used can take various forms. It can be individual work for the student, with orders given by the teacher, or an activity followed by a whole class on several machines, or the presentation of a notion to a group of students with a display screen. Depending on the level of knowledge of the students and on the time devoted to the activity, the teacher may provide the tools, in the form of figures or macro-constructions elaborated beforehand, or may have them realised by the students. Cabri has an understanding of geometry that goes well beyond the dynamic and immediate construction of geometric figures. It allows one to visualise the loci of points by materialising the trajectory of a chosen point while moving another point according to the specifications of the figure. It also allows one to measure lengths, angles or areas and to observe their evolutions in real-time during the figure modifications.

The program exists in two versions, CABRI II plus and CABRI 3D. We discuss only the use of CABRI II plus in this contribution as we concentrated on the secondary school geometry. Cabri enables to solve geometric constructions using four basic objects - point, segment, line and circle. For work with these objects we can use built in elementary constructions such as the construction of the perpendicular line to a given line passing through a given point, the construction of the perpendicular bisector of a segment, the construction of the intersection point of two objects etc. An indisputable advantage of Cabri is its dynamics given by the program's ability to change the appearance of the picture by dragging individual points of the geometric figures. It is also very useful to represent loci of points by tracing the path of the selected point as it moves.

We deal with the usage of these features of the program in this contribution. The investigation of loci of points is one of the main ways of solving constructional problems. As the studied curves are immediately illustrated on the screen students or teachers can easily formulate the hypotheses about the studied loci of points also in the cases when their intuition, supported only by drawing sketches on paper, fails. Many interesting problems about loci of points can be found for example in book [2]. Now, we discuss finding of loci of points in some problems which we solved with our students.

HYPOTHESIS ABOUT LOCI OF POINTS

The following examples involve the situations in which during the formulation of the hypothesis about loci of points the usage of Cabri is really efficient. We describe those stages of the solution when Cabri was used in the commentary to the individual examples. We state the appropriate hypothesis and prove the hypothesis in some cases.

Example 1. There are given two concurrent lines p, q and their intersection point R. There is given the point D on one of the bisectors of the concurrent lines p, q in the distance of v from the point R. Through the point D the line d is passing and is rotating around the point D. The points P, Q are intersection points of the line d and the lines p, q. Find the locus of midpoints X of all segments PQ.

To solve this problem we formulate the following questions to students:

1. How do we have to construct the line d to be rotatable?
2. By which procedure can the midpoint X be constructed?
3. How do we display loci of points X?
4. Formulate hypothesis about the type of loci of points.

Answers:

1. The point D is one of the points of the line d. We draw the other point Y of the line d in the free space of the screen.
2. Using of the procedure "MIDPOINTS" in menu "OPERATIONS"
3. We draw the searched locus of points by dragging the point Y on the drawing area. Because the point D is fixed, our dragging of the point Y causes the rotation of the line d around the point D.
4. The searched locus of points is a hyperbola which vertices are the points R, D and the asymptotic lines are parallel to the lines p, q (figure 1).

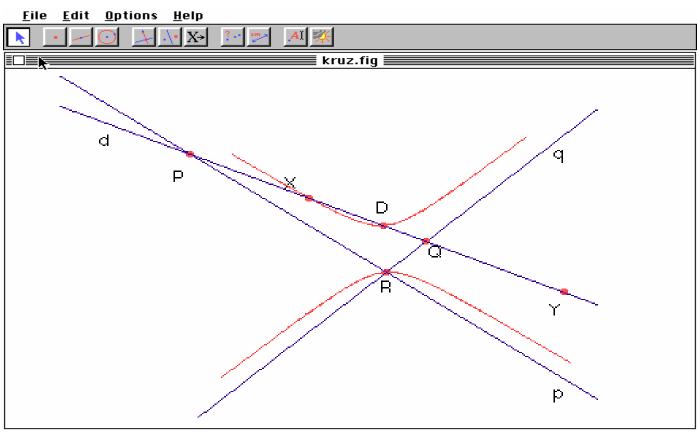

Figure 1. Locus of points for two concurrent lines p, q and point D

The following series of examples is interesting as it shows how easily new problems investigating the loci of points can be created by modifying the parametres of the basic problem.

