Pricing and hedging GDP-linked bonds in incomplete markets

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Abstract

We model the super-replication of payoffs linked to a country's GDP as a stochastic linear program on a discrete time and state-space scenario tree to price GDP-linked bonds. As a byproduct of the model we obtain a hedging portfolio. Using linear programming duality we also compute the risk premium. The model applies to coupon-indexed and principal-indexed bonds, and allows the analysis of bonds with different design parameters (coupon, target GDP growth rate, and maturity). We calibrate for UK and US instruments, and carry out sensitivity analysis of prices and risk premia to the risk factors and bond design parameters. We also compare coupon-indexed and principal-indexed bonds.

Results shed light on the policy question whether the risk premia of these bonds make them beneficial for sovereigns. The findings from UK and US data affirm that both coupon-indexed and principal-indexed bonds can benefit a sovereign, with an advantage for coupon-indexed bonds. This finding is robust, but a nuanced reading is needed due to the many inter-related risk factors and design parameters that affect prices and premia.

Keywords: contingent bonds; debt restructuring; asset pricing; incomplete markets; risk premium; stochastic programming; super-replication.

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1 Introduction

An old idea for sovereign contingent debt was revived at the G20 meeting in Chengdu, China, in July 2016. There are theoretical arguments in favor of contingent debt for sovereigns, originating in the works of Froot et al. (1989); Krugman (1988), and the International Monetary Fund was asked to analyze "technicalities, opportunities, and challenges of state-contingent debt instruments, including GDP-linked bonds"[1]. In this paper we contribute a model for pricing and hedging GDP-linked bonds, and use it to estimate risk premia and compare two competing bond designs.

GDP-linked bonds make debt payments contingent on a country's GDP, thereby ensuring that debt can always be serviced. Linking instruments to GDP growth was first suggested during the debt crisis of the 1980's in the context of debt restructuring. Concerned about a country's growth prospects in the aftermath of the crisis, creditors and debtor governments sought instruments for risk sharing to increase debt resilience to macroeconomic shocks. A brief history of these instruments is given in Borensztein and Mauro (2004). Early attention was restricted to warrants that provide additional payments if a favorable event occurs, such as an increase in prices of commodities (e.g., oil, copper). Such warrants (also called *value recovery rights*) were offered to investors as part of the Brady restructuring process for Mexico, Nigeria, Uruguay, and Venezuela. Value recovery rights indexed to GDP growth were also included in debt instruments of Costa Rica and Bosnia and Herzegovina (Borensztein et al., 2004), and Bulgaria issued indexed debt with a call option in 1994. More recent examples are Argentina in 2005, Greece in 2012, and Ukraine in 2015. The Argentinean GDP-linked warrants received attention in the academic literature and Datz (2009); Guzman (2016) document significant gains for investors in these instruments, which, in turn, implies smaller haircuts for investors who accepted these instruments as part of a debt restructuring deal. In these early instruments sovereigns are funded mostly by plain vanilla bonds and detachable linked instruments served as sweeteners in debt restructuring. The recent G20 interest is in cash instruments, that pay both on the upside and downside, albeit differentiating based on growth. It is these instruments we call GDP-linked bonds. They have not been issued yet and the debate on their benefits for sovereigns is ongoing. It is for these instruments that we develop pricing and hedging models, and estimate risk premia.

GDP-linked bonds provide insurance from negative growth shocks (Froot et al., 1989), provide long-term investors an instrument to hedge income risks and invest in the "wealth of nations" (Shiller, 1993), allow governments to smooth taxation over the economic cycle (Barro, 2003), and reduce reliance on large-scale official sector support programs, thereby improving the functioning of the international financial system (Barr et al., 2014). Concerns that countries would manipulate GDP, or, *in extremis*, suppress growth to avoid debt repayment, were countered by proposals to link bond payments to exogenous factors that affect a country's macroeconomic conditions, such as commodity prices (Caballero and Panageas, 2008; Froot et al., 1989; Krugman, 1988). The earlier GDP-linked warrants pay only on the upside, so they are distant relatives to the new instruments, and Borensztein and Mauro (2004); Obstfeld and Peri (1998) suggest that governments reduce their idiosyncratic GDP risks by issuing such warrants.

Potential pitfalls in emitting GDP-linked bonds —moral hazard, imprecise, erroneous and/or manipulated statistics, call options— and ways to avoid them are summarized in Benford et al. (2016); Borensztein and Mauro (2004); Brooke et al. (2013). These papers argue in favor of GDP-linked bonds, although Brooke et al. (2013) also argue for a complement to GDP-linked bonds in the form of sovereign contingent convertible debt, a suggestion advanced in the S-CoCo instrument of Consiglio and Zenios (2015) who propose a design with contingent payment

[1] Paragraph 11 of Communiquè G20 Finance Ministers and Central Bank Governors Meeting, G20 Web Site, 28 July 2016, available at http://www.g20.org/English/Documents/Current/201607/t20160728_3091.html

standstill and develop the pricing (Consiglio et al., 2016b) and risk management models.

Neither GDP-linked bonds nor GDP per se are currently traded. Hence, the markets are incomplete and there is a dearth of pricing and hedging models for these instruments. The lack of a pricing model is not necessarily an obstacle to issuing GDP-linked bonds —stocks and options were traded before Black-Scholes/Merton developed their formulas— but availability of such models will encourage the development of a market (Borensztein et al., 2004; Griffith-Jones and Sharma, 2006). Bank of England authors (Benford et al., 2016) adopt a Darwinian stance towards the development of GDP-linked markets by suggesting the commission of a set of "rival pricing models". The primary contribution of our paper is a model for pricing GDP-linked bonds in incomplete markets, using stochastic linear programming to compute a super-replicating portfolio. As a byproduct of the model we obtain a hedging portfolio for investors in these novel instruments, and using linear programming duality we compute risk premia for bonds of different designs.

Sovereigns will benefit from GDP-linked bonds if they are not too expensive, and several researchers are asking "At what premium do the benefits of GDP-linked debt payments outweigh the burden of issuing more expensive debt?". Benford et al. (2016) give an overview of existing answers, and we use the pricing model to calculate risk premia and answer the policy question whether GDP-linked bonds are beneficial for sovereigns.

In the computational part of the paper we apply the model to price instruments for UK and US, and study the sensitivity of prices and risk premia to the risk factors and bond design parameters. We also estimate risk premia for different GDP-linked bond designs, and comparing with premium thresholds from the literature we draw conclusions on designs that are beneficial for sovereigns. The model is also used to compare two bond designs that compete for attention in the current debate, namely *coupon-indexed* and *principal-indexed* bonds.

The paper is organized as follows. Section 2 reviews literature for pricing GDP-linked bonds and pricing in incomplete markets, develops the model for pricing and hedging GDP-linked bonds, and shows how to estimate the risk premium. Section 3 describes the calibration and reports numerical results. Section 4 concludes.

2 The pricing model

The market for GDP-linked bonds is incomplete in the sense defined in Pliska (1997). Not every GDP contingent claim can be generated by some trading strategy using market instruments. There are many source of market incompleteness, the most significant for the present study being the lack of instruments trading in GDP. Even when a suitable proxy is identified —such as, for instance, the use of copper prices for the Chilean economy (Caballero and Panageas, 2008; Froot et al., 1989; Krugman, 1988)— other sources of market incompleteness remain, such as jumps in the underlying stochastic process due to exogenous shocks to the economy or market crashes for the proxy, and heteroskedasticity of the GDP process. (Other sources of market incompleteness, such as frictions arising from transactions costs and the use of discrete hedging, are not unique to the GDP bond markets.)

Assuming market completeness, Kruse et al. (2005) develop a Black-Scholes type pricing model using a single-factor stochastic model when GDP follows a log-normal distribution and interest rates are deterministic, and they use it to estimate returns on linked and plain bonds for Indonesia and Venezuela. Miyajima (2006) develops pricing models for GDP-linked warrants using a Brownian motion for the GDP and Monte Carlo simulations on foreign currency and inflation conditions, and studies the sensitivity of prices to exchange rate and GDP shocks. This model assumes risk neutral investors. Kamstra and Shiller (2009) price their version of GDP-linked bond (the "trill") using a fundamental valuation dividend-discounting method to estimate prices and yields for GDP-linked bonds and, applying portfolio diversification, they conclude that long-term investors will hold a portfolio of 28% bonds, 38% S&P500 and 34%

linked bonds. This model needs an exogenously determined risk premium and the authors assume a value 350bp, which is the risk premium used to discount risky equities. Bowman and Naylor (2016) use CAPM (and downside-CAPM) to obtain a range of premia for G20 countries and contribute to the policy debate on the benefits of these instruments (although their empirical findings do not lead to clear conclusions). These market-based premia estimates could be used in a model such as the one of Kamstra and Shiller (2009) to price the instruments.

