SOLVING NONLINEAR DYNAMICAL EQUATIONS OF FUSION SYSTEMS USING RUNG - KUTTA 45 AND LSODE COMPUTER CODES

M. R. Eskandari

ABSTRACT
In fusion system dynamics, there are most complicated coupled non-linear equations, which to be able to consider the physical dynamics of these systems, they have to be solved . In this paper the applications of computer codes Rung-Kutta 45 and Lsode in solving these dynamical equations are shown. The dynamical equations at the time range of microseconds are analyzed and the results are compared with available experimental values. It is shown that they are in a very good agreement, showing the power of used computational method.

KEYWORDS
Non linear dynamical equations, RUNG - KUTTA, LSODE, computational programs

INTRODUCTION

Muon catalyzed fusion (μCF) systems have attracted the greatest interest because of their most favorable physics. A homogenous mixture of mostly gas or liquid, hydrogen isotopes has been used traditionally, as inhomogenous targets also are used in recent researches.

A muon injected into the hydrogen isotopes will slow down and form a small muonic atom, μx by replacing the electron in atom where x is hydrogen isotopes of proton, p, deuterium, d or tritium, t. The μx will then collide with molecules like D_2, T_2 or DT and form complex muonic molecule in non-resonance [1] and resonance [9] as:

$$\text{(non-resonance)} \quad \mu x + D_2 \xrightarrow{\lambda_{dx\mu}^{nr,F}} [(dx\mu)de]^+ + \bar{e} \tag{1}$$

$$\text{(resonance)} \quad \mu x + (D_2)_{v_i,k_i} \xrightarrow{\lambda_{dx\mu}^{r,F}} [(d\,x\mu)dee]^* \tag{2}$$

Molecular formation occurs predominantly via resonant mechanism, in which the energy released from the formation of the $dx\mu$ molecule is absorbed by the rotational and vibrational excitation of the molecular complex $[(dx\mu)dee]$ where the compact object $dx\mu$ acts as a pseudonucleus. Muonic molecule formation can also occur by releasing the energy via the Auger process (non-resonant formation), but this rate is much smaller than resonance formation $(\lambda_{dx\mu}^{nr,F} << \lambda_{dx\mu}^{r,F})$. If a μx is formed, the muon will be transferred to a y isotope (y isotope being heavier than x), forming a more tightly bound μy with transfer rate of λ_{xy} as:

$$\mu x + y \xrightarrow{\lambda_{xy}} \mu y + Q_{xy} \quad (3)$$

where Q_{xy} is the energy released from reaction. An example of reaction type (3) is :

$$\mu d + t \xrightarrow{\lambda_{dt}} \mu t + d + 48.1\ eV \quad (4)$$

$\lambda_{dt} = 3 \times 10^8\ s^{-1}$ [8] and the relative isotope concentration of p, d and tritium, t and other existing isotopes in plasma are given by $C_p, C_d, C_t \ldots\ldots$ respectively, where:

$$C_p + C_d + C_t + \cdots = 1$$

For the muonic molecules,atoms or any particle density we will use $N_i(t)$, which is number of particles per unit volume (cm^3) at time t and i can be pdμ, ptμ, dtμ, ddμ, ttμ,..... or pμ, dμ, tμ,.... or particles of p,d,t,...., μ,(muon). The production rate for any particles i, is $+\lambda_i$ and its decay is $-\lambda_i$. Muon itself has decay constant of λ_μ=0.455 $\times 10^6\ s^{-1}$, therefore all muonic atoms or molecules will convert to electronic atoms or molecules at this time. Because of the size of the muonic molecules is smaller than ordinary molecules by its mass ratio (m_μ / m_e) in zeroth order, the internuclear distance in $dx\mu$ is small enough that fusion takes place with λ_i^F (fusion rate). After fusion, the muon is released more than 99% of the time, but a small probability exists for a process known as sticking in which the muon becomes attached to the charged fusion product. In the following sections, after brief introduction of fusion media, the muon cycling and sticking is discussed followed by the brief introduction on the most recent research on this matter. Then the general form of point dynamical equations for these systems are written. Finally the used two important computational methods of Rung Kutta 45 and Lsode are introduced, showing the importance of used codes in solving such difficult non-linear equations.

FUSION MEDIA

Fusion media is the media consisting hydrogen isotopes at temperature T (Kelvin), and providing fusion conditions, with ion density of $N_o\phi$, where $N_o = 4.25 \times 10^{25}\ cm^{-3}$ and is liquid hydrogen particle density and ϕ is called relative ion density. All formation and decay constants are dependent on T and ϕ.

