Modelling and Forecasting Brent Crude Oil Returns

Dissertation submitted

by

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ABSTRACT

This dissertation was produced as a component of the "Monetary and Finance Economics" postgraduate program at the University of Cyprus. Its primary objective is to examine the time series data of weekly returns on crude oil prices to forecast their future fluctuations. The study proposes a methodology for predicting oil returns movements utilizing the Box-Jenkins approach, a widely recognized method in time series analysis. The Box-Jenkins methodology involves the identification, estimation, and diagnostic checking of a suitable autoregressive integrated moving average model for the time series data.

In addition to the theoretical framework, the study will also include empirical validation of the forecasting model using historical crude oil price data. A theoretical overview is presented, elucidating the crude oil concept and its significance within the global market and financial domain. Subsequent chapters delve into empirical analysis, employing suitable methods to effectively model and forecast crude oil returns. Specifically, autoregressive moving average (ARMA) models and hybrid models within the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) family are constructed utilizing the statistical software EViews 9. By comparing the forecasted values with actual returns movements, the model's performance will be evaluated, thereby contributing to the practical applicability of the Box-Jenkins methodology in the context of crude oil returns forecasting. By shedding light on the predictability of oil returns and the factors driving their movements, this study aims to provide valuable insights that can aid in risk management and strategic planning in the energy and financial sectors.

Keywords: Brent crude oil, prices, returns, time-series, Box-Jenkins, Eviews 9, hybrid models

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List of Abbreviations

ACF: Auto-Correlation Function

ADF: Augmented Dickey Fuller

ANN: Artificial Neural Network

APARCH (or APGARCH): Asymmetric Power ARCH

AR: AutoRegressive

ARCH: Autoregressive conditional heteroskedasticity

ARIMA: Auto Regressive Integrated Moving Average

ARMA: Autoregressive Moving Average

Bcm: Billion Cubic Meter

DM: Diebold Mariano

DF: Dickey Fuller

ECB: European Central Bank

EGARCH: Exponential Generalized Autoregressive Conditional Heteroskedastic

FIGARCH: Fractionally Integrated Generalized Autoregressive Conditional Heteroscedastic

GARCH: Generalized Autoregressive Conditional Heteroskedasticity

GAS: Generalized Autoregressive Scpre

GJR GARCH: Glosten, Jagannathan, and Runkle Generalized Autoregressive Conditional Heteroskedasticity

GT: Gamma TestA

IEA: International Energy Agency

IGARCH: Integrated Generalized Autoregressive Conditional heteroskedasticity

IMF: International Monetary Fund

IOC: International Oil Companies

LM: Lagrange Multiplier

MA: Moving Average

MAE: Mean Absolute Error

MAPE: Mean Absolute Percentage Error

MS: Markov Switching

NYMEX: New York Mercantile Exchange

OPEC: Organization of the Petroleum Exporting Countries

PACF: Partial Auto-Correlation Function

QML: Quasi Maximum Likelihood

RMSE: Root Mean Square Error

VAR: Value at Risk

CHAPTER 1: INTRODUCTION

1.1 PURPOSE

Crude oil, a global energy powerhouse, wields a profound influence over the economic landscape, drawing the attention of businesses, investors, and governments. In this ever-shifting environment, companies in the electricity market rely heavily on returns forecasting techniques to navigate market volatility. These forecasts, which are not just crucial but often the lifeblood of commodity purchases and portfolio management, play a pivotal role in risk mitigation and formulation of energy strategy. However, the art of returns forecasting is a labyrinth, intricately woven with a myriad of factors such as weather conditions, business cycles, and international trade dynamics, all vying for attention.

This paper aims to employ a range of econometric models drawn from extant literature to identify the most suitable model for forecasting weekly returns of Brent crude oil. Following the compilation of data about weekly returns on Brent crude oil prices, this time series dataset was utilised to construct an Autoregressive Moving Average (ARMA) model using the Box-Jenkins methodology alongside four hybrid models within the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) family. The objective behind applying these econometric models was to identify the model that most accurately captures the characteristics of the underlying time series. With the aid of evaluation criteria, the most appropriate model was selected, facilitating the generation of oil returns forecasts by utilising the statistical software EViews 9.0. Crude oil is a significant player in the global energy industry and significantly impacts the economy, attracting the attention of businesses, investors, and governments. Electricity market companies rely heavily on forecasting techniques to navigate market volatility. These forecasts are crucial for commodity purchases and portfolio management and play a key role in risk mitigation and energy strategy formulation. However, forecasting is a complex task, influenced by several factors such as weather conditions, business cycles, and international trade dynamics.