Our methodological approach is distinct from these earlier works and allows us to overcome their limiting assumptions. We account for the fact that markets are incomplete, estimate prices and premia that are internally consistent and externally consistent with market data, and provide a hedging portfolio for GDP-linked bond investors. The model does not need assumptions on investor risk aversion (Barr et al., 2014), or exogenous estimation of a premium (Kamstra and Shiller, 2009).

In the case of market incompleteness there may not exist a unique price for a contingent claim, but the absence of arbitrage specifies a range of prices. Following King (2002) we use stochastic linear programming to compute a super-replication strategy on a discrete market model. A positive optimal objective value for the stochastic program identifies an arbitrage strategy that begins with a portfolio of value zero, makes self-financing trades at each time step, has non-negative terminal values in every scenario, and has a positive expected value at maturity. Hence, the optimal value of the stochastic program is the price of the contingent claim that must be paid up front so that the portfolio value is non-zero, thus precluding arbitrage. The least cost of super-replication is the seller's price, whereas the greatest amount a buyer could pay for the contingent claim without negative terminal wealth is the buyer's price. The theory for obtaining price bounds on a contingent claim in incomplete markets is developed in King et al. (2005). In this section we formulate a stochastic programming model for computing bid and ask prices for GDP-linked bonds.

The methodology we adopt and the models we develop can be applied to other derivatives linked to economic activity. For instance, (Baron and Lange, 2007, pp. 36-37) report that Goldman Sachs and Deutsche Bank completed in September 2002 parimutuel auctions of options on the US Bureau of Labor Statistics' release of change in US Nonfarm Payroll data, and in April 2003 they hosted auctions for 3- and 6-month options on the European Harmonized Index of Consumer Prices. Our work shows that we can rigorously price such derivatives in the absence of a complete market, even if the prices are estimated within a bid-ask spread.

There are two types of GDP-linked bonds and both can be priced with our framework:

Coupon-indexed bonds link the coupon to GDP growth (Borensztein and Mauro, 2004) by

$$c_t = \max\left[c_0 + (g_t - \bar{g}), 0\right], \tag{1}$$

where g_t is the real growth rate, $\bar{g}$ is a contractually specified target growth rate, and c_0 is the baseline coupon rate. If growth exceeds the target, the coupon will increase from the baseline, otherwise coupon payments decrease with a floor at zero.

Principal-indexed bonds pay principal at maturity (Kamstra and Shiller, 2009) according to the formula

$$B_t = B_0 \frac{Y_t}{Y_0}, \tag{2}$$

where B_0 is the original amount issued, typically set at 100, and Y_0, Y_t are the nominal GDP values at the issuing date and t, respectively. Coupon payments satisfy $(1+c_0)B_t = (1+c_0)B_0\frac{Y_t}{Y_0}$, so the *effective coupon rate* per 100 units of debt is given by

$$1 + c_t = (1 + c_0)\frac{Y_t}{Y_0}. \tag{3}$$

There are several variations of these two instruments (Kruse et al., 2005), and all can be priced using our model. The specifications of each instrument are given by some functions $c_t = \Phi(g_t)$ or $c_t = \Phi(Y_t)$ that are input to the pricing model. For instance, the Greek GDP-linked security, issued as part of debt restructuring in 2012, stipulates a function with coupons equal to 1.5 times the excess GDP growth over a target $\bar{g} = 2.9\%$ until 2020, and 2% afterwards.

2.1 Model preliminaries

We assume that the time horizon of the investor consists of a finite set of *decision stages* $\mathcal{T} = \{0, 1, 2, \ldots, T\}$, and that the market consists of $J+1$ securities with prices $S_t = \left(S_t^0, S_t^1, \ldots, S_t^J\right)$ at each $t \in \mathcal{T}$. Without loss of generality we use S_t^0 for the price at time t of 1€ invested in the money market at time 0, and express security prices in terms of the *numéraire* $\beta_t = 1/S_t^0$. The *discounted price process*

$$Z_t = \frac{S_t}{S_t^0} = \beta_t S_t \tag{4}$$

denotes prices relative to the numéraire. Expressing prices relative to a numéraire, or as a discounted price process, is not done only to account for time value. Under certain hypotheses it can be proved that the stochastic process Z_t is a *martingale* with respect to a measure Q. In continuous time, this property is expressed by

$$Z_t = \mathbb{E}_Q \left[Z_T | \mathcal{F}_t \right], \tag{5}$$

where $\mathbb{E}$ is the expectation operator and $\mathcal{F}_t$ is a filtration.

For discrete time and discrete probability space $\Omega = \{\omega_1, \omega_2, \ldots, \omega_L\}$, the evolution of the risky asset price S_t can be represented using *scenario trees*. A scenario tree is a set of states and interconnecting links denoting possible transitions between states. In particular, at each non-final state corresponds one, and only one, ancestor state $a(n)$, and a non-empty set of child states $\mathcal{C}(n)$. In this structure, the filtration $\mathcal{F}_t$ is a partition of Ω into sets $\mathcal{A}_t$, with $\mathcal{A}_t \subseteq \mathcal{A}_{t+1}$, with one-to-one correspondence between $\mathcal{A}_t$ and the set of states of the tree $\mathcal{N}_t$, see Figure 1. The martingale property on a tree is a conditional expectation with respect to the set of states at t:

$$Z_t = \mathbb{E}_Q \left[Z_T | \mathcal{N}_t \right]. \tag{6}$$

Following King (2002), we assume that the random process S_t is *measurable* with respect to $\mathcal{N}_t$, meaning that S_n is the value taken by S_t at each state $n \in \mathcal{N}_t$. If we denote by $\mathcal{K}(n)$ the set of states $m \in \mathcal{N}_T$ that can be reached from n ($\mathcal{K}(n) \subseteq \mathcal{N}_T$), equation (6) becomes

$$Z_n = \sum_{m \in \mathcal{K}(n)} \frac{q_m}{q_n} Z_m. \tag{7}$$

Note that for Z_0, $q_0 = 1$ and $K(0) = \mathcal{N}_T$. The set of weights $q_n > 0$ attached to each terminal state $n \in \mathcal{N}_T$ form the *risk neutral* probability distribution Q. Q is equivalent to the *objective* probability measure P —in the sense that P and Q both agree on the set of events with null probability— identified by the set of weights $p_n > 0$, $n \in \mathcal{N}_T$, with

$$\sum_{n \in \mathcal{N}_T} p_n = \sum_{n \in \mathcal{N}_T} q_n = 1. \tag{8}$$

The existence of a risk neutral measure Q ensures that arbitrage is ruled out, and Q is unique in *complete* markets.

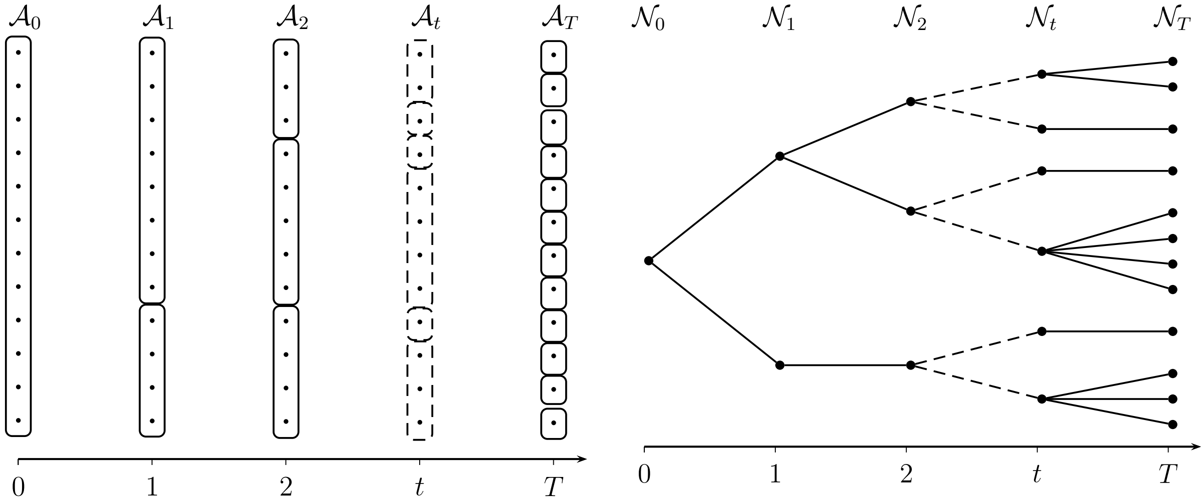


Figure 1: A finite filtration (left panel) and its associated tree (right panel).

