Risk management optimization for sovereign debt restructuring

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Abstract

Debt restructuring is one of the policy tools available for resolving sovereign debt crises and, while unorthodox, it is not uncommon. We propose a scenario analysis for debt sustainability and integrate it with scenario optimization for risk management in restructuring sovereign debt. The scenario dynamics of debt-to-GDP ratio are used to define a tail risk measure, termed *conditional Debt-at-Risk*. A multi-period stochastic programming model minimizes the expected cost of debt financing subject to risk limits. It provides an operational model to handle significant aspects of debt restructuring: it collects all debt issues in a common framework, and can include contingent claims, multiple currencies and step-up or linked contractual features. Alternative debt profiles —obtained by maturity rescheduling, interest payment concessions or nominal value haircuts— are analyzed for their expected cost–risk tradeoffs. With a suitable re-calculation of the efficient frontier, the risk of debt un-sustainability of alternative risk profiles can be ascertained with a given confidence level. The model is applied to Greece sovereign debt crisis analyzing the suitability of various proposals to restore debt sustainability.

Keywords: sovereign debt; debt restructuring; scenario analysis; portfolio optimization; stochastic programming; Value-at-Risk; conditional Value-at-Risk; Greek crisis.

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1 The sovereign debt restructuring challenge

On May 20, 2013, the Executive Board of the International Monetary Fund discussed developments in sovereign debt restructuring and the implications for the IMF legal and policy framework (IMF, 2013). There had not been any discussions at the IMF about sovereign debt restructuring since 2005, and twelve years had passed since Anne Krueger's signature proposal on a sovereign debt restructuring mechanism (SDRM) was rejected under strong opposition from creditor countries (Krueger, 2002). The new IMF policy considers the possibility of maturity extensions for countries whose debt is considered non-sustainable. Debt rescheduling and, perhaps, restructuring will become an official policy tool.

The renewed interest was prompted by a realization, in the aftermath of the Greek sovereign debt crisis of 2011-2012 that "debt restructurings have often been too little and too late, thus failing to re-establish debt sustainability and market access in a durable way"(IMF, 2013, p. 15). The IMF report was prescient, and two years later (February 2015) there was again talk about the country defaulting on its debt obligations with spreads of the five-year credit default swaps on Greek government bonds exceeding 1,200, from 465 in September 2014.

Sovereign debt defaults are prevalent as shown in two databases compiled recently (Beers and Nadeau, 2014; Trebesch, 2011). Figure 1 summarizes statistics on debt default worldwide during 1975–2013. Greece holds the record with the largest sovereign debt restructuring in history; other recent restructurings include Belize (in 2007 and 2013), Jamaica (in 2010 and 2013), and St. Kitts and Nevis (2012). Litigation against Argentina in New York courts by holdouts from the restructuring of 2005 is expected to have significant ramifications for future sovereign debt restructurings, and the ruling against Argentina in June 2014 pushed the country back into (technical) default. In Sept. 2014 the U.N. General Assembly adopted a resolution to "negotiate and adopt a multilateral legal framework for sovereign debt restructuring"[1]. In Sept. 2015 the General Assembly adopted draft resolution A/69/L.84 on "Basic Principles on Sovereign Debt Restructuring Processes"[2]. Li (2016) discusses the U.N. experience.

The eurozone crisis highlighted that sovereign defaults are not the privilege of emerging markets. It happens to the best economies as Sturzenegger and Zettelmeyer (2006) show studying ten years of crises. Reinhart et al. (2015) discuss policy tools for re-normalizing excessive debt, and they classify orthodox methods (enhanced growth, primary budget surpluses, privatisation of state-owned assets) and heterodox methods (unexpected inflation, taxing private wealth, financial repression and debt restructuring).

The factors contributing to sovereign defaults, reviewed in the next section, create negative feedback loops and heterodox methods may be needed to reverse the vicious circle. Given the complexity of inter-related risk factors in sovereign debt, there is need for advances in risk management in this area. In general, it has been assumed that markets will impose discipline on sovereigns and keep their debt sustainable. However, for many reasons, this does not happen. In the case of Greece, the Troika of IMF-EC-ECB demanded involvement of private creditors before the official sector would bail out the country. The Greek debt restructuring with Private Sector Involvement (PSI) saw €106bn transferred from private creditors to Greece together with an international assistance package of €164bn. The later came under strict austerity measures that pushed the Greek economy into recession for five consecutive years with 26% cumulative GDP contraction and unemployment rising to 27%.

While Greece has been an extreme example of debt crisis (Zettelmeyer et al., 2013) it is not a unique case of advanced economies facing debt crises. Figure 2 illustrates the increase of sovereign debt ratios of selected countries after the 2008 international crisis. The current average indebtedness of eurozone countries exceeds the debt-to-GDP ratio of 60% prescribed

[1]In favor 124 member States, 11 against and 41 abstained, see, e.g., http://www.reuters.com/article/2014/09/09/argentina-debt-un-idUSL1N0RA1X120140909

[2]In favor 136 member States, 6 against and 41 abstained, see http://unctad.org/en/pages/newsdetails.aspx?OriginalVersionID=1074

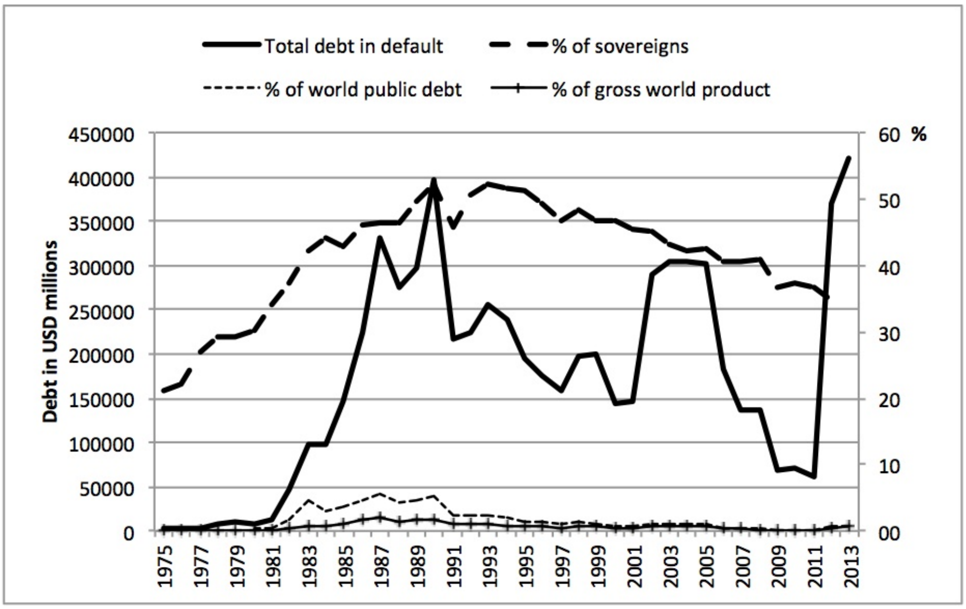

Figure 1: Sovereign debt default worldwide. (Data from Beers and Nadeau (2014).)

in the stability and growth pact of the European Union. At 96% it exceeds the 90% threshold of Reinhart and Rogoff (2010), and while this rigid threshold was recently rejected by Herndon et al. (2014), the hypothesis that excessive debt is a drag on growth is supported by other studies (Cecchetti et al., 2011; Jorda and Taylor, 2015).

Debt is fragile and when a sovereign loses market access —it has a “Minsky moment” to quote the popular term (McCulley, 2013)— the results are not easily reversed. Austerity is a popular tool in dealing with debt overhang, but as Jorda and Taylor (2015) show austerity is always a drag on growth especially in depressed economies. A 1% of GDP fiscal consolidation translates into a loss of 4% of real GDP over five years when implemented in a slump, and the debt overhang worsens. This is where some form of debt relief comes into play. Recognising the severity of debt crises in eurozone countries, proposals have emerged for debt restructuring, see, e.g., Gianviti et al. (2010); Paris and Wyplosz (2014).

In this paper we propose a risk management approach to re-profile public debt. We use the broader term *re-profile* to include both *rescheduling* and *restructuring* of debt whereby creditors may suffer losses in present or nominal value. Risk management goes beyond the static analysis of debt sustainability. It requires policymakers to postulate plausible scenarios and develop risk measures that ensure high probability of debt sustainability. We adopt for sovereign debt crisis management the same quantitative approach used for asset/liability management of financial institutions (Zenios and Ziemba, 2007). Other recent works also suggest probabilistic debt sustainability analysis (Guzman and Heymann, 2016; Lukkezen, 2015). Our work contributes the use of scenarios and goes beyond debt sustainability analysis to risk management optimization.

Of course, risk management failures abound and we do not imply that what we suggest is foolproof. Nevertheless, it is a much richer framework. Our approach is in line with recent trends in public debt management under non-crisis conditions, that integrate simulations with optimization models (Balibek and Köksalan, 2010; Bolder and Rubin, 2011; Consiglio and Staino, 2012).

We start with a discussion of debt restructuring issues in section 2 and identify a gap in the literature: operational risk management models are missing. The scenario analysis for debt re-profiling is in Section 3 where we develop the stochastic programming model. We also discuss how various issues arising in debt restructuring can be addressed either by our model

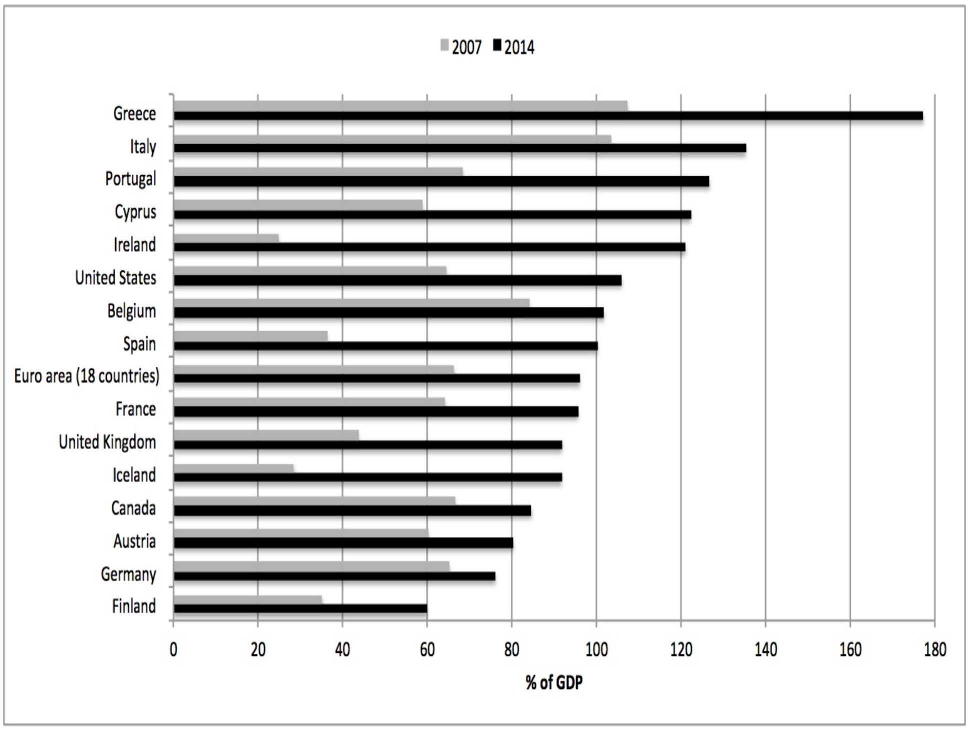

Figure 2: Sovereign debt of selected eurozone and major economies before and after the crisis. (Data from AMECO.)

or by extensions. The model is applied to Greece sovereign debt crisis in section 4 to analyse alternative restructuring proposals. Section 5 concludes.