The significance of this paper lies in its quest to identify the most suitable model for forecasting weekly returns of Brent crude oil by employing a range of econometric models. The dataset of weekly returns on Brent crude oil prices was used to construct different models, such as the Autoregressive Moving Average (ARMA) model, and four hybrid models within the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) family, using the Box-Jenkins methodology. The objective of using these econometric models was to identify the model that best captures the characteristics of the underlying time series. With the help of rigorous evaluation criteria, the most appropriate model was selected, and the statistical software EViews 9.0 was used to generate oil returns forecasts.

1.2 SUBJECT

Given the aforementioned rationales, this study undertakes an examination of the weekly returns of Brent crude oil prices through the application of time series modelling and forecasting methods. In particular, academic attention towards forecasting crude oil returns has escalated in the preceding decade. Within recent literature, numerous investigations regarding return prediction and risk evaluation have shifted emphasis towards methodological performance comparisons rather than exploring interrelationships among input variables.

1.3 BRIEF DESCRIPTION OF THE RESEARCH

This paper, composed of seven chapters, drafts an approach to forecasting Brent crude oil returns utilising the Box–Jenkins methodology, a widely utilised technique in time series analysis. The initial chapter delivers a theoretical framework, enclosing the definition of Brent crude oil and its key role within the global market and financial aspect. This theoretical illustration highlights the significance of crude and its pricing dynamics. Moreover, a historical backwards-looking of critical crises impacting the commodity is detailed, examining factors that substantially influence its pricing trends. Furthermore, several econometric models are applied based on existing literature. The prior objective of this study is to conduct forecasts of Brent crude oil returns within the global market context.

Following the introductory chapter, Chapter 2 presents a more comprehensive context of crude oil, clarifying its economic significance and pervasive effect on everyday life. Thereafter, Chapter 3 synthesises relevant elements from prior literature, outlining the models and methodologies employed in the modelling and forecasting crude oil returns. The subsequent chapters comprise the empirical segment of this study. Chapters 4 and 5 illuminate the theoretical reinforcements of statistical and econometric components essential for modelling. Additionally, fundamental concepts and features of time series are expounded upon as the principal forecasting methodologies alongside an evaluation metrics synopsis. Moreover, Chapter 6 analyses the examination of stationary and non-stationary processes. Together, within the same chapter, a brief overview of the econometric software EViews 9 is provided, facilitating the performance of econometric analyses.

Chapter 7 provides a comprehensive version of the weekly pricing dynamics of crude oil and the data sources which support this study. It examines weekly returns transiting a 5-year period from 2014 to 2019. This thorough research process forms the bedrock for applying specific econometric models, which are instrumental in forecasting the returns of the commodities under investigation. The journey includes a strict stationarity test, model construction, and heteroscedasticity assessment, culminating in selecting a model documented in existing literature. Once the model is developed, we generate and evaluate forecasts of crude oil returns. Finally, Chapter 8 encapsulates the conclusions from the preceding econometric analysis and outlines avenues for prospective research endeavours.

CHAPTER 2: THEORETICAL FRAMEWORK OF CRUDE OIL

2.1 ORGANIZATION OF PETROLEUM EXPORTING COUNTRIES (OPEC)

Crude oil has shaped the world economy. By the early 1970s, this important energy commodity shifted consumption to nearly 50% of global energy consumption. The current daily production, which is approximately 96.3 million barrels (U.S. Energy Information Administration, 2017), keeps us informed about the present state of crude oil production.

At this point, it is essential to refer to the geopolitical importance of crude oil. Since the beginning of the oil industry, its strategic importance has been evident, endowing countries possessing it with substantial national power on both political and economic fronts. In terms of global crude oil production structure, the Organization of the Petroleum Exporting Countries (OPEC)member nations collectively hold 81% of the world's reserves, accounting for 40% of global oil output (OPEC Annual Statistical Bulletin, 2016). Middle Eastern nations such as Saudi Arabia, Iran, Iraq, and the United Arab Emirates control the largest share of OPEC production and reserves, thereby consolidating wealth and influence within their domain (Hamilton, 2008).

The OPEC is an international economic consortium that was established in Baghdad in 1960. Its main objective is to coordinate policies among member states to ensure equitable and stable crude oil prices for producers. Moreover, it strives to achieve efficient, economical, and stable crude oil supplies while seeking fair returns on investment capital within the industry. As of 2016, OPEC comprises 13 member states, including the five founding nations—Iran, Iraq, Kuwait, Saudi Arabia, and Venezuela—and subsequent additions such as Algeria, Angola, Ecuador, Gabon, Libya, Nigeria, Qatar, and the United Arab Emirates (OPEC, 2012).