2.2 The super-replication pricing model

Let us assume that we are supplied with a scenario tree endowed with both risk neutral Q and objective P probability measures, such as the one calibrated in Consiglio et al. (2016a), and let S^1 denote the GDP level of a given country. If the market is complete, then we can price any claim with payoffs contingent on the GDP through some function $\Phi(S^1)$, by discounting its expected payoff under Q. This price, however, would be incorrect for GDP-linked bonds. The underlying assumption is that the payoff $\Phi(S_n^1)$ is attainable by means of a self-financing portfolio, which implies that it is possible to trade the GDP, or that there exists an asset (e.g., a future on GDP) that reproduces GDP dynamics. If such an asset does not exist, then the price of the GDP contingent claim, which is the cost of the replicating portfolio, can not be obtained. In particular, either the buyer or the seller will incur higher costs. The seller asks a price equal to the lowest cost for hedging the payoff stream with instruments not perfectly replicating the GDP dynamics. Similarly, the buyer bids at a price that is the highest cost of a portfolio producing at least the same payoff as the GDP contingent claim. The equilibrium price of the asset lies between the buyer and seller prices, and the distance between the two is the *bid-ask spread*.

A way to model buyer and seller behavior is to select a portfolio that super-replicates the payoff. The lower, in absolute value, the correlation of assets with the GDP, the higher will be the cost (for buyer and seller) to hedge the cashflows, and the wider the bid-ask spread. Following (Consiglio and De Giovanni, 2010; King, 2002) we formulate the super-replication problem as a stochastic program on the scenario tree. Seller's price V is the minimum amount that an issuer asks for selling the cashflows $\Phi(S^1)$ without risk of negative terminal wealth, and is the solution of:

$$\underset{V,\theta}{\text{Minimize}} \; V \tag{9}$$

s.t.

$$Z_0 \cdot \theta_0 = V, \tag{10}$$

$$Z_n \cdot (\theta_n - \theta_{a(n)}) = -\beta_n \Phi(S_n^1), \quad n \in \mathcal{N}_t, t \geq 1, \tag{11}$$

$$Z_n \cdot \theta_n \geq 0, \quad n \in \mathcal{N}_T, \tag{12}$$

$$\theta_n^1 = 0, \quad n \in \mathcal{N}, \tag{13}$$

where $\theta_n = (\theta_n^1, \theta_n^2, \ldots, \theta_n^J)$ is the hedging portfolio at state n. Constraints (13) take into account the non-tradeability of GDP (S_n^1) by fixing the corresponding weights (θ_n^1) to zero.

Similarly, the super-replication model for the buyer's price computes the maximum amount the buyer is willing to bid to purchase $\Phi(S^1)$ without risk of falling short at maturity:

$$\underset{V,\theta}{\text{Maximize}} \; V \tag{14}$$

s.t.

$$Z_0 \cdot \theta_0 = -V, \tag{15}$$

$$Z_n \cdot (\theta_n - \theta_{a(n)}) = \beta_n \Phi(S_n^1), \quad n \in \mathcal{N}_t, t \geq 1, \tag{16}$$

$$Z_n \cdot \theta_n \geq 0, \quad n \in \mathcal{N}_T, \tag{17}$$

$$\theta_n^1 = 0, \quad n \in \mathcal{N}. \tag{18}$$

In complete markets, the bid and ask prices coincide, and the pricing mechanism simply reduces to discounting the expected cashflows with respect to Q.

It is instructive to model the contingent claim price as the dual of (9)–(13). The price will be the same, but the dual also provides the risk neutral measure Q for an incomplete market, which is used to estimate risk premia in Section 2.3. With endogenous risk premia the model is internally consistent. Following King (2002), the dual problem of (9)–(13) is:

$$\underset{q}{\text{Maximize}} \quad \sum_{t=1}^{T} \sum_{n \in \mathcal{N}_t} q_n \beta_n \Phi(S_n^1) \tag{19}$$

s.t.

$$q_n \geq 0, \quad n \in \mathcal{N}_T, \tag{20}$$

$$q_0 = 1, \tag{21}$$

$$\sum_{m \in \mathcal{C}(n)} q_m Z_m^j = q_n Z_n^j, \quad j \neq 1, n \in \mathcal{N}_t, t = 0, 1, \ldots, T-1. \tag{22}$$

Unlike King (2002), we introduce the martingale measure q_n for the discounted process Z_n. Constraints (22) ensure that q_n is a martingale measure, cf. eqn. (7). To take into account the non-tradeability of GDP, martingale equations are defined for all risk factors except Z_n^1. q_n is associated with the incomplete market price of the GDP-linked bond, and, as explained next, there is close relation between risk neutral probabilities and the stochastic discount factor.

2.3 Estimating the risk premium

To assess the merits of GDP-linked bonds we need to examine two counteracting effects. Decrease in default risk of GDP-linked financed sovereigns lowers risk premia, whereas the systematic risk of bonds linked to a volatile GDP commands a premium. Several researchers estimate a risk premium threshold that makes these instruments attractive for sovereigns. Barr et al. (2014) study the effect of GDP-linked bonds on the maximum sustainable level of debt and the probability of default of a sovereign. Modeling the cost of the risk premium and the lowered default probabilities of GDP-linked bonds they conclude that there are welfare gains for risk premium lower than 350bp. (Recall that risk premia are negative and in this paper we refer to absolute values.) Blanchard et al. (2016) simulate the effects of plain bonds and GDP-linked bonds on default probabilities and find that GDP-linked bonds dominate when the risk premium is 100bp, but at 200bp plain bonds are favored. (The threshold depends on the original indebtedness.) Benford et al. (2016) report similar findings. The take from these papers is that for risk premia above 350bp the GDP-linked bonds are too expensive[2]. Values in the range 100bp to 350bp (250bp for more conservative estimates) indicate that GDP-linked bonds benefit sovereigns. For premia lower than 100bp the probability of insolvency is reduced significantly by issuing GDP-linked debt to compensate for the systematic risk of GDP volatility. These results serve as thresholds when assessing the viability of financing sovereigns with GDP-linked bonds. We use 250bp as a tight threshold and 350bp as a relaxed threshold.

The studies cited above do not estimate what the premium will be but what it should be for the issuing sovereign to benefit. Our interest is to estimate the risk premium and compare it to the thresholds. To do so requires that several risk factors need to be disentangled. Borensztein et al. (2004); Griffith-Jones and Sharma (2006) identify the risk factors which are summarized by Blanchard et al. (2016) as follows:

Novelty premium for buying a new and unfamiliar investment product.

Liquidity premium for converting the asset into cash at fair market value in thin markets.

Default premium for the risk that the debtor will not make the required repayments. (This premium is positive if GDP-linked bonds make debt more sustainable.)

Growth risk premium for exposure to a country's economic growth uncertainty.

The first two risk premia are transitory. Novelty and liquidity premia decline as the markets deepen. For instance, Costa et al. (2008) find the novelty premium on Argentina's GDP-linked warrants declined by about 600bp during the first 18 months after issuance. It is hard to model these premia and the (limited) research on pricing GDP-linked bonds focuses on estimating the growth risk premium. We show how to obtain the risk premium from our model.

Consider a stochastic payoff x linked to the GDP (the precise form of indexation $x = \Phi(S^1)$ is not important) with price $\mathcal{P}_0$ obtained from (Cochrane, 2005, eqn. (1.9)):

$$\mathcal{P}_0 = \frac{\mathbb{E}_P(x)}{1 + r_f} + \mathrm{cov}(m, x). \tag{23}$$

r_f is the risk free rate of return and m is the stochastic discount factor, with $1 = \mathbb{E}_P((1+r_f)m)$. The first term on the right is the discounted expected value under the objective probability measure and the second term is a risk adjustment. Assets with payoffs negatively correlated to the discount factor have lower prices and, therefore, have higher excess return over the risk free rate. Debt with payoffs negatively correlated to the discount factor will be more expensive.