2 Sovereign debt re-profiling: facts, why and how

We discuss here the issues relating to sovereign debt re-profiling. By re-profiling we mean a change of the terms of the debt obligation that allow the debtor to continue servicing it. For instance, maturities may be extended, interest rates renegotiated or nominal values written down. When creditors do not suffer losses of nominal value then we talk about debt rescheduling, while debt restructuring implies haircut in nominal value. Re-scheduling may involve losses in present value or it may be loss-free to allow the debtor to ride past concentration of payments around specific years. Figure 3 illustrates an IMF example of debt restructuring with nominal debt reduction of 33% and lengthening of maturities (Das et al., 2012). The literature review reveals an important gap that justifies the model, but we start with the known facts.

2.1 The facts

Fact 1. Sovereign debt re-profiling (rescheduling or default) is prevalent. In Figure 1 we see that up to 55% of sovereigns had been in default at some point since 1975. The amounts involved are in the hundreds of billions USD but they represent only a small percentage (average 1.8%) of global sovereign debt, and a minuscule proportion (average 0.6%) of gross world economic activity. While sovereign defaults are prevalent, any debt restructuring affects a small faction of the global debt market and the world economy.

Debt restructuring is called a heterodox method for dealing with debt crises by Reinhart et al. (2015), but the authors point out that "advanced countries have relied far more on these approaches than many observers choose to remember". The literature documents

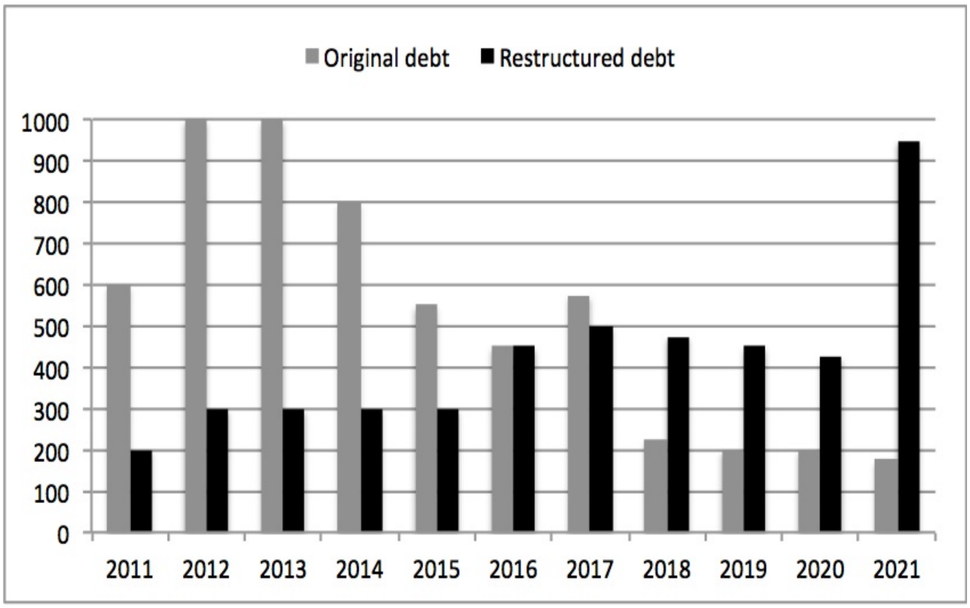

Figure 3: IMF example of debt with and without restructuring from Das et al. (2012). Vertical axis is nominal value of debt in mil. USD.

several instances where debt restructuring and financial repression were part of the resolution of debt crisis in advanced economies. Capital controls in Cyprus in 2013 (Zenios, 2015) and the Greek PSI (Zettelmeyer et al., 2013) are the most recent episodes.

Many factors contribute in pushing a distressed sovereign towards default, and the seminal paper to allow repudiation of sovereign debt is Eaton and Gersovitz (1981). There are sudden increases in interest rates once credit markets recognise that the sovereign is distressed, the effect of debt servicing on economic growth leading to the debt deflation spiral of Fisher (1933), the bank-sovereign "diabolical loop" (Brunnermeier et al., 2011; Mody and Sandri, 2012). The eurozone periphery also experienced capital flight in the absence of automatic stabilisers (De Grauwe, 2012). Aguiar and Gopinath (2006) provide a quantitative model to explain empirical observations on the counter-cyclicality of interest rates and net exports, and the correlation between current account balances and interest rates as drivers of default. Reinhart and Rogoff (2009); Sturzenegger and Zettelmeyer (2006) provide qualitative interpretations based on extensive empirical observations of defaults.

Fact 2. The haircuts involved are substantial. Historical experience shows that until the 1980's debt write-downs were barely implemented, while the majority of more recent deals include nominal value reductions (Wright, 2012). The recent cases of Argentina and Greece show that creditor losses (or "haircuts") can be significant: Argentinean debt was exchanged for 30 cents to the dollar, and Greek debt write-down was estimated at 53%.

Systematic studies of haircuts are reported in Benjamin and Wright (2009); Cruces and Trebesch (2013). The former study compiles a database covering 90 defaults by 73 countries during 1989–2006, and the data show haircuts of roughly 40%. The later compiles a database covering 180 restructuring cases in 68 countries during 1970–2010, and finds average haircut 37%. Reinhart and Trebesch (2015) find debt relief averaging 36-43% of external government debt (exceeding 40% of GDP in some cases) of debtor countries, following World War I debts in the 1930's and emerging markets debt during 1978-2010.

Fact 3. Defaults carry both benefits and penalties. Why should a sovereign avoid default? While there are obvious benefits from having debt eradicated, defaulting sovereigns face severe penalties: sanctions including loss of market access, litigation costs, reputation punishment etc. The theoretical economics literature is mixed whether benefits accrue to a defaulting sovereign. Bulow and Rogoff (1989) provide a no-reputation contracts theorem, which is “robust to many considerations” and conclude that “debts which are forgiven will be forgotten”. On the other hand, Cruces and Trebesch (2013) consider it a “main puzzle” that defaults have minor effects on subsequent borrowing costs and access to credit, and show that restructurings involving higher haircuts are associated with higher subsequent bond yield spreads and longer periods of capital market exclusion. Their analysis casts doubt on the belief that credit markets forget.

In the absence of consensus on theory, researchers turn to empirical investigations. Alas, the empirical literature faces a methodological difficulty that makes it problematic to compare research findings. The counterfactual to a defaulting sovereign would be a non-defaulting sovereign with similar economy under identical debt conditions, and such counterfactuals are hard to identify as each country's case is different. In any event, if debt is unsustainable there is no other option and debt relief must be sought in a manner that preserves value for both debtor and creditors. Wright (2012) argues that better outcomes can be achieved for both parties from the restructuring process. Reinhart and Trebesch (2015) show that “the economic landscape of debtor countries improves significantly after debt relief operations, but only if these involve debt write-offs. Softer forms of debt relief, such as maturity extensions and interest rate reductions, are not generally followed by higher economic growth or improved credit ratings”. Arslanalp and Blair (2005) provide evidence that both borrowers and lenders benefit from debt relief: stock markets of Latin American countries involved in the Brady Plan appreciated by an average of 60% in real dollar terms (without significant market increase for a control group of countries without Brady agreements), while market capitalization of U.S. commercial banks with developing country exposure rose by $13bn. However, not all Brady deals were successful as escalating interest rates after a grace period led to relapse of the crises when the grace period ended (Das et al., 2012). This is an example of myopia in debt restructuring that would be overcome with a multi-period model such as the one developped in this paper.

Fact 4. Re-profiling sovereign debt is a complex problem. Several challenges are raised in a default situation. First, of course, is the issue of when should a sovereign default. Benjamin and Wright (2009) found that average default takes almost eight years to resolve and leaves the sovereign country more highly indebted than when it entered default. They posit a theory of sovereign debt renegotiation to explain delays as a result of commitment problems in sharing the surplus generated from the restructuring.

Once the parties engage in default negotiations, significant legal and policy issues need to be resolved (Das et al., 2012; Wright, 2012). There is no universally applicable legal framework for dealing with sovereign defaults –akin to Chapter 11 for corporations and Chapter 9 for municipalities in US insolvency procedures– and coordination between creditors and debtor is a major challenge in sovereign debt restructuring. Nevertheless, some principles have emerged from experience to ensure stable capital flows, and the Paris and London Clubs facilitate creditor-debtor coordination. Collective Action Clauses have been introduced to coordinate *ex post* multiple creditors, and Consiglio and Zenios (2015c) suggest sovereign contingent debt to *ex ante* price risk-sharing between creditors and debtor.

Recently we have seen renewed debate on the process[3]. For instance, Gianviti et al. (2010) suggest an adaptation of SDRM with a special chamber of the Court of Justice of the Eu-

[3] See the collection of papers in (Guzman et al., 2016) and the papers presented at the conference hosted on March 27, 2015, by Imperial College London and the University of Chicago Booth School of Business that brought together lawyers and economists from IMF, the United Nations, law firms and academia, at `http://research.chicagobooth.edu/igm/events/conferences/conference-on-sovereign-debt-restructuring.aspx`.

ropean Union to deal with negotiations between a sovereign and its creditors. Reforms in sovereign debt contracts were suggested by the International Capital Markets Association[4] and Buchheit et al. (2013) proposed modifications to the European Stability Mechanism treaty so that debt restructuring becomes a condition of assistance. Guzman and Stiglitz (2016) discuss the creation of a functional framework for sovereign debt restructuring.

Fact 5. An operational model is missing. Assuming that coordination is achieved and legal obstacle can be overcome, then we need to address operational issues. Das et al. (2012) discuss the key parameters of the problem:

1. Face and market value of bonds or loans
2. Interest rate and coupon (fixed or flexible, step-up or linked features)
3. Amortization schedule (bullet or amortization, existence of a sinking fund)
4. Currency of denomination of the instruments (local and/or foreign currency)
5. Enhancements, including embedded options or collateral
6. Legal clauses (CACs, non-default clauses, exit consents)

Taking these parameters as input, the negotiating parties need to decide principal payments rescheduling, interest payment concessions, grace period or haircuts, and ensure that the rescheduled debt can be financed in a sustainable way. There is paucity of literature on operational issues, and risk management in particular has not been part of restructuring analyses. This is where our paper makes its contribution.

2.2 The need for a model

Issues relating to practical implementation of debt restructuring are discussed by Wright (2012). Perhaps his most important suggestion, and the one most relevant to our work, is the development of "criteria for an "optimal" debt restructuring process". "Optimal" is put within quotes by the author, recognizing that in such a complex setting there is no unique optimality criterion. We cannot expect a mathematical optimum for complex institutional and legal issues. However, there is need for a model to coordinate the process and provide decision support. Such a model would capture the tasks of the "international debt referee", address "the greatest problem which is restricted to restructuring a single debt" and accommodate the "central role [...] of state contingent debt".