MUON CYCLING AND STICKING COEFFICIENT

Fig.1, illustrates a greatly simplified scheme of muon catalyze fusion (μCF) cycle in D/T mixture.

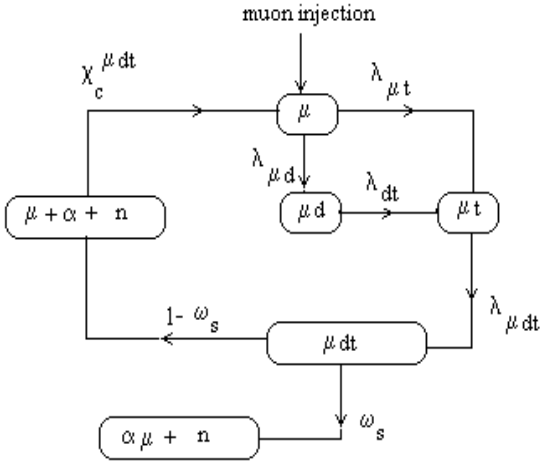

Figure 1. Greatly simplified cycle of muon catalyzed fusion in D-T mixture

A muon injected into the hydrogen target will slow down and form muonic deuterium (μd) or muonic tritium (μt) by replacing the electron in atom. The formed (μd) will transfer to μt according to Fig.1 and Eq.4. The μt will then form $dt\mu$ molecule on collision with deuterium molecule, which itself makes fission by two ways of producing free muon to recycle again or bounded muon to produced α particel with probability ω_s (sticking coefficient), and almost ending muon cycling. $X_c^{dt\mu}$ is muon cycling and in general is defined as the number of catalysis for an injected muon in its lifetime τ_μ= (2.2 μsec.). Therefore we have:

$$X_c^{dt\mu} = \int_0^{\tau_\mu} \lambda_{dt\mu}^F N_{dt\mu}(t)\, dt \qquad (5)$$

Where $N_{dt\mu}(t)$ is the $dt\mu$ molecules density at time t. Smallest ω_s results larger $X_c^{dt\mu}$, which is one of greatly important research still left to be worked .

PROGRESSED RESEARCH

The cycling rate $X_c^{()}$ in various fusion media depends on particle relative densities, $c_p, c_d, c_t, \ldots$ and the relative density of media ϕ, polarization of nucleus, sticking coefficient and media temperature. Most of these parameters dependencies are considered in past decades. In mixture of three hydrogen isotropes of H/D/T various papers are published in controdiction of each other [2]. In D/T and D/T/H homogeneous and inhomogenouus systems, muon cycling rete are calculated and compared to each other [3]. It is shown that muon cycling rate in D/T increases in condition of resonance molecule formation and also in polarization of fusion making nuclide [4-6]. As it mentioned, sticking probability is an important parameter in muon cycling and is dependent on ϕ and T[7]. In most cases, the obtained theoretical results are compared with available experimental results and they are in good agreement, showing the accuracy of used computational methods, and will be discussed in detail after introducing the form of written equations for μCF systems.

μCF SYSTEM EQUATIONS

The mathematical dynamical equations for μCF systems are written based on formation and decay of particles, muonic molecule and atom densities. These equations are non-linear coupled time dependent equations called point dynamical equations. As a general form, let us write time rate of change of these particle densities as:

$$\frac{dN_{\mu x}(t)}{dt} = \lambda_{\mu x}\phi N_\mu(t) C_x - \lambda_\mu N_{\mu x}(t) - \Sigma_y \lambda_{xy} N_{\mu x}(t)\phi C_y - \Sigma_y \lambda_{\mu xy}\phi C_y N_{\mu x}(t)$$

$$\frac{dN_{\mu xy}(t)}{dt} = -\lambda_\mu N_{\mu xy}(t) + \lambda_{\mu xy} N_{\mu x}(t)\phi C_y - \lambda_{\mu xy}^F N_{\mu xy}(t)$$

$$\frac{dN_y(t)}{dt} = \lambda_\mu N_{\mu x}(t) - \sum_x \lambda_{\mu xy}\phi C_y N_{\mu x}(t) - \Sigma_x \lambda_{xy}\phi N_{\mu x}(t) C_y$$

$$\frac{dN_\mu(t)}{dt} = S_\mu - \lambda_\mu N_\mu(t) - \Sigma_x \lambda_{\mu x}\phi N_\mu(t) C_x + \sum_{x,y} \lambda_{\mu xy}^F N_{\mu xy}(t) \sum_i (1-\omega_{si}) p_i$$