The interpretation of the second term as a risk premium is better understood if, following (Cochrane, 2005, eqn. (1.10)), we write

$$\mathcal{P}_0 = \frac{\mathbb{E}_P(x)}{1 + r_f} + \frac{\mathrm{cov}(\beta \mathcal{U}'(C_{t+1}), x_{t+1})}{\mathcal{U}'(C_t)}, \tag{24}$$

where β is the subjective discount factor capturing impatience, $\mathcal{U}(\cdot)$ the investor utility function, and C_t, C_{t+1} the consumption at t and $t+1$. The second term is negative if the payoff covaries negatively with the marginal utility, and since marginal utility declines as consumption rises, the term is negative if payoff covaries positively with consumption. This is key to understanding the risk premium: if payoff is correlated with consumption it makes consumption more volatile, and investors demand a price discount to buy the asset. Equivalently, investors expect higher return for holding the asset.

[2] In a recent presentation, J.D. Ostry and J.I. Kim of the IMF put the acceptable premium in the range 153bp–260bp for low and high growth uncertainty countries, respectively. Their estimates are in broad agreement but somewhat more conservative than previous studies.

The risk premium can be obtained from our pricing model. The price $\mathcal{P}_0$ is computed under the risk neutral probability measure (Cochrane, 2005, section 3.2) by

$$\mathcal{P}_0 = \frac{\mathbb{E}_Q(x)}{1 + r_f}, \tag{25}$$

and combining (23) and (25), we write the risk premium as

$$\text{cov}(m, x) = \frac{\mathbb{E}_Q(x)}{1 + r_f} - \frac{\mathbb{E}_P(x)}{1 + r_f}. \tag{26}$$

The first term on the right is the price of a risk averse investor, calculated from our pricing model by discounting the payoff under the risk neutral measure Q. The second term is the price of a risk neutral investor, computed by discounting under the objective measure P.[3] Alternatively, we can calculate the covariance of the payoffs with the discount factor that is also obtained from the calibrated scenario tree by $m = \frac{q}{p(1+r_f)}$.

It is worth noting that the risk premium calculation is endogenous to the model, so that prices and premia are internally consistent. The model does not need any assumptions on investor level of risk aversion (Barr et al., 2014), or exogenous estimation of a premium (Kamstra and Shiller, 2009). Market information is conveyed through the calibrated tree, making only an arbitrage-free assumption, and is used to estimate both prices and risk premia.

3 Numerical results

We calibrate the models for GDP-linked bonds for UK and US in year 2013, and use them to (i) study the effect of risk factors and bond design choices on the prices of different coupon-indexed bonds, (iii) estimate risk premia, and (iii) compare the coupon-indexed and principal-indexed bonds. We calibrate the arbitrage-free tree of Consiglio et al. (2016a) for each country using asset returns from the Dimson-Marsh-Staunton Global Returns Data (Dimson et al., 2002) and GDP data from Schularick and Taylor (2012). For UK we use as traded assets UK T-bills, bonds and equity indices, and World bills, bonds and equity indices. The database uses US T-bills as the World bills index, so for the US model we use the three US assets plus the World (ex-US) bonds and equity indices. For risk free rate we use the yield curve on treasury bonds from Bank of England and US Department of Treasury of December 31, 2013. Means, standard deviations and correlations of the time series are estimated for the time windows 1983-2013, 1993-2013, and 2003-2013, see Appendix A. We also use IMF projections for GDP growth (1.5% for UK and 2.3% for US) with the 2003-2013 calibrated asset data. Data used to estimate moments are in nominal values.

Pricing proceeds in two steps. First, a tree is calibrated on estimated market moments, and, second, the stochastic programs are solved to determine seller and buyer prices. The stochastic program is implemented in the algebraic modeling language GAMS (GAMS Development Corporation, 2016) and optimized using solver `CPLEX`. To illustrate the computational demands of the procedure we summarize in Table 1 the total execution time of both steps, and the sizes and solution times of the stochastic program. All models are solved on a machine with an Intel Xeon processor with 32 Gbytes RAM. The most time consuming part is for tree building, however once a tree is built it is used repeatedly to price multiple instruments and calculate risk premia. The large execution time is due to the need to store the tree in a database for the extensive numerical experiments performed. Storing the tree in RAM speeds up computations, although the size of the tree grows exponentially with time to maturity and limits the use of

[3] For coupon bearing bonds the calculations involve summations over the decision stages to compute the price $\mathcal{P}_t$ of payoff x_{t+1} recursively until we get the price $\mathcal{P}_0$ at the origin. These recursive discounted summations are standard and we do not give them here.

Maturity (years)	Tree building time (h:m:s)	Stochastic program size			Stochastic program solution time (h:m:s)
		Equations	Variables	Nonzeros	
2	0:00:01	72	63	560	0:00:00
3	0:00:02	584	511	4,592	0:00:01
4	0:00:13	4,680	4,095	36,848	0:00:01
5	0:07:11	37,448	32,767	294,896	0:00:09
6	6:52:10	299,592	262,143	2,359,280	0:07:23

Table 1: Execution times for tree calibration and solution of the pricing model. The size of the stochastic program grows with bond maturity. Better memory management can reduce tree building times, even though for longer maturities coarser discretization might be needed.

Calibration window	Buyer price	Seller price
UK reference bond		
2003-2013	0.982	1.000
1993-2013	0.965	0.968
1983-2013	0.996	1.023
IMF	0.962	0.964
US reference bond		
2003-2013	0.980	0.983
1993-2013	0.985	0.996
1983-2013	0.976	0.982
IMF	0.980	0.981

Table 2: Buyer and seller prices for coupon-indexed bonds using different calibration windows.

RAM. Larger models can be solved using either coarser time discretization or high-performance computers.

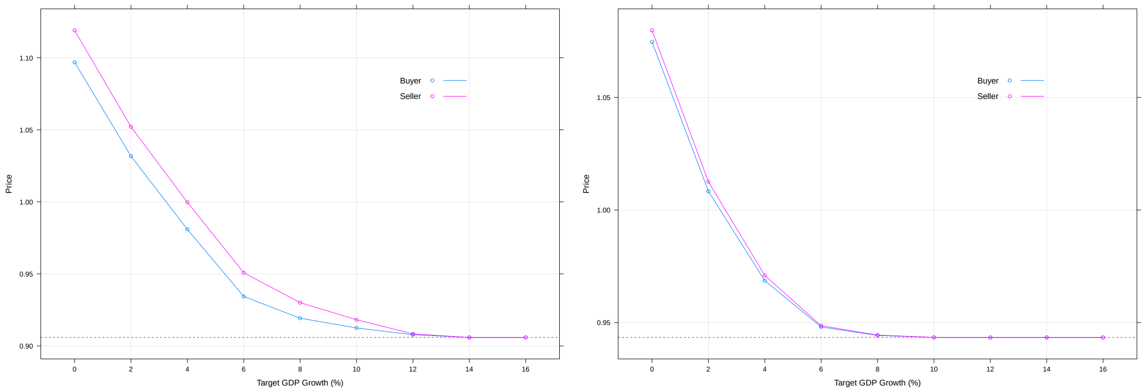
3.1 Pricing calibrations

We apply now the models to price coupon-indexed bonds with different design characteristics. We also give some results with the pricing of principal-indexed bonds, but defer a comparison of the two types of bonds to section 3.3. We consider reference GDP-linked bonds for UK and US with 5 year maturity, base coupon rate equal to the risk free rate 2% and 1.17%, respectively, and $\bar{g}$ the expected value of GDP growth. The base scenario tree is calibrated on the time window 2003-2013, with $\bar{g}$ equal to 3.97% and 3.79% respectively[4].

In Table 2 we summarize buyer and seller prices for the reference bonds using different time windows for the calibration, and with IMF projections. Bid-ask spreads remain consistently less than 3bp. Buyer prices vary by up to 3%, and seller prices by up to 6%, with the time window. These results are encouraging for the bonds priced here, but we emphasize that prices depend on asset moment estimates so they can be quite different for the bonds of other countries.