The model we develop handles the operational aspects of debt restructuring. First, it collects all debt issues in a common framework and under a set of scenarios for all relevant risk factors. Second, it develops optimal financing strategies for alternative debt profiles and trades off their cost and associated risks using an optimality criterion that reflects debt sustainability. Third, contingent claims can readily be modelled conditioned on the scenarios. Fourth, the currency of denomination can be modelled via exchange rate scenarios. Fifth, we can model sinking funds and step-up or linked contractual features using the multi-period nature of the model. We formulate a model for the first two aspects and discuss extensions to the rest.

The operational model is one of the innovations of this paper. The other and, in our opinion, most important contribution, is in introducing risk management in debt restructuring. The model goes beyond the standard debt sustainability analysis to introduce risk measures akin to those used in portfolio analysis and the risk management of financial institutions. Debt sustainability analysis, while usually develops a set of scenarios, eventually makes policy recommendations based on the mean. This shortcoming manifested itself in the Greece crisis with IMF projections proven repeatedly excessively optimistic, a point highlighted from the analysis of

[4] Gelpern, A., August 29, 2014, *A sensible step to mitigate sovereign bond dysfunction*, Peterson Institute for International Economics, available at `http://blogs.piie.com/realtime/?p=4485`.

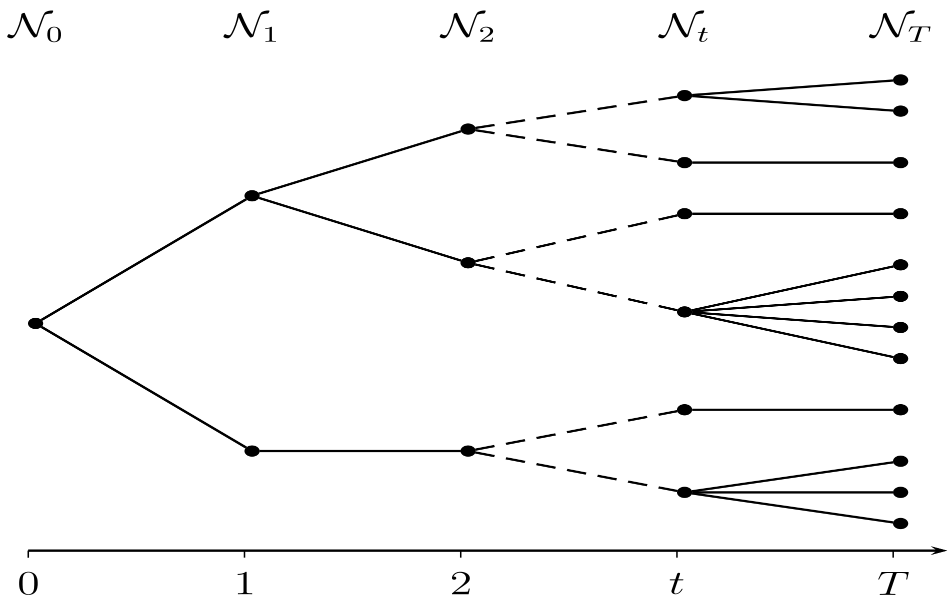

Figure 4: A scenario tree.

Lukkezen (2015). See Guzman and Heymann (2016) for a discussion of IMF debt sustainability analyses and its (missed) projections for European economies in distress. A risk management approach requires policymakers to face the reality of adverse outcomes that, though not the most likely, may occur with high probability.

3 Scenario analysis for optimal debt restructuring

3.1 The choice of a model

We adopt the discrete state-space, discrete time-space of multi-period stochastic programming, see, e.g., (Zenios, 2007, chap. 5). Decisions are made *here-and-now* based on all available information, and anticipating future uncertain information. As new information is obtained and uncertainty resolved we have *recourse* decisions that are conditioned on the realised scenario and previous decisions. Stochastic programming models have a rich history in financial management over the last twenty years; see (Zenios and Ziemba, 2007) and the extensive bibliography therein. These models received attention for public debt management in the work of Balibek and Köksalan (2010) for the Turkish Ministry of Finance, and Consiglio and Staino (2012) for the Italian Ministry of Treasure. A multi-period model captures important features of debt re-profiling problems: it integrates multiple debt issues with different maturities and accounts for clustering of maturities around specific dates with the associated roll-over risk. Clustering of debt maturities, usually after elections, is prevalent in sovereign debt management.

3.2 The scenario setting

We consider the setting where the key economic and financial variables evolve according to some stochastic processes, possibly correlated. In scenario modeling we have a discrete set of time-stages when decisions are made, $\mathcal{T} = \{0, 1, 2, \ldots T\}$. We denote by $t \in \mathcal{T}$ the time index, where $t = 0$ indicates here-and-now and T is the risk horizon. Market data evolve on a scenario tree, such as the one illustrated in Figure 4, whereby data take values from a set indexed by the set of *nodes* $\mathcal{N}_t$ for each time instance t. Each node $n \in \mathcal{N}_t$ represents a possible state of the economy at time t. Not all nodes at t can be reached from every node at $t-1$ and we define *paths* from the *root node* 0 to some final node in the set $\mathcal{N}_T$ to denote the unique way of reaching a particular node. Each of these paths is a *scenario*. The example of Figure 4 has 12 scenarios, two possible states at $t = 1$, three possible states at $t = 2$ and six at t. We denote by $\mathcal{P}(n)$ the set of nodes on the unique path from the root node to $n \in \mathcal{N}_t$, and by $p(n)$ the unique predecessor node for n, with $p(0)$ being empty. For any given node n all information at nodes of the path $\mathcal{P}(n)$, including of course the predecessor $p(n)$, is known.

With this notation we define state-dependent fiscal variables for each node $n \in \mathcal{N}_t$:

GDP, denoted by G^n in nominal value with growth rate g^n.

Debt, denoted by D^n in nominal value and d^n as ratio to GDP.

Interest rate on debt, denoted by r^n. For countries under an assistance program the interest may be fixed at r^0 for all time periods until the end of the program or even beyond.

Government net budget, denoted by NB^n. The following relation applies:

$$NB^n = GT^n - GE^n, \tag{1}$$

where GT denotes government revenues, typically taxes, and GE denotes government expenditures excluding debt servicing costs. For states of the economy with $NB^n > 0$ the government is running a primary surplus that can be used to pay down debt or accumulate reserves. Conversely, $NB^n < 0$ denotes primary deficit that increases debt.

Stock flow adjustment of debt, denoted by SF^n in nominal value and sf^n as ratio to GDP.

We assume that debt is in domestic currency for ease of notation and discuss extensions later.

3.3 Scenario arithmetic for fiscal dynamics

The general debt stock recursive equation is as follows, see, e.g., Ley (2010):

$$D_t = (1 + r_t)D_{t-1} - NB_t + SF_t. \tag{2}$$

Ley uses seignorage as SF_t. We use stock flow adjustment to represent adjustments to the debt profile via restructuring or rescheduling. If a sovereign collects seignorage, then SF is split in a seignorage term and a debt restructuring term, and seignorage modeled exogenously. Similarly, we can model proceeds from privatization in the stock flow adjustment.

In the scenario setting the debt dynamics are conditioned on the nodes:

$$D^n = (1 + r^n)D^{p(n)} - NB^n + SF^n. \tag{3}$$

This equation can be solved recursively for each path leading from the root to each terminal node $n \in \mathcal{N}_T$. While there is only one solution to (2) for the deterministic case, in the scenario setting there are as many solutions to (3) as there are paths.

We express debt dynamics as a ratio to GDP to account for the improvement or deterioration of the debt situation of a country by growth or contraction of the economy. GDP growth is a significant risk factor in debt crises and debt-to-GDP ratio stock dynamics are expressed conditioned on the nodes:

$$\frac{D^n}{G^n} = (1 + r^n)\frac{D^{p(n)}}{G^{p(n)}}\frac{G^{p(n)}}{G^n} - \frac{NB^n}{G^n} + \frac{SF^n}{G^n}. \tag{4}$$

GDP growth is given by

$$g^n = \frac{G^n - G^{p(n)}}{G^{p(n)}}, \tag{5}$$

and we express the debt dynamics in proportional growth instead of nominal value by

$$d^n = \frac{1+r^n}{1+g^n} d^{p(n)} - nb^n + sf^n. \tag{6}$$

We can use this equation to derive conditions for debt sustainability and answer questions such as "How can a government maintain a constant debt-to-GDP ratio?", or, "How to reduce debt-to-GDP ratio below a sustainable threshold?". For instance, debt is stable if $d^n = d^{p(n)}$ for the paths to all terminal nodes, and the primary surplus to ensure this is given by

$$\hat{nb}^n = \frac{r^n - g^n}{1+g^n} d^{p(n)} + sf^n. \tag{7}$$

When primary balance satisfies $nb^n > \hat{nb}^n$, debt will be reduced in direct proportion to the primary balance. Assuming no debt restructuring, i.e., $sf^n = 0$ and balanced budget, i.e., $nb^n = 0$, the debt is stable if growth equals effective interest rate on debt $r^n = g^n$. Growth is suppressed during crises, requiring strictly positive primary balance to maintain constant debt. Fiscal consolidation, in turn, exerts downward pressure on GDP exacerbating the crisis (Jorda and Taylor, 2015). Hence, there is a limit to how much can be achieved with surplus-generating austerity once a country is in crisis. That is when debt restructuring enters as a policy option. The question is then to find a combination of primary surplus and debt restructuring that will reduce debt by a proportion β, and bring it to sustainable level.

To reduce debt by $d^n = \beta d^{p(n)}$ we obtain from (6) the following debt stock equation:

$$nb^n = \left[\frac{1+r^n}{1+g^n} - \beta\right] d^{p(n)} + sf^n. \tag{8}$$

This equation provides the relationship between the policy variables of primary surplus and debt restructuring. Using scenarios uncovers the difficulty in making deterministic statements for these variables. The equation provides different values for each scenario and we need a risk measure, such as the worst case value or some acceptable quintile. We return to this issue later.

3.4 Scenario optimization

3.4.1 Model equations

We consider now the funding of government debt with a sequence of decisions $x = \{x_t\}_{t=0,1,\ldots,T}$. These decisions could include borrowing from the international market, loans of different maturities and contractual obligations from international organizations as part of an assistance program, or bi-lateral government agreements. x_0 is the *here-and-now* decision and x_t are the *recourse* decisions that are conditioned on the state of the economy at time period t. x_t is a set of possible decision vectors $\{x_t^n\}_{n\in\mathcal{N}_t}$. Without ambiguity we drop the time index from x^n since each node n takes values from a time-indexed set $\mathcal{N}_t$. We assume there are J available options for funding debt, and the decision vector at each node is given by $x^n = (x^{n1}, x^{n2} \ldots x^{nJ})$.