Example 2. There is given a circle k, a point A lying on the circle k and a point S lying outside the circle. Let us consider the square ABCD with centrepoint S. Let us find the loci of all vertices B, C, D of such squares having the centrepoint S and the vertex A on the circle k in common.

We formulate the following questions to the students to solve this problem:

1. How do we have to construct point A?
2. How do we display loci of points B, C, D?
3. Formulate hypothesis about the type of loci of points B, C, D.
4. Prove hypothesis about the type of loci of points B, C, D.

Answers:

1. The point A is constructed by the procedure “POINTS ON OBJECT”.
2. We draw the searched loci of points B, C, D by dragging the point A along the circle k.
3. The searched loci of the points B, C, D are the congruent circles k_B, k_C, k_D. The circle k_C is the image of the circle k in the symmetry with respect to the centrepoint S. The circles k_B, k_D are the images of the circle k in the rotations around the point S where the angle of the measure $+90^o$ and -90^o, respectively, is used.
4. The proof of the hypothesis: In every square ABCD the vertices A, C are symmetrically conjugated with respect to the centrepoint S of the square. We get the vertices B and D by the rotation of the vertex A around the centrepoint S, where the oriented angle of the measure $+90^o$ and -90^o, respectively, is used, because it holds that $|\angle ASB| = |\angle ASD| = 90^o$.

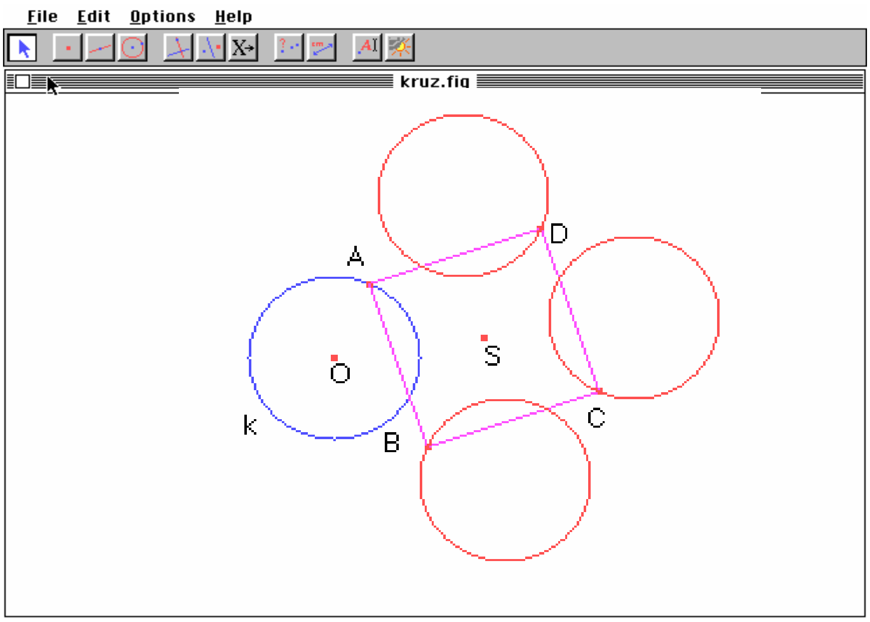

Figure 2. Loci of vertices B, C, D of the square

We gave the task to the students to create some analogous examples. They modified this basic problem by choosing the other point of the square which is fixed on the circle. The students constructed several types of problems. The following examples are two from variations of this basic problem found by students.

Example 3. There is given a circle k, a point S in the circle k and a point A outside the circle k. Let us find the loci of the vertices B, C, D of all squares ABCD where the vertex A is fixed and the point S moves along the circle k.

We formulate the following questions to the students to solve this problem:

1. How do we have to construct point S?
2. How do we display loci of points B, C, D?
3. Formulate hypothesis about the type of loci of points B, C, D.
4. Prove hypothesis about the type of loci of points B, C, D.