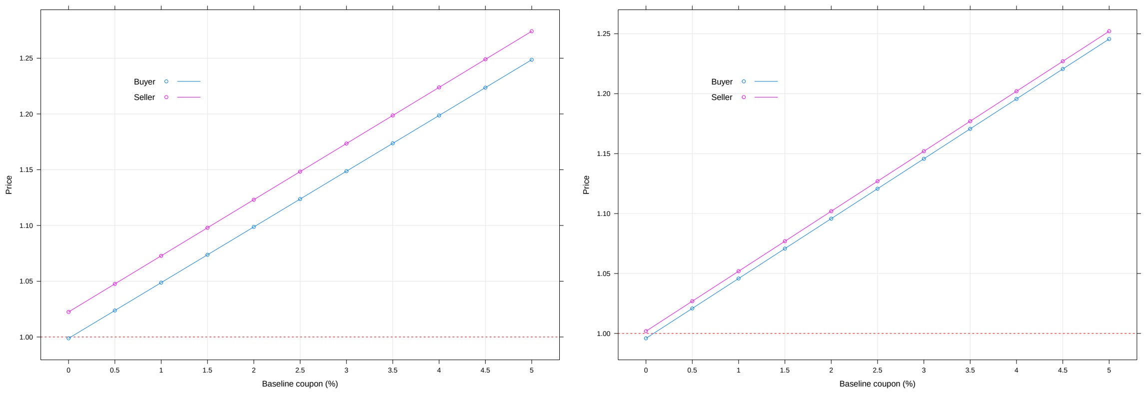
The prices depend on the characteristics of the specific bond, and we carry out sensitivity analysis on the design parameters of base coupon and target growth. The effect of varying target growth values is illustrated in Figure 2. Prices decrease linearly for a broad range of target growth, but for higher target growth they tapper off and converge to that of a zero

[4]The careful reader should be aware that mean GDP growth rates in the appendix are reported to three decimal points and read as 0.040 and 0.038 respectively.



(a) UK (b) US

Figure 2: Buyer and seller prices for coupon-indexed bonds decrease with higher GDP target growth threshold, and converge to the price of a zero coupon bond.



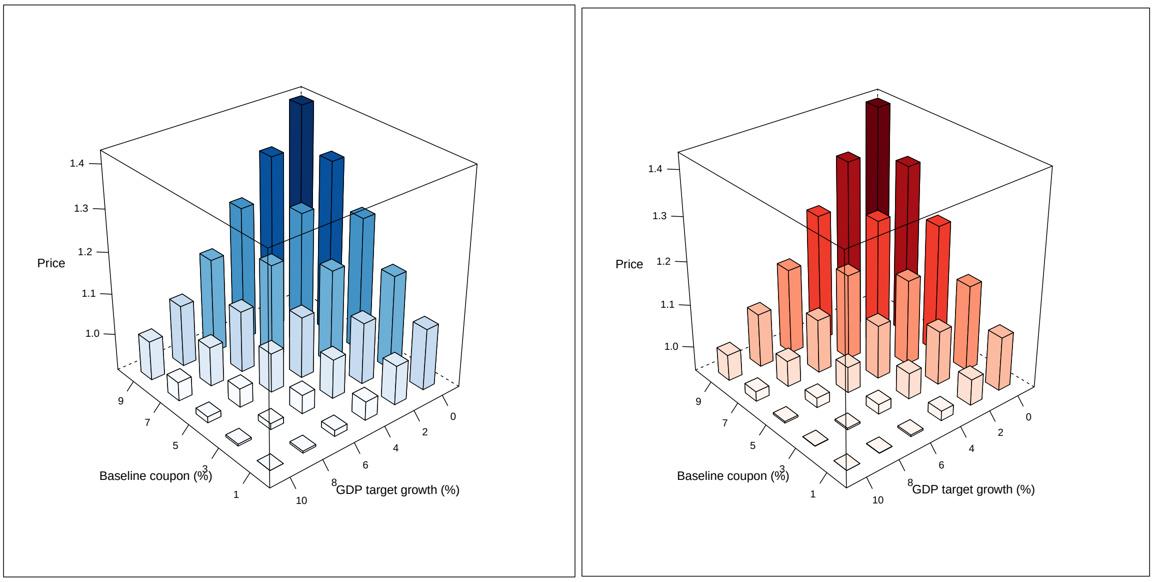
(a) UK (b) US

Figure 3: Buyer and seller prices for coupon-indexed bonds increase with the base coupon rate.

coupon bond. This is a consistency check for the model, since for large $\bar{g}$ the indexed coupon is determined by the floor at zero. The coupon payment of the coupon-indexed bond is option-like, cf. eqn. (1), with $\bar{g}$ being the strike price, and for large values of the strike price the option is out of the money. We also note from the figure that the GDP-linked bond is priced above or below par, depending on parameter settings. This is understood as follows. For coupon equal to the risk free rate and zero target growth $\bar{g}$, we have an instrument with upside potential but no downside risk and it is priced above par. For large $\bar{g}$ there is downside risk that coupon payments could decrease to zero and prices are below par. For some intermediate value the upside potential is equal, in expectations, to the downside potential, and the bond prices at par.

Figure 3 shows buyer and seller prices for different base coupon rates. The linked bonds become linearly dearer for higher base coupon rates. In Figure 4 we parametrize the design space and illustrate the changes of bid prices with joint changes in base coupon and target growth. The price is equally sensitive to changes of base coupon or target growth.

The effect of bond maturity on prices is illustrated in Figure 5. *Ceteris paribus* bond prices decline with maturity, indicating that investors expect a higher excess return for the longer maturity bonds. This price-maturity relationship is affected by the shape of the underlying yield curve, as illustrated in the same figure by pricing the UK bond using, first, the upward



(a) UK (b) US

Figure 4: Buyer price sensitivity to joint changes in base coupon and target growth for coupon-indexed bonds.

sloping yield curve of December 2013 and, then, a constant spot rate 1.47%. We also note an increase in the bid-ask spread for longer maturities, although it remains small.

To assess the accuracy of the models in the presence of market incompleteness we look at the bid-ask spreads. In all experiments reported above the spread is less than 4bp, indicating that for these two countries and the design parameters of the reference bonds, the chosen assets span accurately the countries' GDP. This may not be the case for other countries and different choices of assets, or for different bond design parameters, or when using different time periods to calibrate the tree. Figure 6 illustrates the case for UK, Italy, and Portugal, with base coupon in the range 1–10% and target growth in the range 0–10%. Spreads for UK and Italy are up to 15bp, and for Portugal up to 25bp. These spreads are small and lend credibility to the model.

Finally, we use the model to compare prices for the two bond designs, see Figure 7. The price of principal-indexed bonds for different starting base coupons exhibits the same pattern as coupon-indexed bonds. Bid-ask spreads are 3bp for UK and 1bp for US, and coupon-indexed bonds are cheaper than principal-indexed bonds due to their limited downside risk.

3.2 Risk premium calibrations

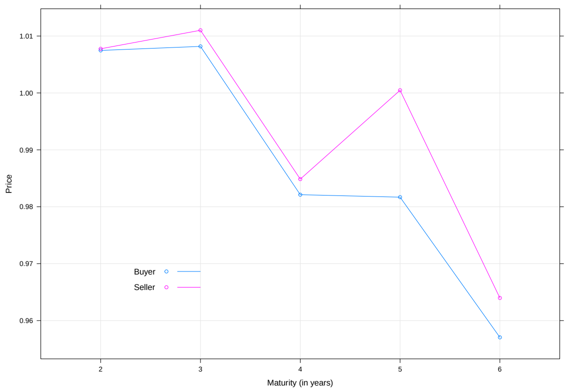
Risk premia can be calculated from eqn. (26) using quantities that are available from the primal and dual pricing models [5]. We summarize first premium estimates from existing literature that provide some reference values, and then discuss our own findings.

3.2.1 Premium estimates from the literature

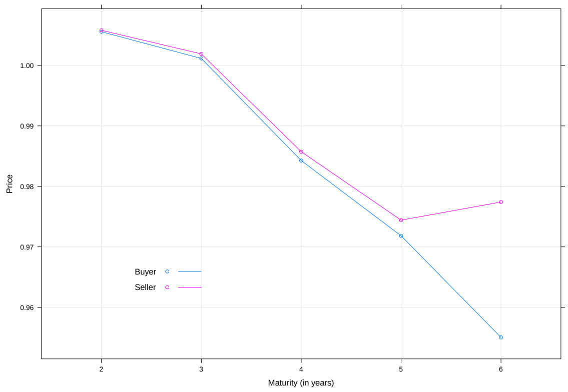
Kruse et al. (2005) use their pricing model to estimate return spreads[6]. They find positive premia for Indonesia, in the range 76bp–200bp, and negative for Venezuela, in the range 24bp–264bp. Higher premia are not always obtained for more volatile GDP returns, and the authors

[5] We estimate premia using both the left and right sides of eqn. (26) and obtain identical results. The premium definition in the equation is for a single-period model, and when calculating premia for bonds of different maturities all quantities are annualized.