We express now debt dynamics on the scenario tree as functions of the funding decisions. For each $t = 1, \ldots, T-1$, and each $n \in \mathcal{N}_t$ we have

$$O^n = \sum_{m\in\mathcal{P}(n)} \sum_{j=1}^{J} x^{mj} \mathrm{CF}^j(n,m), \tag{9}$$

where $\mathrm{CF}^j(n,m)$ denotes cash flows at node n for debt with jth maturity issued at some node $m \in \mathcal{P}(n)$.[5] O^0 is the nominal debt due here-and-now. This equation accounts for the total

[5] The calculations of CF are tedious. They take into account coupon and principal payments, perhaps adjustable rates, contingency provisions and so on. However, all these are exogenous to the debt funding decision.

debt to be covered at each node n due to decisions made at previous time periods.

The debt stock equation aggregates existing debt with all obligations created by previous funding decisions, and finances this stock with new funding decisions:

$$\sum_{j=1}^{J} x^{nj} = D^n + O^n. \tag{10}$$

At the end of risk horizon T and for each $n \in \mathcal{N}_T$ the cost of debt financing decisions is:

$$C^n = D^n + O^n + \sum_{m \in \mathcal{P}(n)} \sum_{j=1}^{J} x^{mj} P^j(n, m). \tag{11}$$

$P^j(n, m)$ is the state-dependent value of outstanding debt and is given in nominal value, if we follow the accounting standards for sovereign debt reporting, or by market value if we are interested in fair valuation for the sovereign's creditors. The use of market values requires contingent re-pricing of debt at the different states as discussed in the extensions later. Choosing between book and market valuation is a prevalent issue in sovereign debt management literature, so much so that there are two approaches of computing haircuts (Das et al., 2012). Market valuation is appropriate for debt buyback, while book values apply for contractual restructuring.

3.4.2 Objective function

We postulate an objective function on the debt-to-GDP ratio $c^n = C^n/G^n$ which is the key indicator for debt sustainability (Sturzenegger and Zettelmeyer, 2006, p. 308-313). Debt-to-GDP is a random variable whose (discrete) distribution depends on debt financing decisions, on the schedule of existing debt, on any debt restructuring, and the economic and financial random variables. The decision maker wants to shape the distribution of this random variable to match some views. For instance, they may want to tradeoff expected value against volatility, or limit the Value-at-Risk (Jorion, 2006) or limit the coherent risk measure of conditional Value-at-Risk (CVaR) (Artzner et al., 1999). A public debt management office may have multiple, possibly conflicting, objectives and wish to trade-off expected cost, duration of debt, volatility of cost and so on. For instance, the public debt management office of the Turkish government adopts a multi-objective formulation (Balibek and Köksalan, 2010), while Consiglio and Staino (2012) optimizes conditional Cost-at-Risk for the Italian Ministry of Treasure.

Static debt sustainability analysis establishes a threshold debt-to-GDP ratio below which the sovereign can service its debt without resorting to additional borrowing beyond what is needed to rollover existing debt. For instance, 120% is considered a threshold for advanced economies, and this was used for the Greek debt restructuring. These targets are based on a unique projection of debt-to-GDP. In a scenario setting this would be the mean value and for stress testing this would be the worst case scenario. To assess the risk of deviation from a sustainable debt-to-GDP threshold we need a risk measure.

We define the *stress debt* for each terminal state of the economy as the non-negative difference of the state-dependent debt-to-GDP ratio from its expected value. Stress debt is a signal of problems as debt-to-GDP ratio deviates upwards from its mean value and if the mean is unsustainable then, at the risk horizon, a sovereign will find its debt in states of the world that are unsustainable with high probability. However, it might very well be the case that the mean is sustainable and still have a high probability of debt being unsustainable, where "high" may be considered a probability of 5% or even 1%. Hence, we formulate the conditional Value-at-Risk of stress debt, which we call *conditional Debt-at-Risk*.

Once a scenario tree is built and a path $\mathcal{P}(n)$ specified CF is obtained using standard cash-flow calculators; see (Consiglio and Staino, 2012, eqn. 6) for cash-flow calculations of government bonds.

Let the stress debt be

$$sd^n = c^n - \mathbb{E}[c], \tag{12}$$

where $\mathbb{E}[c]$ is the expected value of the terminal final debt-to-GDP ratio. That is,

$$\mathbb{E}[c] = \sum_{n \in \mathcal{N}_T} \pi^n c^n, \tag{13}$$

where π^n are the probabilities of the terminal states. For uniform distributions of child nodes at each non-terminal node, the probabilities are given by $\frac{1}{|\mathcal{N}_T|}$ for symmetric trees. For non-symmetric trees, such as the one of Figure 4, terminal probabilities are easily computed as the joint probability of all nodes on the path $\mathcal{P}(n)$. The conditional Debt-at-Risk (CDeaR) is the expected value of the stress debt, conditioned on the stress test being in excess of its Value-at-Risk at a given confidence level α. Following Rockafellar and Uryasev (2000) CDeaR can be modeled in a portfolio selection problem by including the following equation and inequalities:

$$\begin{aligned} \text{CDeaR} &= \zeta + \frac{1}{1-\alpha} \sum_{n \in \mathcal{N}_T} \pi^n y_+^n, && (14) \\ y_+^n &\geq sd^n - \zeta, && (15) \\ y_+^n &\geq 0, && (16) \end{aligned}$$

where y_+^n is a dummy variable denoting the non-negative values of debt in excess of ζ. ζ is the Value-at-Risk of debt, which we call *Debt-at-Risk* (DeaR), i.e., the lowest possible value so that the probability of excess debt not exceeding DeaR is $(1 - \alpha)$.

3.4.3 Model

We have expressed debt dynamics and conditional Debt-at-Risk in terms of exogenous GDP dynamics and debt restructuring choices, and endogenous debt financing decisions. The optimization model given below minimizes the expected cost of debt with limits on the risk measure CDeaR. A summary of all variables and model data is given in the Appendix.

$$\text{Minimize } \mathbb{E}[c] \tag{17}$$

s.t.

$$O^n = \sum_{m \in \mathcal{P}(n)} \sum_{j=1}^{J} x^{mj} \mathrm{CF}^j(n,m) \quad \text{for all } n \in \mathcal{N}_t, t \in \mathcal{T} \backslash 0, \tag{18}$$

$$D^n + O^n = \sum_{j=i}^{J} x^{nj}, \quad \text{for all } n \in \mathcal{N}, \tag{19}$$

$$C^n = D^n + O^n + \sum_{m \in \mathcal{P}(n)} \sum_{j=1}^{J} x^{mj} P^j(n,m), \quad \text{for all } n \in \mathcal{N}_T, \tag{20}$$

$$c^n = C^n / G^n, \quad \text{for all } n \in \mathcal{N}_T, \tag{21}$$

$$sd^n = c^n - \mathbb{E}[c], \quad \text{for all } n \in \mathcal{N}_T, \tag{22}$$

$$y_+^n \geq sd^n - \zeta, \quad \text{for all } n \in \mathcal{N}_T, \tag{23}$$

$$\zeta + \frac{1}{1-\alpha} \sum_{n \in \mathcal{N}_T} \pi^n y_+^n \leq \rho, \tag{24}$$

$$x^n, O^n, c^n, y_+^n \geq 0, \quad \text{for all } n \in \mathcal{N}. \tag{25}$$

Varying the parameter ρ we trace an *efficient frontier* of expected cost *vs* CDeaR and use it to evaluate different restructuring policies and identify those that are sustainable at the α confidence level. We point out that CDeaR bounds DeaR from above, cf. eqn. (14), however the DeaR corresponding to a given CDeaR is not necessarily minimal and hence the frontier of expected cost *vs* DeaR is not efficient.

3.4.4 Extensions

D^n is the exogenous debt, either in its original form or under alternative restructured forms. This is the parameter that changes with debt restructuring by rescheduling maturities, offering interest payment concessions or imposing haircuts on nominal value. In the simplest case this parameter is time-dependent but not state-dependent i.e., $D^n = D_t$ for all $n \in \mathcal{N}_t, t \in \mathcal{T}\backslash 0$. State-dependence is introduced when issuing state-contingent debt such as sovereign coco's or GDP-linked bonds. State-dependent prices can be computed, e.g., using the analysis of Heath et al. (1992) for bond price dynamics in a continuous setting or Mulvey and Zenios (1994) for fixed-income state-dependent pricing on binomial lattices. These settings can be incorporated in our model using contingent re-pricing of debt at the different states. In Consiglio and Zenios (2015c) we make the case for sovereign contingent debt with payment standstill, and develop the extended risk optimization model.

Accommodating sinking funds and step-up contractual features is straightforward within the multi-period structure of the model. The cashflow parameters $\mathrm{CF}^j(n, m)$ can capture sinking funds and step-up features at different stages m on the path leading to node n. Similarly we can capture time-dependent features of debt such as a grace period for repayments. These features are exogenous to the optimization model and are derived from the scenario generator.

Technical modifications of the cashflow accounting equations can account for foreign debt and exchange rate risks. See (Sturzenegger and Zettelmeyer, 2006, eqns. (A.10)-(A.18)) for multi-currency dynamics in a deterministic setting and (Topaloglou et al., 2002, eqns. (13)-(14)) for multi-currency scenario setting that includes also hedging of exchange rate risk; this is critical for countries borrowing in foreign currency.

Can the model be extended to endogenize debt restructuring decisions? In principle the multi-period multi-state setting allows this, but several issues need to be resolved. Endogenizing the debt restructuring decisions is done by introducing a variable to fund (part of) debt by borrowing from the end of the horizon at zero cost. This would be a variable denoting "forgiven debt". However, to implement the model a realistic estimate of the cost of forgiveness is needed. For instance, if this cost would be exclusion from market access, it can be modeled using binary variables but such variables significantly complicate the model. Also, would a debt restructuring decision be made simply based on costs or on some threshold that renders debt unsustainable at certain states of the economy? Many open issues remain to be addressed for debt restructuring to be endogenized within an operational model such as the one we propose in this paper. Benjamin and Wright (2009) endogenize debt restructuring decisions using a game-theoretic framework which is useful in explaining stylized facts on creditor-debt behaviour, but it is not an operational model.