Although the crude oil market was dominated by multinational oil companies with American interests, nowadays, OPEC exercises considerable power over the crude oil market, with control over approximately 40% of global oil production and 81% of proven reserves worldwide (OPEC Annual Statistical Bulletin, 2016). The establishment of this organization marked a climactic shift in the national sovereignty over natural resources.

In summary, OPEC operates as an entity leveraging its substantial oil reserves to influence the global crude oil market by adjusting production quotas in line with its interests. Despite a decline in its share of the global oil supply market since the beginning of the 21st century, OPEC retains a considerable stake, ensuring that the market will remain susceptible to the organization's decisions and interventions.

2.2 OIL CRISIS

In recent years, a significant number of researchers have highlighted the correlation between oil shocks and the real economy. These studies have established oil shocks not only as key factors influencing oil prices but also as noteworthy economic indicators. For instance, Hamilton's (1983) analysis during the years 1948 to 1980 reveals a consistent pattern of preceding recessions in the American economy; there were typically concurrent increases in oil prices. This finding emphasizes the conclusion that substantial fluctuations (or shocks) in oil prices have a concrete impact on economic activity. To further illuminate this, a collection of the most consequential oil crises in the post-World War II era will be presented, highlighting their interconnectedness with fluctuations in crude oil prices.

The crude oil crisis spread in late 1973, surrounded by escalating geopolitical tensions. This disruption was triggered by the Yom Kippur War, a conflict that saw Israel encircled by Egypt, Syria, and other Arab nations. In response to the Western support extended to Israel, Iran and several Arab exporting nations imposed an oil embargo on nations supporting Israel.

This strategic manoeuvre, coupled with OPEC's announcement of a production reduction, precipitated a sharp spike in crude oil prices, soaring from $2.50 to $12. The historical context of this crisis, marked by geopolitical tensions and strategic moves, adds a layer of intrigue to its economic implications.

Subsequently, the world was rocked by a second crude oil shock in 1979 and 1980 sparked by Iran and Iraq. The cessation of oil production in Iran and diminished exports from Iraq catalyzed a surge in crude oil prices, overlapping with a global economic downturn. As a result, this led to a significant spike in prices, also worsening the economic downturn. Post-1980, oil prices began a downward course in response to declining demand and a simultaneous uptick in production from other nations (U.S. Energy Information Administration, Thomson Reuters).

As the new millennium began in 2000, European nations attempted to diminish their reliance on crude oil sourced from Arab nations, seeking alternative energy sources. Nevertheless, despite these efforts, oil remained the superior energy resource in Europe by a notable margin (Hamilton, 2013).

Since 2011, oil prices have experienced an upward movement, reaching levels slightly exceeding $100 until early 2014. However, in the latter half of 2014, there was a downturn in oil prices attributable to the magnified shale oil production in the United States and the reduced demand from European countries and China (Hamilton, 2013).

2.3 DETERMINING FACTORS OF THE PRICE OF OIL

Crude oil price dynamics are sensitive to various disruptive events, including geopolitical crises or significant weather phenomena, which can hamper its distribution. Such events may engender uncertainty regarding future demand or supply. Consequently, causing higher volatility in crude oil prices. A significant portion of crude oil reserves is situated in regions historically tolerant to political unrest or prostrate to production disruptions due to political disruptions. Under these circumstances, the market perpetually evaluates the likelihood of future disruptions and their prospective ramifications. Also, it jointly measures the availability and capability of remaining producers whose actions are important to mitigate potential supply losses (U.S. Energy Information Administration, 2016).

Supply

Oil supply is a major determinant of crude oil prices, containing the outputs of both OPEC and non-OPEC nations (OPEC, 2016). OPEC's supply holds significant power over crude oil price dynamics, underscored by historical evidence. OPEC member countries collectively contribute 40% of global crude oil production and account for 60% of global oil trade. The organisation actively regulates production levels among its members, establishing quotas and demonstrating a correlation between reduced production targets and subsequent oil price increases. Additionally, the indicator of OPEC's excess production capacity assumes consequences in potential crude oil organisations' crises, as it recalls the market's resilience.

Conversely, non-OPEC countries contribute 60% of global crude oil production, with significant production hubs in North America, the former Soviet Union territories, and the North Sea. Unlike OPEC's centralised management structure, non-OPEC nations' production activities are predominantly overseen by International Oil Companies (IOCs). IOCs operate in decentralised terms of production volumes. A considerable portion of non-OPEC production occurs at a higher cost than OPEC counterparts, driving these nations to explore frontier areas such as deepwater and unconventional sources like oil sands.

Demand

Similar to supply, the demand for oil stands as a crucial determinant of oil prices, which can be categorized into demand from member nations of the Organization for Economic Cooperation and Development (OECD) and demand from non-OECD countries.