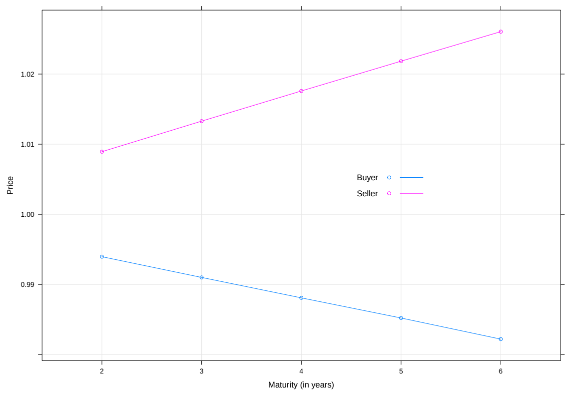
[6] Computed as the difference of the returns (not log returns) from Tables 1 and 2 of the reference.



(a) UK



(b) US



(c) UK

Figure 5: Price vs. maturity for coupon-indexed bonds. The top panels use the yield curve of December 2013 for discounting, the bottom panel uses a flat yield curve at 1.47%, with all other data calibrated for 2003–2013.

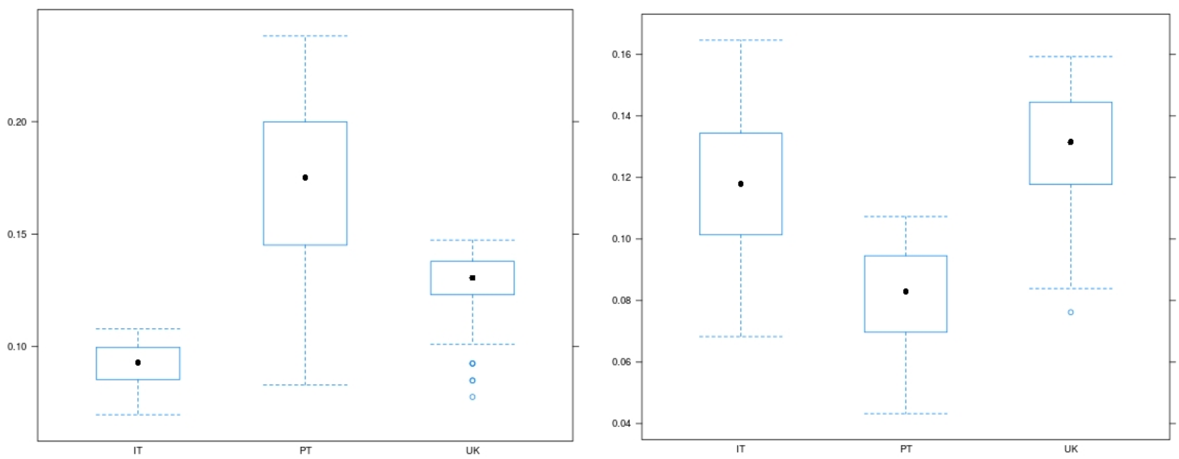
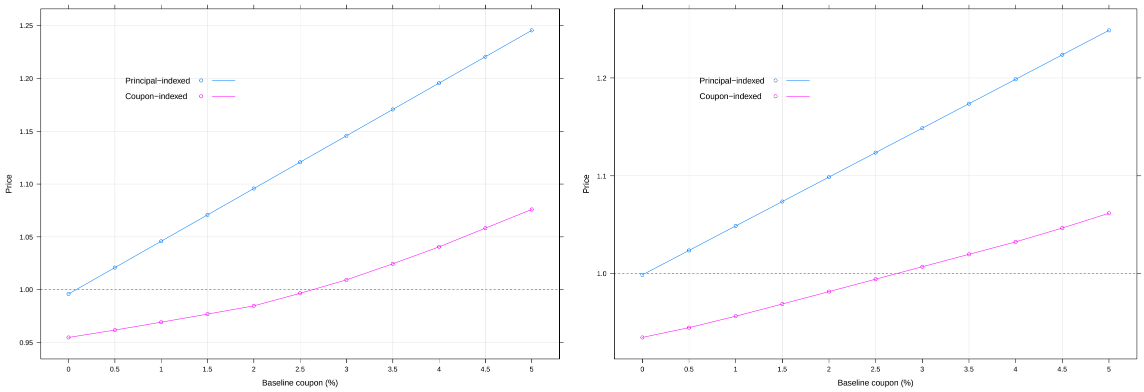


Figure 6: Box-Whisker plots of bid-ask spreads for UK, Italy, and Portugal coupon-indexed bonds for base coupon 1–10% and target growth 0–10%. Left panel uses US assets and right panel uses German assets to calibrate a tree.



(a) UK (b) US

Figure 7: Comparing buyer prices of principal-indexed and coupon-indexed bonds. (Tree calibrated for 2003–2013. For principal-indexed bonds the coupon rate on the x-axis is the effective coupon.)

conclude that “comparisons between the GDP-linked bonds and the underlying government bond, from which the prices of the GDP-linked bonds are derived, show no clear patterns”. Bowman and Naylor (2016) use CAPM and downside-CAPM to obtain a range of premia for G20 countries. Their results show that the choice of the market portfolio has a large effect on the estimate of the risk premia across countries, and do not provide a conclusive answer to the policy question. The authors point out that “the cost of borrowing using GDP-linked bonds is highly uncertain, largely due to the wide range of estimates for the growth risk premium” and they conclude that “[f]or almost half of the countries examined, the highest estimated cost would be large enough to make the issuance of GDP-linked bonds undesirable”. Kamstra and Shiller (2009) calibrate a CAPM model of GDP growth on the S&P 500 and estimate a premium of 150bp. Borensztein and Mauro (2004) estimate risk premia using an equilibrium model and find premia in the range 35bp–370bp for different levels of investor risk aversion. Barr et al. (2014) use CAPM to estimate risk premium for Argentina close to 100bp. Therefore current literature is ambiguous on whether the anticipated risk premia will render GDP-linked bonds beneficial for issuing sovereigns. Table 3 summarizes the premia obtained by different authors and their underlying assumptions. Our model sheds light on this ambiguity.

3.2.2 Premium estimates from the model

We now estimate risk premia for coupon-indexed bonds. Results are summarized in Figure 8 for a broad range of base coupon or target growth, so these figures also provide sensitivity analysis on the design parameters. As expected, premia decrease with higher target growth rate, and increase with base coupon. If the target growth is very high the option will never be exercised and the instrument has, essentially, no risk premium. On the other hand, if the base coupon rate is high there is more negative impact from low growth rates until the floor of zero is hit, and hence the risk premium is higher. Our results are within the ranges estimated by Barr et al. (2014); Bowman and Naylor (2016) for US, or UK, or advanced economies, but our model provides tighter estimates. Whereas their highest premia exceed both the 250bp tight threshold and, in some instances, the 350bp relaxed threshold, our estimates are below the threshold. Barr et al. (2014) argue that for reasonable risk aversion parameters the premium is less than 350bp, and our work shows that it is below 250bp. Bowman and Naylor (2016) conclude that, due to the wide range of premia, the cost would be large enough to make the issuance of GDP-linked bonds undesirable, and call for “further investigation”. Our work reduces the range and

Reference and method	Bond design and country	Market assumptions	Premium estimates (bp)
Borensztein-Mauro CAPM	Coupon-indexed Unspecified params. Argentina	$r_f = 3\%$ Exp. return 8%	100
Kruse et al. Options pricing on underlying GDP	Coupon-indexed and Principal-indexed Indonesia, Venezuela	Historical from Frankfurt Exchange	76–200* (Indonesia) 24–264 (Venezuela)
Barr et al. Equilibrium model with risk averse investors	Principal-indexed $c_0 = 0$ Advanced economies	$r_f = 3.4\%$ Exp. return 2.1%	35–370 depending on risk aversion
Bowman-Naylor CAPM Downside-CAPM	Unspecified params. G20 countries	$r_f = 0\%$ Exp. return 6.5%	> 350 for half of the G20 0–400 for UK and US
Kamstra-Shiller CAPM	Principal-indexed $c_0 = 0$ US	Not provided	150

* Risk premia for Indonesia are positive. All other premia are negative, and following our convention are given in absolute value.