4 Applications to the case of Greece sovereign debt

We analyse several policy options suggested in dealing with the Greece debt crisis. Not all details of the debt instruments are modelled, and further validation is needed to draw robust recommendations for the country. Nevertheless, the risk profiles of debt financing are captured, and the examples illustrate how risk profiles change with alternative debt restructuring policies. We use the model to identify restructuring policies that restore debt sustainability with high probability. In a companion paper (Consiglio and Zenios, 2015a) we use the model to develop

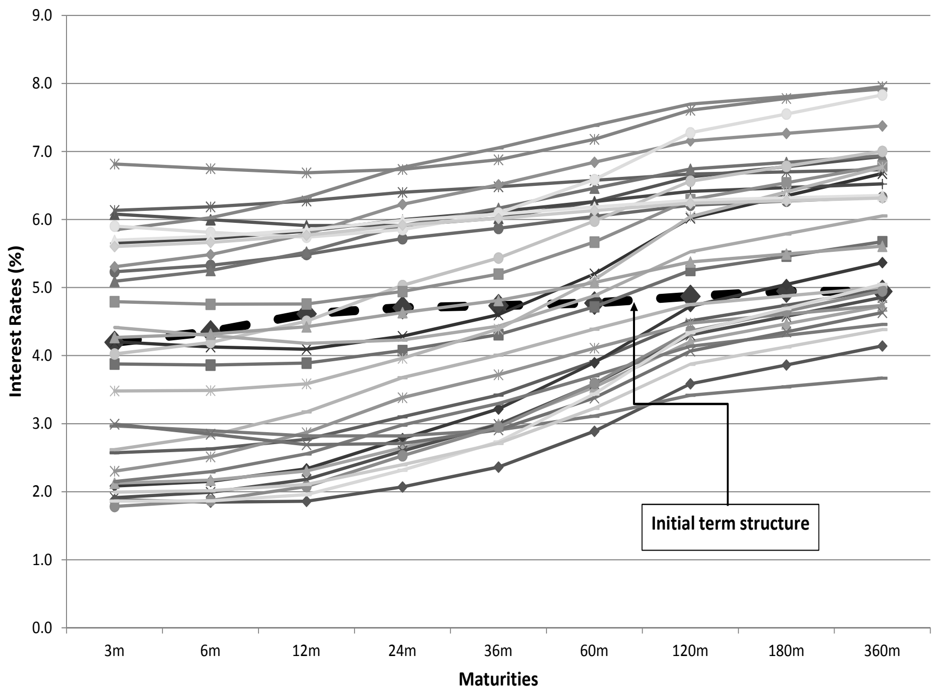

Figure 5: Sample scenarios and the input term structure for the implementation.

risk profiles for Cyprus, Italy and Ukraine.

Interest rate scenarios are generated using the simulator of Bernaschi et al. (2007). The input data, based on 2013, are (i) the current term structure, (ii) the ECB official rate, and (iii) the current inflation rate. Using this information and fitted parameters for the short rate model, the simulator generates a set of scenarios of length 12 months. The next stage scenarios are simulated by starting from the relevant data of the previous month and the procedure is repeated for each scenario and for each stage. Figure 5 illustrates the input term structure and the generated scenarios. The exogenous debt is time-dependent but state-independent, as given in the projected payments shown below. The confidence level α is set to 5%.

4.1 Greece sovereign debt crisis

Greece was the first eurozone country to face a debt crisis during the Great Recession. It entered the global crisis with excessive government debt, but very modest household, corporate and banking debt. The crisis was precipitated in October 2009, when a newly elected Government revealed that the country had understated its debt and deficit figures. The projected budget deficit for 2009 was revised upwards from an estimated 7% to more than 12%. The final deficit was 15.6% and there were repeated rating downgrades of the sovereign. By April 2009 the Greek government bond spreads increased from 300 basis points (bp) to almost 900bp and the country was effectively excluded from the bond markets. The government turned to EU and the IMF and a three-year rescue package was agreed in May 2010 for €80bn in EU loans and €30bn of IMF credit. These were to be paid out in tranches until 2012, conditional on the implementation of a fiscal adjustment package of 11% GDP and structural reforms to restore competitiveness and growth. Following the rescue package and further rescue measures for the eurozone agreed by the EU leaders —creation of the European Financial Stability Facility (EFSF) with a lending capacity of €440bn and the ECB secondary market purchase programme (SMPP) to stabilise sovereign bond yields in secondary markets— the spreads fell sharply. However, in mid-June

Moody's downgraded Greece citing substantial risks associated with the support package and spreads again began to exceed 800bp.

In October 2010, at the Summit in Deauville, President Sarkozy and Chancellor Merkel called for a crisis resolution mechanism with arrangements for "adequate participation of the private sector". This introduced sovereign default risk in EU member states, leading to a widening of bond spreads of peripheral European countries. The prospects of Greece returning to international capital markets by early 2012, as planned by the May programme, looked slim. The IMF review published in mid-July 2011 concluded that Greece debt can not be considered sustainable with high probability. Following this conclusion the country and its private sector creditors entered into debt restructuring agreements, known as PSI and PSI+, concluded in July–October 2011 and March 2012. Overall, €106bn or 54% GDP of sovereign debt owned by the private sector was written-off, but Greek banks suffered losses of €38bn and were recapitalized from an EFSF loan to the Government. The net debt reduction was €68bn or 35% GDP. Zettelmeyer et al. (2013) discuss the Greek debt restructuring.

The Greek debt restructuring of 2011-2012 is what the IMF called "too little and too late" to re-establish debt sustainability. By February 2015 debt-to-GDP ratio exceeded 180% and the Greek economy remained for a sixth consecutive year in recession, with cumulative GDP loss of 25% and unemployment at about the same level. Following the election victory of radical left party SYRIZA in January 2015, discussions on debt relief took again center stage[6].

We use Greek government debt obligations to illustrate the use of the model and analyze the effectiveness of alternative proposals suggested to deal with the crisis. Figure 6 shows the debt schedule, both in its original form and after a hypothetical rescheduling (top). Three- and four- period discretisations are also shown in the figure. The debt schedule is from the Public Debt Management Office[7]. Interest rates on the various debt instruments are obtained from the Wall Street Journal[8] and aggregated on a yearly basis by us. Figure 7 shows projections for GDP growth and primary surplus from the IMF country report[9]. On the same figure we plot the proposal of Prime Minister Tsipras and Minister of Finance Varoufakis (T-V) for primary surplus 1.5% instead of 4.5% stipulated by the IMF. We also illustrate a GDP projection of higher growth under the expanded fiscal space of the T-V proposal. Reducing primary surplus by 3% of GDP starting in 2015, and assuming fiscal multiplier of 0.8, we have higher growth by 2.4% as shown in the figure[10]. Unless otherwise stated all runs are for a 20-year horizon for the discretised debt in the middle diagram of Figure 6.

For comparison purposes we will be referring also to the IMF debt sustainability analyses for Greece carried out in June 2014 (Country Report No. 14/151) and June 2015 (Country Report No. 15/165). We refer to these two analyses as IMF-2014 and IMF-2015, respectively. IMF-2014 concluded "a high probability that [...] debt is sustainable", but within a year reality trampled expectations and IMF-2015 changed its verdict to "not sustainable with high probability". A non-technical summary of policy implications from the model runs we carry out next, and comparisons with the findings of IMF and others are published in Consiglio and Zenios (2015b).

[6]See, for instance, the letter by Nobel laureates Joseph Stieglitz and Christopher Pissarides and other prominent economists published in the Financial Times on Feb. 18, 2015, *Pragmatism is required amid Greek deal stand-off*, available at `http://www.ft.com/intl/cms/s/0/2174cb48-b7c2-11e4-981d-00144feab7de.html#axzz3RyJ9lCxp`, and the editorial in Der Spiegel on Feb. 2, 2015, *It's Time To Compromise on Greece*, available at `http://www.spiegel.de/international/europe/spiegel-editorial-on-need-to-compromise-with-tsipras-and-greece-a-1016275.html`.

[7]Available at `http://www.pdma.gr/index.php/en/public-debt-strategy/public-debt/maturity-profile-en`, accessed on February 22, 2015.

[8]Available at `http://graphics.wsj.com/greece-debt-timeline/`, accessed on February 22, 2015.

[9]IMF (2014), available at `http://www.imf.org/external/pubs/ft/scr/2014/cr14151.pdf`, accessed on February 22, 2015.

[10]We use the average fiscal multiplier reported in the studies surveyed by Baum et al. (2012).

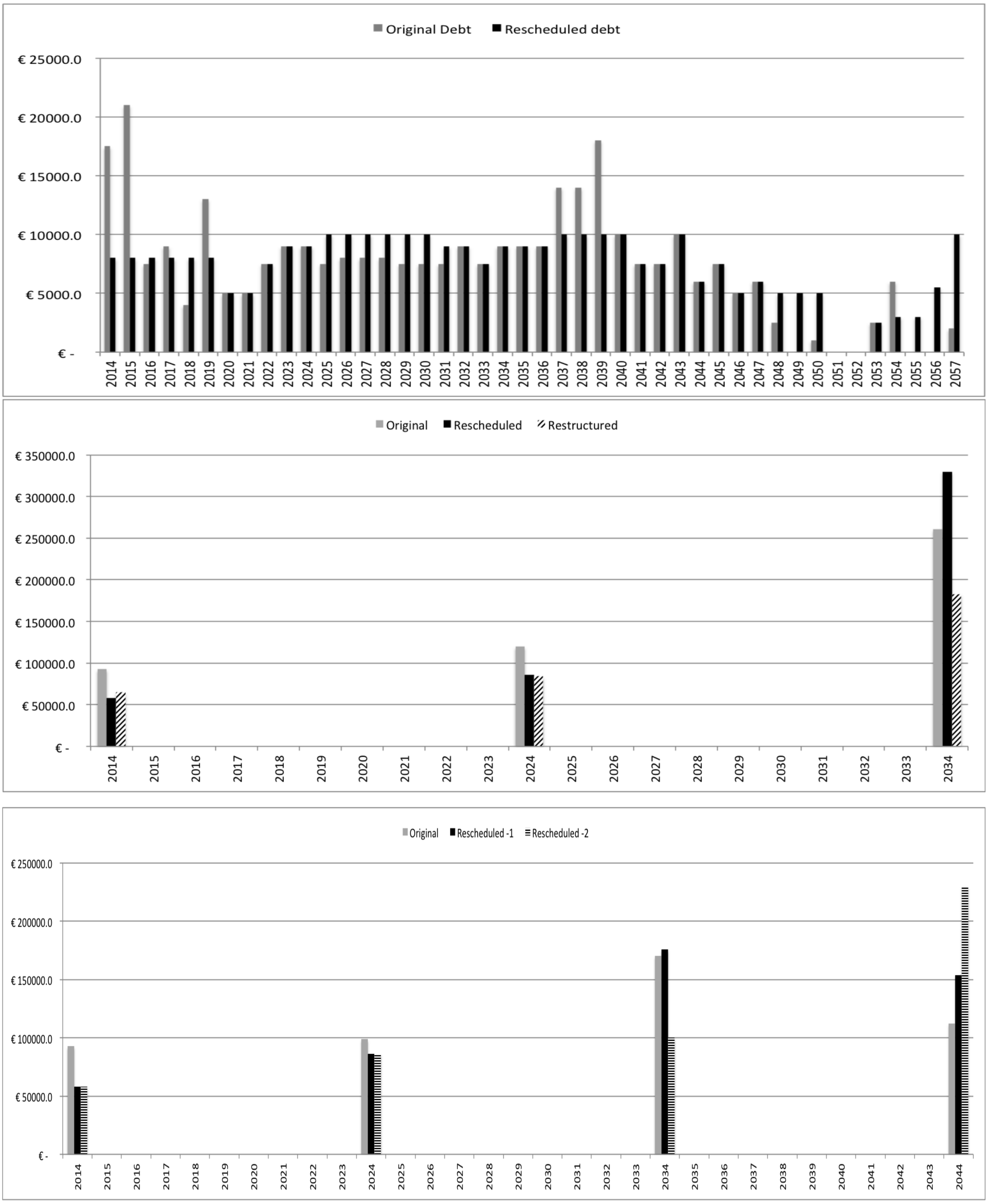

Figure 6: Greece debt profile (top), three-period with rescheduling and restructuring with 30% nominal value haircut (middle), and four-period discretizations with alternative reschedulings and no haircut (bottom).