All other equations for the particle densities are written similar to these equations where x and y can be p, d or t and S_μ is muon injection rate. $\lambda_{(\ )}$ are the formation and decay rates and are dependent on media conditions. $\sum_i (1-\omega_i)p_i$ is the muon release probability from μxy molecule. $p_i = 1$ for $\mu dt, \mu pt, and\ \mu tt$ molecules. ω_{si} is muon sticking coefficient to fusion product of i'th molecule $(i = \mu dt, \mu pt, \mu tt)$. For example for μdd molecule we have:

$$\sum_i (1-\omega_{si})p_i = (1-\omega_{s1})p_1 + (1-\omega_{s2})p_2$$

and ω_{s1}, ω_{s2} are sticking coefficient to tritium (t) and Helium (^3He) respectively. p_1 and p_2 are probability of tritium and ^3He production in μdd fusion.

RUNG KUTTA AND LSODE COMPUTER CODES

To be able to solve these kind of non-linear coupled equations and determine the time dependent particle densities and eventually muon cycling rate, two computational methods exist as Rung Kutta 45 and Lsode, which they are numerical methods.
a) Rung-Kutta, this code operates as:

restart:

$\lambda_\mu := 0.455\ e6: \lambda_{dt\mu} := 3e8: \lambda_{\mu dd} : 3e6: \cdots \lambda_{\mu xy} := \cdots$
$eq1 := diff(N_{\mu x}(t), t) = \lambda_{\mu x} := \cdots : \cdots : \phi := \cdots :$
$eq2 := diff(N_{\mu xy}(t), t) = \cdots$
$eq3 := \cdots$
.....
$eqn := \cdots$
$init := N_\mu(0) = 1, N_{\mu x}(0) = 0, \cdots, \cdots;$
$So := dsolve(\{eq1, eq2, \ldots, eqn, init\}, \{N_\mu(t), N_{\mu x}(t), \cdots\}, numeric, method =$
$rkF45, \max fun = 40{,}000{,}000);$
for i from 0 to 0.0000022 by 0.00000001 so (i): od; with (plots): odeplot (so, [t, $(\lambda^F_{\mu xy} * N_{\mu xy}(t)], 0, \cdots 2.2e-6, color = (black)$;

It starts with "restart" command and then all formation and decay constants, $\cdots, \phi \cdots$ have to be given. All n equations then are written and initial conditions are given by "init" command. So is an arbitrary name in receiver of solution of equations. At the end by running the program, the parameters can be determined at various time $(t = 0\ to\ 2.2\,\mu\sec)$. This is done with respect to three last lines of program. The time dependent $\lambda^F_{\mu xy}\ N^F_{\mu xy}(t)$ are determined and area under that would be muon cycling for μxy molecule;

b) Lsode code: this code operates as:
restart; digits:=15; with (plots);
e1:=diff $(N_{\mu x}(t), t) = \cdots\cdots;$
e2:=diff $\cdots\cdots;$
.....
en:$\cdots\cdots$

sys: = {e.(1⋯⋯n)};
init: = { $N_{\mu x}(0) = 0,\cdots\cdots\}$;
func: = $\{N_{\mu x}(t),\cdots\cdots\}$;
y: = dsolve (sys union init, func, type= numeric,
method= Lsode [adamsfull], output= procedurelist);
odeplot (9,[t,( ) * $N_{\mu xy}(t)$],0⋯⋯$2.2e-6$);

In this code, the constants are introduced in equations e1, e2 , e3, ... , en and sys acts as so in rkf 45. The last line in program plots ( ) $N_{\mu xy}(t)$ versus time (in our case for t=0 to t=2.2 μsec). ( ) is value of $\lambda^{F}_{\mu xy}$. Again, area under this variation is muon cycling for μxy molecule.

CONCLUSIONS

We have seen that in solving a fusion system dynamics, there are most complicated coupled non- linear equations. It is shown here, that how the solution of these equations becomes simple when computer codes like Rung-Kutta 45 or Lsode codes are used. The obtained results in various conditions with this numerical method are compared with avaliable experimental results.As an example the muon cycling rate, λ_c, versus tritium concentration ratio at 3K and ρ = 1.46 liquid hydrogen density (LHD) for solid homogenous D/T system are calculated by the given methods (solid line) and are compared with experimental results (squares) and results are given in Fig .2. The various calculations for most cases are in a very good agreement showing the power of used computational method.

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M.R. Eskandari
Physics Department
Shiraz University
Shiraz 71454, Iran
Email: eskandari@physics.susc.ac.ir