Table 3: Risk premium estimates by different authors.

reaches a different conclusion.

Looking at the slope of the curves in each panel we observe that there is a change of roughly 25bp per 1% change in target growth. Comparing the curves for the same time-window across panels, we observe similar change per 1% change of base coupon. Just like prices, the premia appear equally sensitive to changes of base coupon or target growth.

From the results of the figure we make some observations pertaining to the policy question:

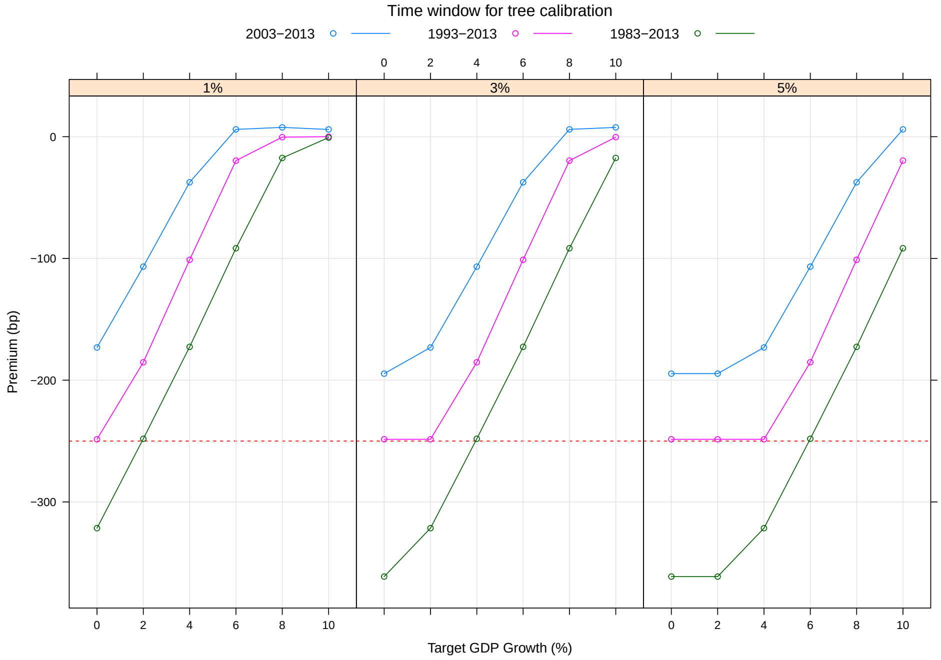
1. The risk premium is sensitive to the parameters of the bond design. Hence, it is hard to interpret current literature that estimates risk premia of GDP-linked bonds without a consensus on the bond design. There is no unique premium, and this may explain the inconclusive results from the literature cited above.

2. There is a broad range of bond design parameters with risk premia attractive for sovereigns. Designs with premia above the 250bp horizontal line are acceptable. For instance, for UK a coupon-indexed bond with base coupon 1% and target growth 4% has risk premium in the range 50bp–175bp, depending on the calibration data. Increasing the base coupon to 3% increases risk premia to 105bp–250bp. For US, a coupon-indexed bond with base coupon 1% and target growth 4% has risk premium in the range 90bp–200bp. These ranges of premia values make the bonds beneficial for sovereigns.

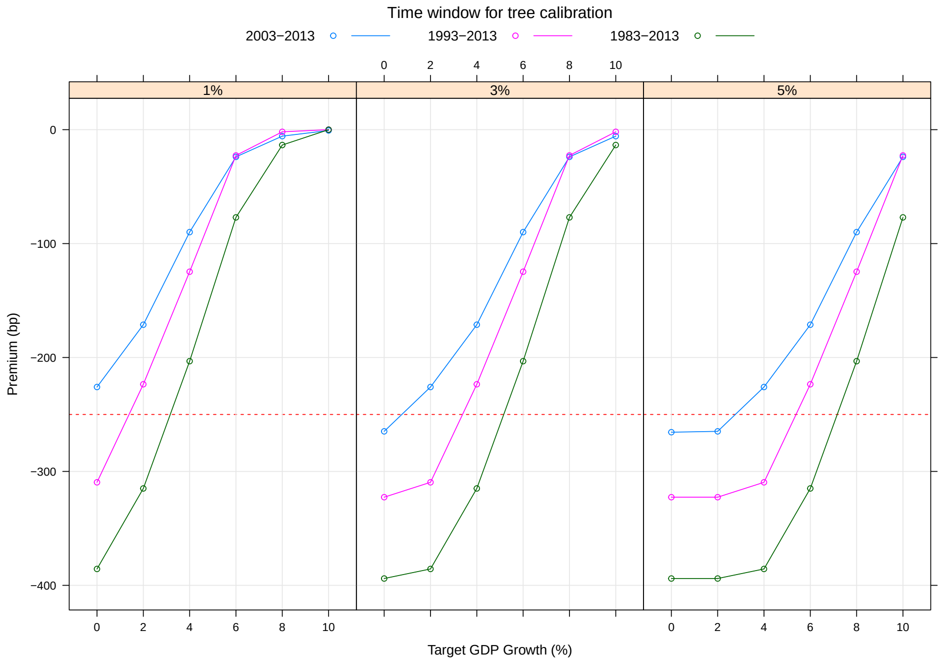
3. There are very few observations with premia exceeding 350bp and there are several with premia less than 250bp. Based on these thresholds our results show that GDP-linked bonds can be beneficial for sovereigns. Although the risk premia, like prices, are sensitive to input data, the conclusion is robust when tested using different calibration windows.

3.3 Comparing coupon-indexed and principal-indexed bonds

We use the model to compare the risk premia commanded by coupon-indexed and principal-indexed bonds. We compute premia for the reference coupon-indexed bonds and principal-

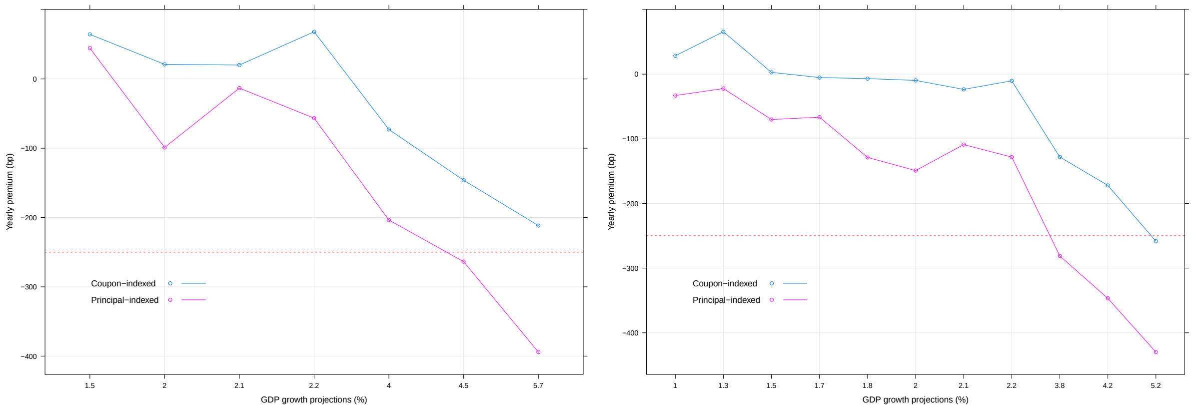


(a) UK



(b) US

Figure 8: Risk premia for various combinations of base coupon and target growth rates for coupon-indexed bonds. Trees are calibrated for different time windows and the three panels in each figure correspond to base coupons 1%, 3%, and 5%. The horizontal dashed line corresponds to the tighter threshold that renders GDP-linked bonds beneficial for sovereigns.



(a) UK (b) US

Figure 9: Risk premia for coupon-indexed and principal-indexed bonds for different expected growth rates.

indexed bonds with zero base coupon. The risk premium depends on data calibration, and the critical parameter for principal-indexed bonds is the expected GDP growth. Hence, we use the 2003-2013 window for volatility and correlations, and compute premia for different expected GDP growth rates, see Figure 9.