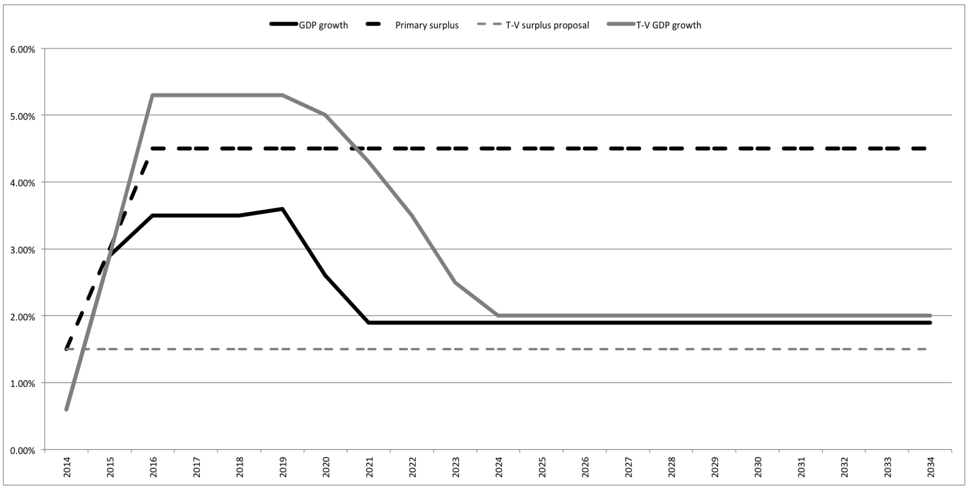

Figure 7: Projections for Greece GDP growth and primary surplus. (Data from IMF (2014) and authors' calculations.)

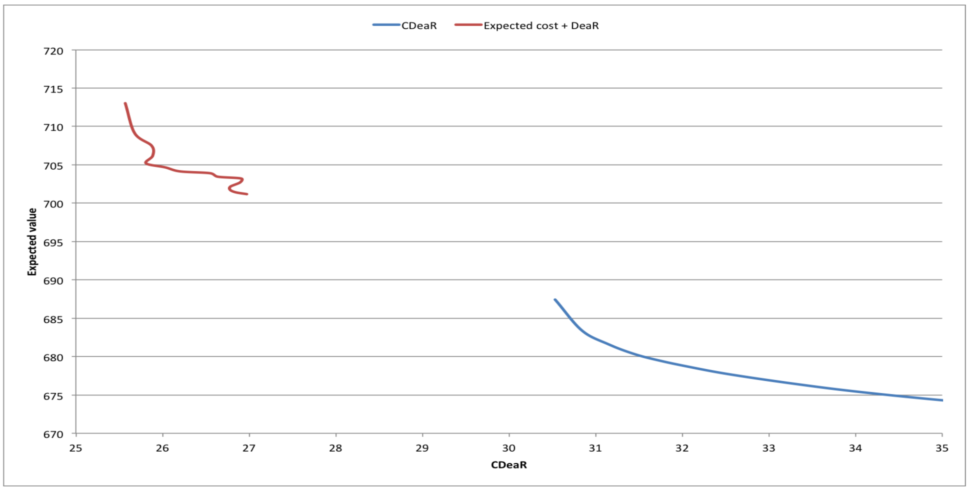

Figure 8: The efficient frontier for financing Greece debt and the 0.95 percentile of cost (expected cost + DeaR) under a balanced budget.

4.2 Sustainability of existing debt

4.2.1 Debt sustainability with balanced budget

We first run a benchmark to finance existing debt assuming the government runs a balanced budget during the 20-year risk horizon. The efficient frontier is shown in Figure 8, together with the corresponding Debt-at-Risk. Under the growth rates projected by IMF, GDP will grow to €289bn by 2034. From the figure we observe that the expected cost and the 0.95 percentile of cost exceed 230% of this amount. If Greece were to return to the markets to refinance its debt under the interest rates shown in Figure 5 then the debt situation will keep deteriorating from its current level of 180% GDP even if the country runs a balanced budget. Under the current status of the Troika program and under market conditions our model finds the debt unsustainable. This finding is in disagreement with the assessment of IMF-2014 and precedes the findings of IMF-2015. Since we use the same data as IMF-2014, this finding provides evidence that the scenario risk analysis we suggest unearthed problems that the IMF analysis could not until a

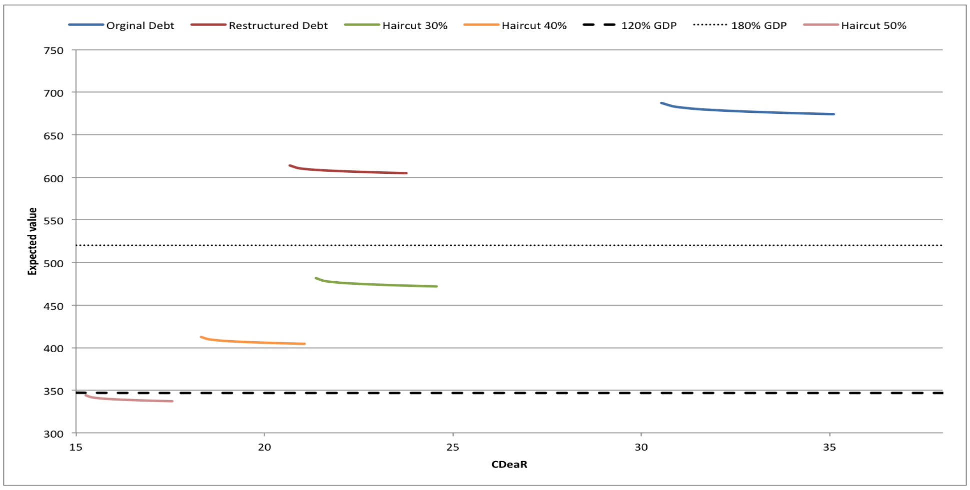

Figure 9: Risk profiles for financing alternative structures of Greece debt under balanced budget.

year later.

We then run the model for the rescheduled debt and for restructured debt with varying percentages of haircut on nominal value, assuming again balanced budget. Results are summarized in Figure 9 where we also plot the 120% and 180% GDP lines; 120% is the debt sustainability threshold for Greece and 180% is the current ratio.

What do these results tell us about debt sustainability for Greece? With debt rescheduling the expected cost of financing remains above 180% GDP. Although the frontier is shifted down significantly, the debt remains unsustainable if it were to be financed by the market. The current situation is improved for haircuts of 30% or higher. With haircut 50% the frontier lies below the 120% GDP threshold and debt appears sustainable.

While the expected cost of debt financing is below the threshold with 50% haircut, this analysis ignores the tail of the distribution. In a crisis situation we are interested in the tail and not in the mean. While the tail reflects rare events, a country going through a debt crisis is experiencing a rare event and the tail is the new normal. We therefore re-calculate the efficient frontier for the 50% haircut by adding to expected cost its associated risk ($DeaR$ and $CDeaR$) for each point of the frontier; Figure 10. These re-calculated risk-adjusted frontiers now lie slightly above the threshold. Hence, while the expected cost of Greece debt refinancing under the 50% haircut is sustainable, neither the .95 percentile of cost nor the expected cost in the extreme 5% are sustainable. Sustainability at the .95 percent confidence level needs a nominal haircut slightly higher than 50%.

4.2.2 Debt sustainability under the IMF program

We run the model for the IMF projections of GDP growth and primary surplus of Figure 7. Under these projections the country generates aggregate surplus exceeding its total debt obligations over the risk horizon. Even with 3.5% surplus the expected cost and risks are very small and debt is sustainable; Figure 11. Basically, restricted by the assumptions of IMF-2014 our analysis reaches similar conclusions like IMF-2014. However, primary surplus sustained at 4.5% GDP over twenty years is unrealistic. It is also unrealistic that the country can sustain 1.9% GDP growth while running large surpluses, so we turn to the proposal of the Greek government.

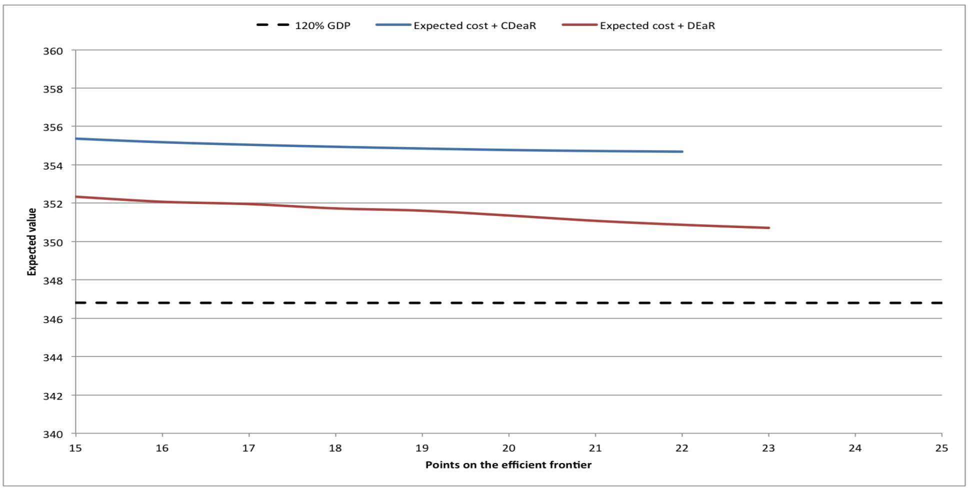

Figure 10: The risk-adjusted cost of financing the Greece debt with a 50% nominal value haircut, for different points on the efficient frontier.

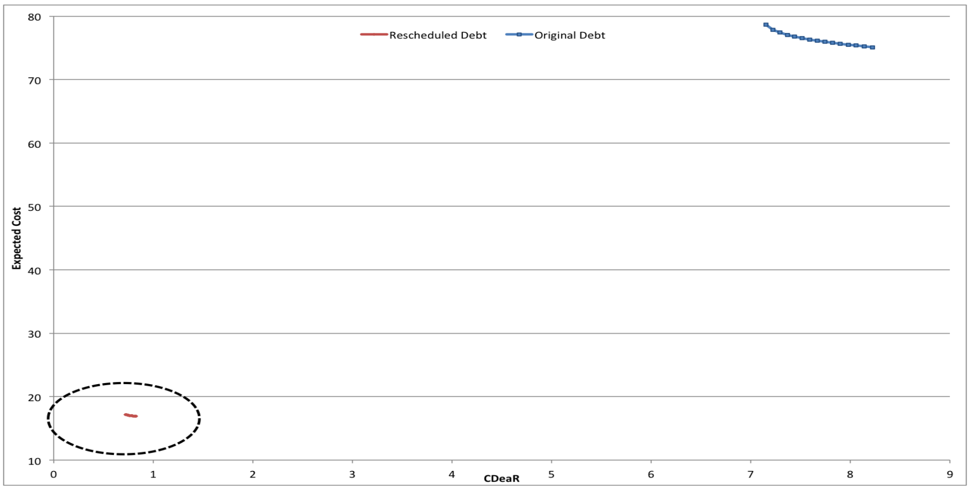

Figure 11: Risk profiles for financing Greece debt under IMF growth and primary surplus projections lie well below the 120% GDP threshold and debt is considered sustainable.

