MECHANICAL OSCILATIONS AND VISUALISATION WITH MATHEMATICAL PROGRAM MAPLE V

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ABSTRACT
This paper engages in using mathematical computer programs in teaching physics at technical universities. As an example there is mentioned making the best of software Maple V for demonstrating forced oscillations and resonance in lectures or in the theoretical lessons.

KEYWORDS
Mathematical program, Maple V, mechanical oscillations, forced oscillations, resonance curves

INTRODUCTION

Maple V is a computer environment developed at the University of Waterloo in Canada in order to use mathematics easier. The title Maple means the authors` wish to maths become a pleasure (Maple – acronym of words MAthematics PLEasure).

Maple includes systems of computer algebra, marked by abbreviation CAS (Computer Algebra Systems). This is an interactive program that makes it possible to perform not only numeric calculation (in contrast to other programs), but also calculation with symbolic expression. It is possible with Maple V to make, by more than 20 different ways, graphs of functions (also 3D) and to model mathematical operations with symbolic expressions.

Maple consists of three parts: point, library and user's interface. The point is written in the programming language C and it incorporates all of the basic calculations of the system. Most of the inner Maple orders are in the Maple library written by Maple's programming language (MPJ). The user's interface makes possible both depicting Maple's functions and the interactive work with orders and procedures in Maple. It is also possible to insert the explanation text between the Maple's orders.

It is possible to use Maple for both easy and difficult calculations (for example derivations, integrals, solution of differential equations etc.), but also for modelling some phenomenon and process and for easy animation. It can also be used in teaching physics (similar tools such as Matlab, Mathematics and Derive are also in use in the Czech Republic).

TEACHING PHYSICS WITH MAPLE V

At our faculty we begin teaching physics in the first study semester (Physics 1), at a time, when students do not have an adequate knowledge of maths. So it is suitable to use some of the available mathematical computer programmes for calculation, for example graphic representation, eventually for easy animation. In this academic year students of Faculty of Electrical Engineering & Computer Science (University of technology in Brno) - FEI VUT - can get acquainted with programming basics by using this mathematical program. It is also appropriate for students to use programs such as MAPLE in their physics classes.

Subject Physics 1 includes these themes: Mechanics, Introduction into the study of special theory of relativity and Oscillations. We find that MAPLE V is particularly useful for teaching forced oscillations. This topic contains complex mathematics and yet it is encountered in the first semester at which point, students` mathematical skills are not matched to the task. The use of MAPLE V facilitates the representation of forced oscillations in such a circumstance.

Lecture supplements are available to students. These supplements show how to use MAPLE V for the representation and analysis of forced oscillations. (It is possible similarly use this product also in theoretical lessons for solution and demonstration of concrete examples from teaching texts that students use in theoretical lessons.)

Students at FEI VUT can now work with the differential equations that are encountered in the physics of oscillations without having first studied them (differential equations) in mathematics.

Aim of the article is not correct introduction of the problem. The aim is to show possibilities of representation some (not traditionally shown) dependencies got by Maple.

FORCED OSCILLATIONS

Forced oscillations are caused for example by performance of the harmonic external force (the driving force) $F_m \sin(\Omega t + \alpha)$ in direction of oscillation. When at the mass point affects in addition the elastic force $F_e = -k\,u$ and the force of resistance $F_o = -R\,v$ (k and R are positive constants and v velocity), then the differential equation is:

```
> f:={diff(u(t),t$2)+2*b*diff(u(t),t)+omega^2*u(t)=(F[m]/m)*sin(Omega*t+alpha),D(u)(0)=v,u(0)=0};
```

$$f := \left\{ \left(\frac{\partial^2}{\partial t^2} u(t)\right) + 2\,b\left(\frac{\partial}{\partial t} u(t)\right) + \omega^2\, u(t) = \frac{F_m \sin(\Omega\, t + \alpha)}{m},\ D(u)(0) = v,\ u(0) = 0 \right\}$$

(where b is the damping coefficient, ω the angular frequency of the own oscillations, Ω the angular frequency of the driving force).

In the first part of this example physics teachers can show students how to write down the differential equation of forced oscillation and find, the common solution of this equation. After assigning the initial conditions it is possible to get solution of the given differential equation where the first member's form that does not express stabilised state is known from textbooks:

```
> rf:=rhs(f);
```