Answers:

1. The point S has to be constructed by the procedure "POINTS ON OBJECT".
2. We draw the searched loci of points B, C, D by dragging the point S along the circle k (see figure 3.).
3. The locus of the vertices C is the image of the circle k in the homothety with respect to the centrepoint A where the factor of the homothety 2 is used. The loci of the vertices B, D are the circles k_B and k_D which are the images of the circle k in the transformations composed of the rotation around the centrepoint A where the oriented angle of the measure -45^o , resp. 45^o , is used and of the homothety with respect to the same centrepoint where the factor of the homothety $\sqrt{2}$ is used.
4. The proof of the hypothesis: In any every square ABCD it holds that |AC| = 2|AS|. It also holds that $|\angle$ SAB | = $|\angle$SAD| = 45^o and |AB| = |AD| = $\sqrt{2}$ |AS|.

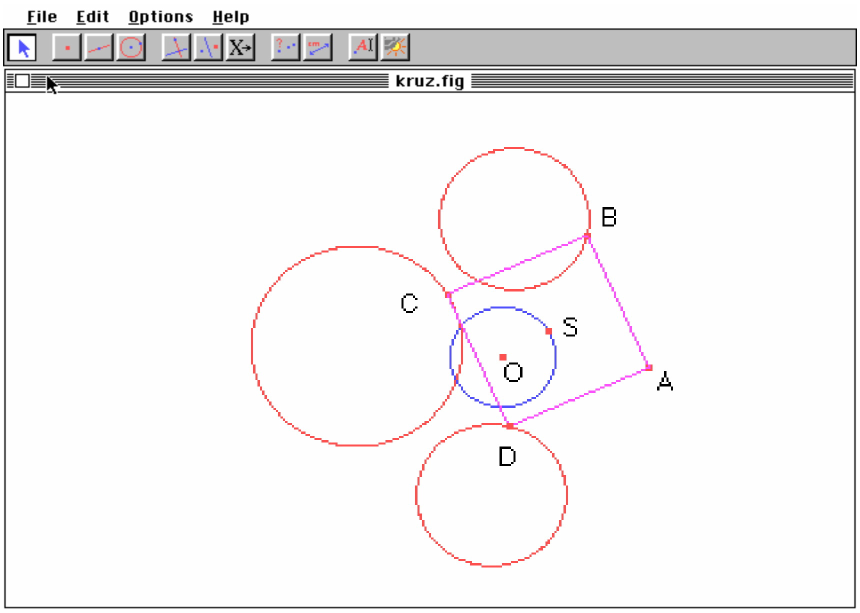

Figure 3. Loci of vertices B, C, D of the square

Example 4. There is given a circle k, a point A on the circle k and the point E outside the circle k. Let us consider the square ABCD. The point E lies on its diagonal and |AC| = 3|AE|. Let us find the loci of the vertices B, C, D and of the centrepoints S of all squares that have the point E in common and the point A moves along the circle k.

We formulate the following questions to the students to solve this problem:

1. How do we have to construct point A?
2. How do we display loci of points B, C, D?
3. Formulate hypothesis about loci of points B, C, D and S.
4. Prove hypothesis about loci of points B, C, D and S.

Answers:

1. The point A has to be constructed by the procedure "POINTS ON OBJECT".

2. We draw the searched loci of the points B, C, D, S by dragging the point A along the circle k (see figure 4.).
3. The loci of the vertices C and the centrepoints S are the circles k_C and k_S which are the images of the circle k in the homothety with respect to the centrepoint E where the factor of the homothety -2 and –1/2, respectively, is used. The locus of the vertices B and D is the circle k_B and k_D which are the images of the circle k in the transformations composed of the rotations around the centrepoint E where the oriented angle $\angle \overline{AEB}$, resp. $\angle \overline{AED}$, is used (congruent opposite orientation angles) and the homothety with respect to the centrepoint E where the positive factor of the homothety k is used.
4. The searched loci of the points C and S are the circles k_C and k_S, because in each square ABCD the relations |AC| = 3|AE|, |AS| = 3|AE|/2 hold and the point E is the inside point of the segment AS. We enumerate the measure of the angle of the rotation and the factor of the homothety. It is obvious that $|\angle \text{AEB}| = \pi - |\angle \text{SEB}|$ and in the right-angled triangle ESB the relation $|\angle \text{SEB}| = \text{arccotg}\dfrac{|ES|}{|BS|} = \text{arccotg}\ \dfrac{1}{3}$ is true. It also holds that $|\angle \text{AEB}| = \text{arccotg}\left(-\dfrac{1}{3}\right)$ (from the property of the function arccotg). Pythagoras's theorem holds for the right-angled triangle *ESB* and therefore we have for the factor of the homothety

$$k = \frac{|EB|}{|AE|} = \frac{\sqrt{|ES|^2 + |BS|^2}}{2\ |BS|/3} = \frac{\sqrt{1/9\,|BS|^2 + |BS|^2}}{2\ |BS|/3} = \frac{\sqrt{10}}{2}.$$

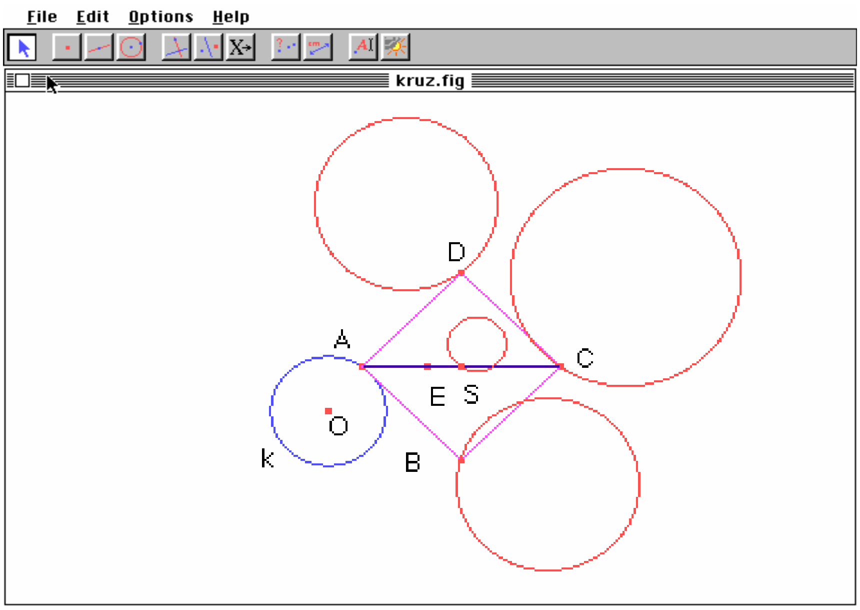

Figure 4. Loci of vertices B, C, D, S of the square.

CONCLUSIONS

The important feature of the preceding problems is the possibility to change the level of the difficulty of the problem, from making easy ones suitable for pupils at the basic schools to those that can be used in the universities. It would be very interesting to study similar loci of points with the help of Cabri, for example in case we use other conic instead of the circle and other type of the polygon (equilateral triangle, rectangle) instead of the square.

The formulation of the hypothesis is clear to be only one part of the solution of the problems concerning the loci of points. However, the experience regarding most difficult problems shows that this task can be rather complicated. The student who is less able to imagine geometrical situation need not be successful even in this phase of the solution of the constructional problem. Cabri should help him in this situation to overcome all the difficulties that can appear. Then, of course, the stated hypothesis should be proved or disapproved by the formal proof.
In our contribution we try to emphasize only one of the most efficient ways of using Cabri Géomètre during the solution of the constructional problems. However, the program offers the user (teacher, student) various ways of usage. The teachers especially are provided with the possibility of creative approach. Having gained sufficient knowledge of the program they can create new problems and prepare methodical material for their teaching and for an individual work of students.

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