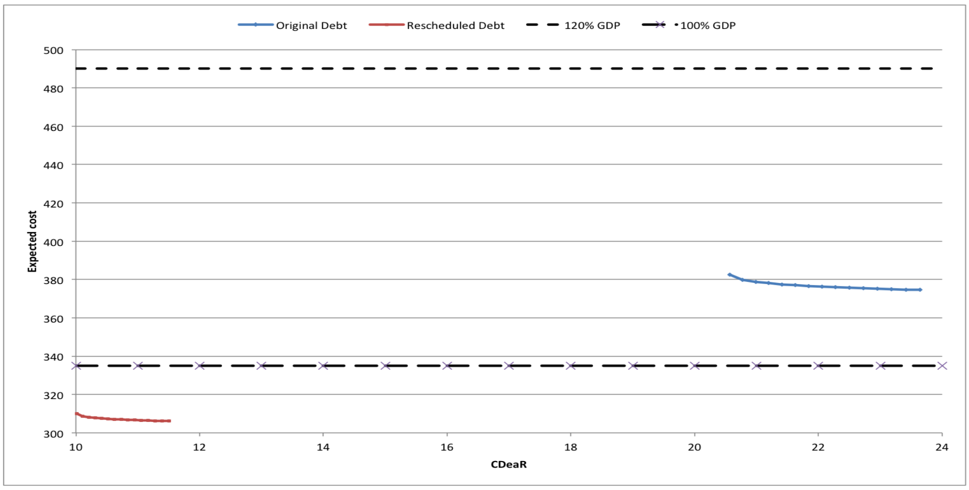

Figure 12: Efficient frontier for financing the Greece debt under the Greek government proposal for primary surplus 1.5% and improved country growth assuming fiscal multiplier 0.8.

4.3 Greek government proposals

We run the model for 1.5% primary surplus and the improved growth using fiscal multiplier 0.8. Results in Figure 12 show that this proposal improves significantly the risk profile below the current 180% ratio although it remains above the 120% threshold. So, whereas the country receives substantial relief with the Government proposed primary surplus, debt sustainability is not restored. Running the model for the rescheduled debt we obtain a risk profile below the debt sustainability threshold. The .95 percentile and the expected cost for the extreme 5% are also below the threshold. The proposal of the Greek government together with debt rescheduling restores debt sustainability without need for nominal value haircuts.

4.4 Interest payment concessions

An alternative approach to debt restructuring is to offer concessions on interest payments. We run the model for three cases whereby the country makes payments only for 90%, 80% or 70% of the contractual interest, keeping primary surplus and GDP growth as per the Greek government projections. Results are shown in Figure 13 and we observe that interest rate concessions significantly improve the current situation. The 70% concessions have expected cost below the 100% GDP. At the bottom figure we plot the extreme frontiers at the 5% confidence level. We observe that the interest rate concessions do not lie below the 100% GDP threshold at the 5% confidence level but they are very close to it and the rescheduled debt does. Hence, interest rate concessions of slightly more than 70% together with the expanded fiscal space suggested by the government restore sustainability even at the 100% GDP threshold.

4.5 Maturity extensions

A multi-stage model allows us to test proposals for maturity extension. In Figure 14 we plot efficient frontiers for the alternative maturity structures illustrated in the four period discretisation of Figure 6. To account for the differences in maturities we estimate, in this figure, all expected costs in their present value instead of the future value we did in previous runs. The present value of GDP is €182bn and the 120% GDP threshold is €218bn. We observe significant improvements in the expected cost, that comes at slight increase in risk, as we extend maturities. The longest maturity extension tested (labeled "Rescheduled debt 2") still does not cross the sustainability threshold and there are diminishing improvements as we extend maturities even

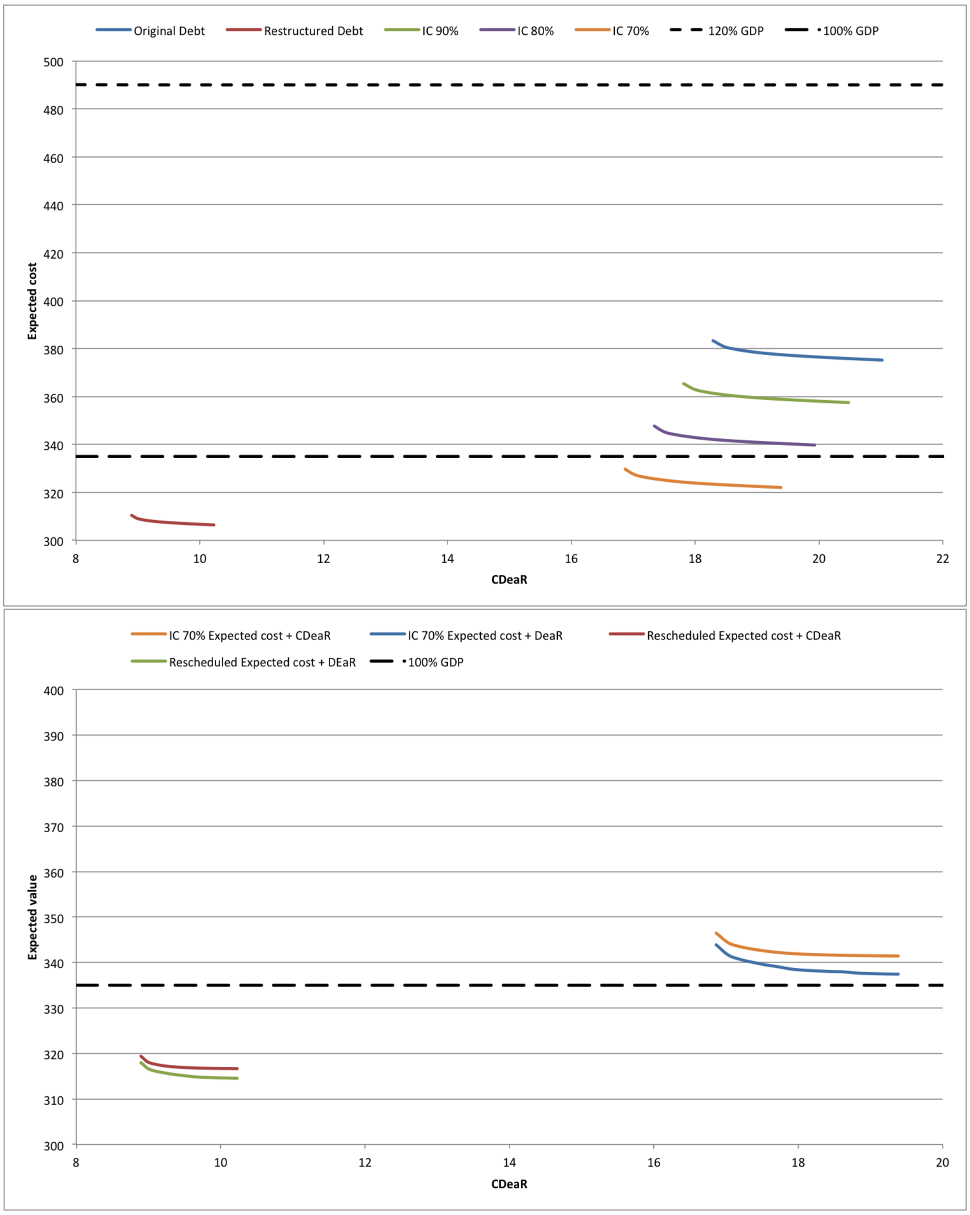

Figure 13: Financing Greece debt under the Greek government projections and for varying concessions on interest payments. Top figure is the frontier of expected cost vs CDeaR, bottom figure are the extreme frontiers for (Expected Cost + CDeaR) vs CDeaR.

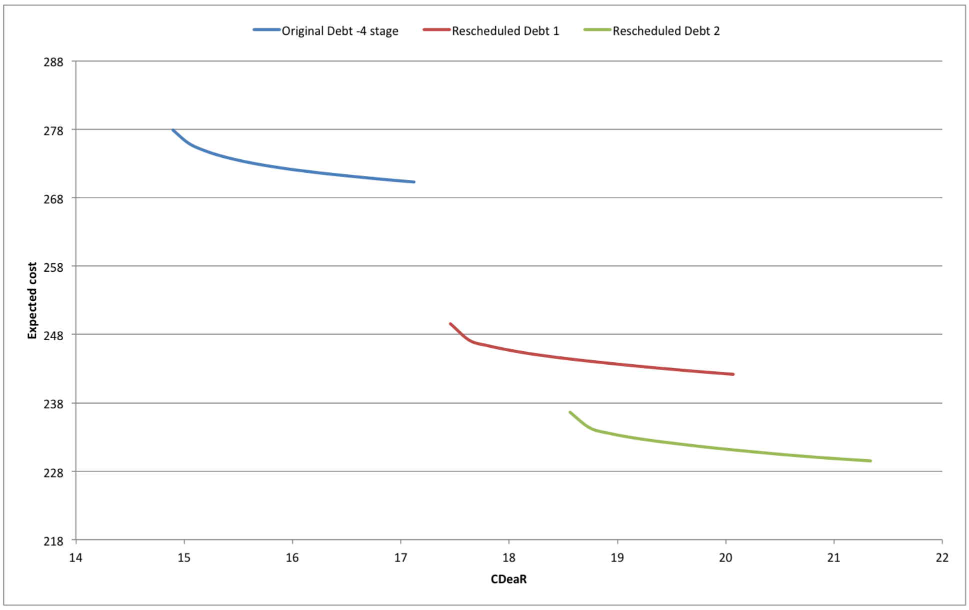

Figure 14: Extending the maturity of Greece debt under balanced budget: frontiers of expected cost vs CDeaR for the 4-stage discretisations and rescheduling options of Figure 6 (bottom).

further. It seems that extraordinary maturity extensions are required for Greece debt to become sustainable under balanced budget; it appears easier to achieve sustainability under the Greek government proposal of reduced primary surplus with modest interest rate concessions.

5 Conclusions

Using scenario analysis and stochastic programming we developed an operational model for debt restructuring. The model introduces risk management in debt restructuring decisions. Current policy debates highlight the need for such models. We have shown how scenario-based debt sustainability analysis can be integrated within an optimization model to trace the tradeoffs between the cost of debt financing and the associated risks. The model allows us to compare alternative debt structures and identify those that are sustainable with a high probability.

The model integrates in a common framework, and under a set of consistent scenarios, multiple debt issues over different time horizons; allows the embedding of contingent claims; can deal with multiple currencies, sinking funds and step-up or GDP-linked bonds. Not all of these features were tested, but modifications required of the base model to do so are possible and we do in the follow-up paper for contingent debt (Consiglio and Zenios, 2015c).