$$
\begin{aligned}
f := \mathrm{u}(t) = &-\frac{F_m\,(2\,\Omega\cos(\Omega\,t+\alpha)\,b-\sin(\Omega\,t+\alpha)\,\omega^2+\sin(\Omega\,t+\alpha)\,\Omega^2)}{m\,(4\,b^2\,\Omega^2+\omega^4-2\,\omega^2\,\Omega^2+\Omega^4)}+\frac{1}{2}\Big((4\,v\,m\,b^2\,\Omega^2 \\
&+2\,F_m\,\Omega\cos(\alpha)\,b^2-F_m\sin(\alpha)\,b\,\Omega^2+2\sqrt{b^2-\omega^2}\,F_m\,\Omega\cos(\alpha)\,b-b\,F_m\sin(\alpha)\,\omega^2+v\,m\,\Omega^4 \\
&+F_m\,\Omega^3\cos(\alpha)+\sqrt{b^2-\omega^2}\,F_m\sin(\alpha)\,\Omega^2-2\,v\,m\,\omega^2\,\Omega^2-F_m\,\Omega\cos(\alpha)\,\omega^2-\sqrt{b^2-\omega^2}\,F_m\sin(\alpha)\,\omega^2 \\
&+v\,m\,\omega^4)\,\mathbf{e}^{(-(b-\sqrt{(b-\omega)\,(b+\omega)})\,t)}\Big)\Big/\Big(\sqrt{b^2-\omega^2}\,m\,(4\,b^2\,\Omega^2+\omega^4-2\,\omega^2\,\Omega^2+\Omega^4)\Big)-\frac{1}{2}\Big((v\,m\,\Omega^4 \\
&+v\,m\,\omega^4-F_m\sin(\alpha)\,b\,\Omega^2-2\,v\,m\,\omega^2\,\Omega^2+4\,v\,m\,b^2\,\Omega^2-F_m\,\Omega\cos(\alpha)\,\omega^2+F_m\,\Omega^3\cos(\alpha) \\
&+2\,F_m\,\Omega\cos(\alpha)\,b^2-b\,F_m\sin(\alpha)\,\omega^2+\sqrt{b^2-\omega^2}\,F_m\sin(\alpha)\,\omega^2-\sqrt{b^2-\omega^2}\,F_m\sin(\alpha)\,\Omega^2 \\
&-2\sqrt{b^2-\omega^2}\,F_m\,\Omega\cos(\alpha)\,b)\,\mathbf{e}^{(-(b+\sqrt{(b-\omega)\,(b+\omega)})\,t)}\Big)\Big/\Big(\\
&\sqrt{b^2-\omega^2}\,m\,(4\,b^2\,\Omega^2+\omega^4-2\,\omega^2\,\Omega^2+\Omega^4)\Big)
\end{aligned}
$$

In second part we can observe the dependence of the displacement u(t) for different values of Ω. Just these dependencies are not usually mentioned in textbooks.

a) $\Omega << \omega$. (Then the forced oscillations are in phase with the exciting power.)

```
> Omega:=0.4*Pi;omega:=3*Pi; v:=2;u(0):=0;F[m]:=5;m:=2;alpha:=Pi/4;b:=0.2;
```

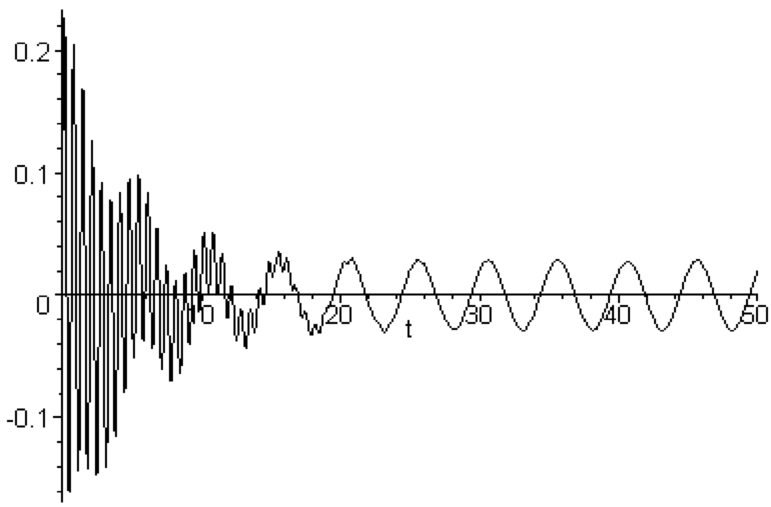

Figure 1. Dependence of the displacement u on the time t for $\Omega << \omega$.