We observe that principal-indexed bonds carry a higher premium than coupon-indexed bonds. The difference can be as high as 200bp for high expected economic growth, but is reduced to 100bp for expected growth about 2.5%. This observation comes with the caveat that results can change by varying the base coupon, and this brings us back to the point that design parameters can have a significant impact. A contribution of the model is to allow the analysis of alternative designs.

For a wide range of expected growth rates the principal-indexed bonds have premia smaller than the threshold and, hence, can be beneficial for sovereigns. For both UK and US the premia are below the 350bp threshold for expected growth below 5.5% (UK) and 4.5% (US). For expected growth rates below 4.5% (UK) and 3% (US) the premium is less than the 250bp tight threshold. From these results and the results in section 3.2.2, we conclude that GDP-linked bonds can be beneficial for sovereigns, in both coupon-indexed and principal-indexed versions. This conclusion holds for different calibration periods and for a broad range of design parameters. Coupon-indexed bonds appear to benefit from lower premia. However, principal-indexed bonds directly stabilize the issuer's debt to GDP ratio but coupon indexed bonds do so only indirectly through lower coupon payments, so the issuer may be willing to pay a higher risk premium on principal-indexed bonds in return for more effective debt stabilization.

4 Conclusions

We have developed a model for pricing and hedging GDP-linked bonds in incomplete markets. We use stochastic programming to sure-replicate the cashflows of the new bond with market traded assets on a discrete scenario tree. As a byproduct of the model we obtain a hedging portfolio for investors in these novel instruments. The dual program provides a risk neutral measure in an incomplete market, which is used to estimate risk premia. The model is applicable for different GDP-linked bond designs, and we used it to price coupon-indexed and principal-indexed bonds. The model is calibrated for UK and US using different calibration windows. The bid-ask spreads, prevalent in incomplete market prices, are found to be small, thus establishing the credibility of the model. Numerical results shed light on the effect of bond design choices

—base coupon, target GDP growth rate, and maturity— on the price.

The model was used to estimate risk premia for coupon-indexed and principal-indexed bonds. Principal-indexed bonds command a higher premium than coupon-indexed bonds. The difference can be as high as 200bp, but is close to 100bp for expected economic growth 2.5%.

Current literature tells us that risk premia above 350bp render GDP-linked bonds too expensive for sovereigns, whereas premia below a 250bp threshold benefit the sovereigns. We have shown that for a wide range of bond design parameters and for different calibration windows, the coupon-indexed bonds will demand premia 50bp–250bp for UK, and 90bp–200bp for the US. Hence, coupon-indexed GDP-linked bonds can benefit sovereigns. For principal-indexed bonds the premia are below the 350bp threshold for expected growth rates below 5.5% (UK) and 4.5% (US), and below the 250bp threshold for expected returns below 4.5% (UK) and 3% (US). Hence, principal-indexed bonds command a higher premium and must be carefully calibrated. Sensitivity analysis using different calibration windows gives us confidence that these observations are robust. Nevertheless, our findings deserve a nuanced reading, since there are many inter-related risk factors and design parameters that affect prices and premia. Furthermore, some investors, especially buy-and-hold long term entities like pension funds, will price these assets differently in the context of their liability portfolio. Our models provide market-based benchmarks.

In conclusion, we have provided evidence in favor of GDP-linked bonds, using results for two advanced economies. An interesting follow-up study would be to calibrate the model for G20 countries or emerging economies of interest to current policy debates. Another promising avenue for further research is to apply the models to price other derivatives linked to economic activity.

A Appendix: Data for tree calibration and yield curves

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Table 4: Mean values, standard deviations, and correlations: UK data for 2003–2013.

	GBGDPN	GBBILLN	GBBONDN	GBEQTYN	WDBILLN	WDBONDN	WDEQTYN
GBGDPN	1.000						
GBBILLN	0.467	1.000					
GBBONDN	0.165	0.071	1.000				
GBEQTYN	0.020	-0.307	-0.558	1.000			
WDBILLN	0.042	0.369	0.460	-0.807	1.000		
WDBONDN	0.113	0.342	0.709	-0.897	0.906	1.000	
WDEQTYN	0.122	-0.222	-0.592	0.926	-0.590	-0.743	1.000
Mean	0.040	0.027	0.052	0.093	0.012	0.064	0.086
Std. dev.	0.023	0.021	0.075	0.167	0.125	0.154	0.124

Table 5: Mean values, standard deviations, and correlations: UK data for 1993–2013.

	GBGDPN	GBBILLN	GBBONDN	GBEQTYN	WDBILLN	WDBONDN	WDEQTYN
GBGDPN	1.000						
GBBILLN	0.522	1.000					
GBBONDN	0.200	0.314	1.000				
GBEQTYN	0.025	-0.073	0.052	1.000			
WDBILLN	0.050	0.367	0.316	-0.408	1.000		
WDBONDN	0.112	0.302	0.672	-0.457	0.780	1.000	
WDEQTYN	0.025	-0.022	0.018	0.925	-0.162	-0.311	1.000
Mean	0.045	0.041	0.077	0.082	0.024	0.074	0.071
Std. dev.	0.018	0.022	0.101	0.169	0.101	0.133	0.150

Table 6: Mean values, standard deviations, and correlations: UK data for 1983–2013.

	GBGDPN	GBBILLN	GBBONDN	GBEQTYN	WDBILLN	WDBONDN	WDEQTYN
GBGDPN	1.000						
GBBILLN	0.702	1.000					
GBBONDN	0.176	0.261	1.000				
GBEQTYN	0.161	0.140	0.126	1.000			
WDBILLN	-0.008	0.202	0.229	-0.008	1.000		
WDBONDN	0.036	0.240	0.590	-0.121	0.772	1.000	
WDEQTYN	0.085	0.014	0.103	0.875	0.278	0.121	1.000
Mean	0.057	0.063	0.089	0.111	0.040	0.093	0.090
Std. dev.	0.027	0.038	0.088	0.158	0.127	0.132	0.173

Table 7: Mean values, standard deviations, and correlations: US data for 2003–2013.

	USGDPN	USBILLN	USBONDN	USEQTYN	WXBONDN	WXEQTYN
USGDPN	1.000					
USBILLN	0.448	1.000				
USBONDN	0.335	0.132	1.000			
USEQTYN	0.080	-0.186	-0.745	1.000		
WXBONDN	0.159	-0.043	0.345	-0.037	1.000	
WXEQTYN	0.179	0.043	-0.743	0.949	0.048	1.000
Mean	0.038	0.015	0.055	0.095	0.073	0.101
Std. dev.	0.023	0.017	0.118	0.197	0.060	0.248

Table 8: Mean values, standard deviations, and correlations: US data for 1993–2013.

	USGDPN	USBILLN	USBONDN	USEQTYN	WXBONDN	WXEQTYN
USGDPN	1.000					
USBILLN	0.574	1.000				
USBONDN	0.199	0.183	1.000			
USEQTYN	0.166	0.052	-0.399	1.000		
WXBONDN	-0.029	-0.093	0.520	-0.003	1.000	
WXEQTYN	0.183	-0.052	-0.555	0.847	0.074	1.000
Mean	0.045	0.028	0.073	0.090	0.081	0.073
Std. dev.	0.019	0.020	0.119	0.189	0.082	0.216

Table 9: Mean values, standard deviations, and correlations: US data for 1983–2013.

	USGDPN	USBILLN	USBONDN	USEQTYN	WXBONDN	WXEQTYN
USGDPN	1.000					
USBILLN	0.719	1.000				
USBONDN	0.205	0.254	1.000			
USEQTYN	0.157	0.134	-0.193	1.000		
WXBONDN	-0.065	0.053	0.444	0.047	1.000	
WXEQTYN	0.209	0.100	-0.253	0.767	0.290	1.000
Mean	0.052	0.041	0.087	0.107	0.097	0.096
Std. dev.	0.022	0.027	0.112	0.170	0.100	0.220

Table 10: Yield curve of spot rates on 31 December 2013.

Term	1yr	2yr	3yr	4yr	5yr	6yr	7yr	8yr	9yr	10yr	
UK	0.37	0.72	1.18	1.62	2.00	2.32	2.59	2.81	3.00	3.16	
Term	1mo	3mo	6mo	1yr	2yr	3yr	5yr	7yr	10yr	20yr	30yr
US	0.01	0.07	0.1	0.13	0.38	0.78	1.75	2.45	3.04	3.72	3.96

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