Application to the Greece case is informative. The model shows that Greek debt is unsustainable even if the government maintains a balanced budget for a very long period, and significant nominal value haircuts are needed to restore debt sustainability. Under IMF projections we find Greece debt sustainable but it is broadly accepted that these projections are unrealistic. We use the model to analyse the request of the Greek government for lower primary surplus and find that the debt situation is improved but debt sustainability is not restored unless additional measures are taken: rescheduling (extending) debt maturities or concessions on interest payments. Nominal value haircuts are not needed to restore sustainability under the Greek government proposal. Our findings are in broad agreement with the IMF analysis of June 2015, and provide additional credibility since they hold with high probability. It is worth noting that our analysis uses 2014 data and, hence, it shows that the IMF erred in declaring debt sustainable in 2014. Our analysis anticipated the subsequent IMF analysis of June 2015 and was not affected by the policies of the new Greek government. Hence, the IMF cannot blame the debtor for the revision of its debt verdict from sustainable to unsustainable. No matter how misguided the negotiating tactics of the new Greek government might have been, debt was unsustainable before they came to power.

A Appendix. Model variables and data

The variables of the optimization model are given below for all $n \in \mathcal{N}$:

x^n Vector of debt financing decisions at node n, with components x^{nj} denoting the nominal amount borrowed at the nth node with j-th maturity, e.g. short-, medium- and long-term.

O^n Accounting variable of payments due at each node n based on borrowing decisions at nodes on the path leading to n.

C^n Accounting variable of total amount due at each node n at the end of the risk horizon. This is the cost variable and it sums up maturing debt, payments due to borrowing decisions at nodes on the path leading to n, and the market or book value of outstanding debt.

c^n Cost variable expressed as a ratio to GDP.

sd^n Accounting variable of stress debt.

y_+^n Dummy variables used to compute conditional Debt-at-Risk (CDeaR).

ζ This variable is the Debt-at-Risk (DeaR).

The model input data are the following:

D^n The exogenous debt to be financed. Debt re-profiling changes the values of this parameter.

$\mathbf{CF}^j(n,m)$ Cash flows at node n for debt with jth maturity issued at node m on the path to n.

$P^j(n,m)$ State-dependent value of outstanding debt at node n at the end of the risk horizon, for debt with jth maturity issued at node m on the path to n. This parameter prices debt maturing past the risk horizon.

References

M. Aguiar and G. Gopinath. Defaultable debt, interest rates and the current account. *Journal of International Economics*, 69:64–83, 2006.

S. Arslanalp and H.P. Blair. Is Debt Relief Efficient? *The Journal of Finance*, 60(2):1017–1051, April 2005.

P. Artzner, F. Delbaen, J. Eber, and D. Heath. Coherent measures of risk. *Mathematical Finance*, 9(3):203–228, 1999.

E. Balibek and M. Köksalan. A multi-objective multi-period stochastic programming model for public debt management. *European Journal of Operational Research*, 205:205–2017, 2010.

A. Baum, M. Poplawski-Ribeiro, and A. Weber. Fiscal multipliers and the state of the economy. WP/12/286, International Monetary Fund, 2012.

D. T. Beers and J-S. Nadeau. Introducing a new database of sovereign defaults. Technical Report No. 101, Bank of Canada, February 2014.

D. Benjamin and M.L.J. Wright. Recovery before redemption: A theory of delays in sovereign debt renegotiations. Technical report, Available at SSRN: `http://ssrn.com/abstract=1392539`, 2009.

M. Bernaschi, M. Briani, M. Papi, and D. Vergni. Scenario-generation methods for an optimal public debt strategy. *Quantitative Finance*, 7:217–229, 2007.

D.J. Bolder and D. Rubin. Optimization in a simulation setting: Use of function approximation in debt strategy analysis. Bank of Canada Working Paper 3, Bank of Canada, 2011.

M.K Brunnermeier, L. Garicano, P.R. Lane, M. Pagano, R. Reis, T. Santos, S. Van Nieuwerburgh, and D. Vayanos. ESBies: a realistic reform of Europe's financial architecture. *VOXeu.org*, October 2011.

L.C Buchheit, A. Gelpern, M. Gulati, U. Panizza, B.W. di Mauro, and J. Zettelmeyer. Revisiting Sovereign Bankruptcy. Technical report, Committee on International Economic Policy and Reform, Brookings, Washington, D.C., October 2013.

J. Bulow and K.S. Rogoff. Sovereign debt: Is to forgive to forget? *American Economic Review*, 79(1):43–50, July 1989.

S.G. Cecchetti, M.S. Mohanty, and F. Zampolli. The real effects of debt. Working paper 352, Bank for International Settlement, September 2011.

A. Consiglio and A. Staino. A stochastic programming model for the optimal issuance of government bonds. *Annals of Operations Research*, 193(1):159–172, 2012.

A. Consiglio and S.A. Zenios. Risk profiles for re-profiling sovereign debt of crisis countries. *The Journal of Risk Finance*, 16(1):2–26, 2015a.

A. Consiglio and S.A. Zenios. Greek debt sustainability: The devil is in the tails. *VOX, CEPR's Policy Portal*, August 2015b. URL `http://www.voxeu.org/article/greek-debt-sustainability-devil-tails`.

A. Consiglio and S.A. Zenios. The case for contingent convertible debt for sovereigns. Working Paper 15-13, The Wharton Financial Institutions Center, Available at `http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2694973`, 2015c.

J.J. Cruces and C. Trebesch. Sovereign Defaults: The Price of Haircuts. *American Economic Journal: Macroeconomics*, 5(3):85–117, 2013.

S.D. Das, M. Papaioannou, and C. Trebesch. Sovereign Debt Restructurings 1950–2010: Literature Survey, Data, and Stylized Facts. Technical Report Working Paper 12/203, The International Monetary Fund, Washington, DC, 2012.

P. De Grauwe. The Governance of a Fragile Eurozone. *Australian Economic Review*, 45(3): 255–268, 2012.

J. Eaton and M. Gersovitz. Debt with potential repudiation: Theoretical and empirical analysis. *The Review of Economic Studies*, 48(2):289–309, 1981.

I. Fisher. The debt-deflation theory of great depressions. *Econometrica*, 1(4):337–357, 1933.

F. Gianviti, A. O. Krueger, J. Pisani-Ferry, A. Sapir, and J. von Hagen. A European mechanism for sovereign debt crisis resolution: a proposal. Blueprint Series 10, Bruegel, Brussels, 2010.

M. Guzman and D. Heymann. The IMF debt sustainability analysis. *Journal of Globalization and Development*, (forthcoming), 2016.

M. Guzman and J.E. Stiglitz. *Creating a framework for sovereign debt restructuring that works*, volume Too Little, Too Late: The quest to resolve sovereign debt crises, chapter 1. Columbia University Press, New York, NY, 2016.

M. Guzman, J.A. Ocampo, and J.E. Stiglitz, editors. *Too Little, Too Late: The quest to resolve sovereign debt crises*. Columbia University Press, New York, NY, 2016.

D. Heath, R. Jarrow, and A. Morton. Bond pricing and the term structure of interest rates: A new methodology for contingent claim valuation. *Econometrica*, 60:77–105, 1992.

T. Herndon, M. Ash, and R. Pollin. Does High Public Debt Consistently Stifle Economic Growth? A Critique of Reinhart and Rogoff. *Cambridge Journal of Economics*, 38(2):257–279, 2014.

IMF. Sovereign debt resturcturing - recent developments and implications for the fund's legal and institutional framework. Technical report, International Monetary Fund, Washington, DC, April 2013.

IMF. Greece: Fifth review. Country Report No. 14/151, International Monetary Fund, Washington, DC, June 2014.

O. Jorda and A.M. Taylor. The time for austerity: Estimating the average treatment effect of fiscal policy. *The Economic Journal*, (available on line doi: 10.1111/ecoj.12332), 2015.

P. Jorion. *Value at Risk: The New Benchmark for Managing Financial Risk*. McGraw-Hill, New York, N.Y., 3rd edition, 2006.

A.O. Krueger. A new approach to sovereign debt restructuring. Technical report, International Monetary Fund, Washington, DC, April 2002.

E. Ley. Fiscal (and External) Sustainability. Technical report, The World Bank, Washington, DC, 2010.

Y. Li. The long march towards an international legal framework for sovereign debt restructuring. *Journal of Globalization and Development*, (forthcoming), 2016.

J. Lukkezen. Greek debt remains unsustainable: Here is why. *VOX, CEPR's Policy Portal*, Available at `http://www.voxeu.org/article/greek-debt-remains-unsustainable`, September 2015.

P. McCulley. The shadow banking system and Minsky's journey. In Arthur M Berd, editor, *Lessons from the Financial Crisis*, pages 49–63. Risk Books, November 2013.

A. Mody and D. Sandri. The eurozone crisis: How banks and sovereigns came to be joined at the hip. *Economic Policy*, pages 199–230, April 2012.

J.M. Mulvey and S.A. Zenios. Capturing the correlations of fixed-income instruments. *Management Science*, 40:1329–1342, 1994.

P. Paris and C. Wyplosz. PADRE-Politically acceptable debt restructuring in the Eurozone. Geneva Reports on the World Economy Special Report 3, International Center for Monteray and Banking Studies, Geneva, Switzerland, 2014.

C. Reinhart and K.S. Rogoff. Growth in a time of debt. *American Economic Review*, 100: 573–578, January 2010.

C.M Reinhart and K.S. Rogoff. *This time is different: eight centuries of financial folly.* Princeton University Press, Princeton, NJ, 2009.

C.M Reinhart and C. Trebesch. Sovereign debt relief and its aftermath. HKS Faculty Research Working Paper Series RWP15-028, Harvard Kennedy School, 2015.

C.M Reinhart, V. Reinhart, and K. Rogoff. Dealing with debt. *Journal of International Economics*, 96(Supplement 1):S43–S55, July 2015.

R.T. Rockafellar and S. Uryasev. Optimization of conditional Value–at–Risk. *Journal of Risk*, 2(3):21–41, 2000.

F. Sturzenegger and J. Zettelmeyer. *Debt defaults and lessons from a decade of crises.* MIT Press, Cambridge, Mass., 2006.

N. Topaloglou, H. Vladimirou, and S.A. Zenios. CVaR models with selective hedging for international asset allocation. *Journal of Banking & Finance*, 26(7):1535–1561, 2002.

C. Trebesch. Sovereign debt restructurings 1950-2010: A new database. Unpublished paper, available at sites.google.com/site/christophtrebesch/data, University of Munich, Munich, Gemany, 2011.

M.L.J. Wright. Sovereign debt restructuring: Problems and prospects. *Harvard Business Law Review*, 2:152–198, 2012.

S.A. Zenios. *Practical Financial Optimization: Decision making for financial engineers.* Blackwell, Malden, MA, 2007.

S.A. Zenios. Fairness and reflexivity in the Cyprus bail-in. *Empirica*, Available at `http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2409284`, (published online 4 September) 2015.

S.A. Zenios and W.T. Ziemba, editors. *Handbook of Asset and Liability Management, Vol. 1. Theory and Methodology, Vol. 2. Applications and Case Studies.* Handbooks in Finance. North-Holland, The Netherlands, 2007.

J. Zettelmeyer, C. Trebesch, and M. Gulati. The Greek debt restructuring: an autopsy. *Economic Policy*, 28(75):513–563, July 2013.