We can see in Fig. 1 how the amplitude is decreased at first (it is called transient phenomenon) in comparison to the stabilized oscillations (in this case damping oscillations are not practically of use anymore).

b) $\Omega >> \omega$. (Forced oscillations and disturbing power are almost in contrary phases (see Figure 2).)

```
> omega:=Pi;Omega:=9*Pi;v:=2;u(0):=0;F[m]:=5;m:=2;alpha:=Pi/4;b:=0.2
```

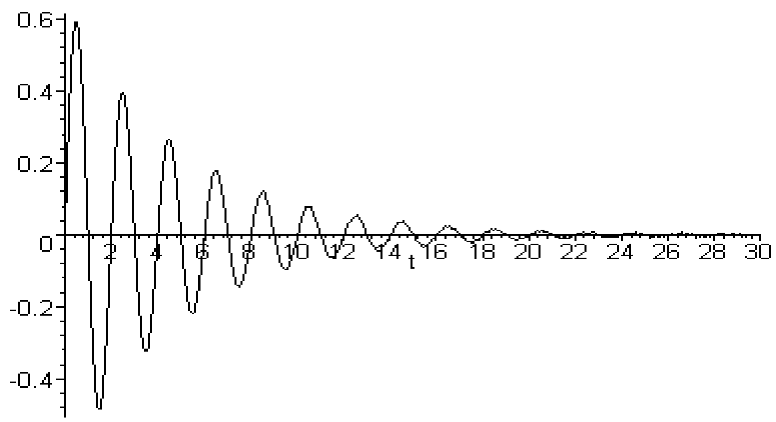

Figure 2. Dependence of the displacement u on the time t for $\Omega >> \omega$

c) $\Omega \cong \omega$. (It comes to the resonance (Fig.3) of the angular frequency.)

```
> omega:=Pi;Omega:=0.999*Pi;v:=2;u(0):=0;F[m]:=5;m:=2;alpha:=Pi/4;b:=0.099;
```

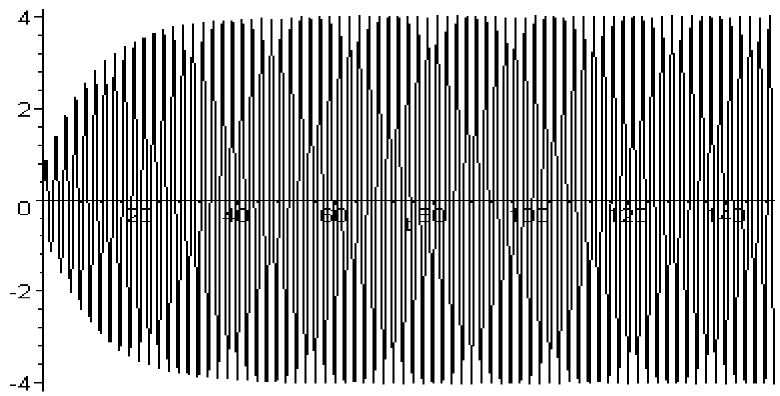

Figure 3. Dependence of the displacement u on the time t for $\Omega \cong \omega$

The third part of the show is given to demonstrate resonance curves (the dependence of the forced oscillation amplitude u_m on angular frequency of the driving force Ω) for different values of the damping coefficient b.

The amplitude of forced oscillations is

$$u_m = \frac{F_m}{m\sqrt{(\omega^2 - \Omega^2)^2 + 4b^2\,\Omega^2}}$$

and for the resonance angular frequency is valid

$$\Omega_{res} = \sqrt{\omega^2 - 2b^2}\ .$$

The graphs show the dependence u_m (Ω) on different values of b. The Figure 4 shows that for very low damping the resonance is pronounced. In contrast, when the damping is large, the resonance is not pronounced.

Teachers (eventually students) can change the constants and see how the situation changes (each graph is displayed in a different color).

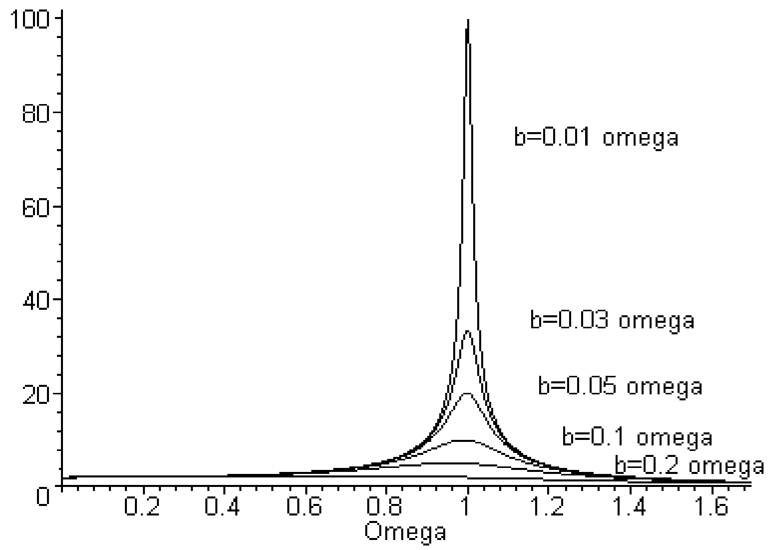

Figure 4. Dependence of the amplitude of the forced oscillations u_m on the frequency of the driving force Ω at different values of dumping coefficient b.

CONCLUSION

Maple V can really help students to understand oscillations because it can used to demonstrate each step in the solution of the related differential equation.

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