The OECD comprises the United States, European nations, and other developed countries, collectively accounting for 53% of global oil consumption in 2010 (EIA, Short Term Energy Outlook, Thomson Reuters, 2016). The economic structure of each nation variably influences the relationship between oil prices and consumption. Developed nations are characterized by high levels of per capita vehicle ownership, resulting in the transportation sector's significant share of total oil consumption. Moreover, OECD member countries implement policies such as increased fuel taxation and the promotion of fuel efficiency and biofuel consumption, leading to a decline in oil consumption despite economic growth.

Conversely, non-OECD countries exhibit a substantial increase (40%) in crude oil consumption, spearheaded by China, India, and Saudi Arabia, reflecting their rapid economic development. These nations utilize crude oil across various sectors, including construction and electricity production, a trend also compounded by population growth. Notably, China's recent economic surge has forced it to become the world's largest energy consumer, thereby significantly contributing to the global increase in crude oil consumption.

Inventories

Inventories are a crucial factor in shaping oil prices. They are functioning as a stabilizing factor between supply and demand dynamics. During periods of surplus production relative to consumption, crude oil can be stored for future utilization. This has been evidenced during the 2008 crisis, where a decrease in oil consumption led to the accumulation of record-level inventories in the United States and other Organization for Economic Cooperation and Development (OECD) countries. Conversely, when consumption outpaces production rates, existing reserves are tapped in order to meet the demand for consumption (EIA, Short Term Energy Outlook, Thomson Reuters, 2016).

Weather

Like many commodities, crude oil is subject to seasonal fluctuations, resulting in amplified demand and consumption during specific periods. For instance, increased heating oil consumption occurs during winter, while greater utilization of diesel is observed in summer due to heightened travel activity. Although market participants anticipate high demand and consumption periods, they often overlap with upward price pressures, which tend to normalize towards the season's finale. Moreover, extreme weather events can disrupt crude oil production facilities, drizzling supply upsets and subsequent price escalations (Breitenfellner et al., 2009).

Expectations

Oil prices are influenced not only by prevailing price levels, demand, and supply dynamics but also by global expectations and investor sentiments regarding future trends in these indicators. For instance, between 2005 and 2008, production reports for non-OPEC countries consistently revealed lower production levels than forecasted, leading to an unforeseen surge in production by OPEC member nations. Consequently, this exerted pressure on their production capacity margins, causing upward tension in crude oil prices. The equilibrium between current and anticipated future prices constitutes a pivotal linkage between investors and trading entities engaged in futures contracts (Levin et al., 2014).

CHAPTER 3: LITERATURE REVIEW

Theoretical discussions on the interplay between crude oil prices and crucial economic factors also have significant practical implications. Numerous researchers, including Hamilton (1983), have argued that oil prices correlate with key economic factors. Hamilton's study, for instance, scrutinizes the interaction between WTI crude oil prices and crucial indicators of the American economy, such as Gross National Product (US GNP), unemployment rates, hourly earnings, and imported product prices. These discussions outline the indirect mechanisms through which crude oil prices influence economic activity, providing a theoretical framework that can inform real-world economic decisions.

In recent years, various methodologies have been utilized in research to forecast prices in electricity markets. These methodologies encompass a range of econometric models, such as Automatic Regressive Integrated Moving Average (ARIMA), Function Transfer, Artificial Neural Networks (ANN), Value-at-Risk, and Stochastic Linear Regression models like GARCH. The volatility of prices, particularly during specific periods, presents a significant challenge for techniques like the Fourier Transform and stochastic modelling (Garcia et al., 2005).

Saltik, Degirmen, and Ural (2016) investigated the spot price volatility of crude oil and other commodities, employing various formulations of the GARCH model. Specifically, they employed the GARCH, IGARCH, GJRGARCH, EGARCH, FIGARCH, and FAPARCH models. Their aim was to assess the accuracy of linear and non-linear asymmetric models in predicting volatility. The study showed that asymmetric and integrated models outperformed during two periods. Moreover, the FIGARCH model demonstrated ideal performance during the initial period, whereas the EGARCH model was deemed more suitable for capturing the volatility of both commodities in the subsequent period.

A study authored by Faith Wacuka Ng'ang'a and Meleah Oleche (2017) examines various volatility models utilized for forecasting crude oil price volatility. The research assesses the performance and efficacy of diverse GARCH, EGARCH, and IGARCH models in predicting oil price volatility, utilizing empirical analysis. The findings indicate that the IGARCH T-distribution model is the most effective in forecasting Brent crude oil price volatility and Value at Risk (VaR) estimations.

The study by He et al.(2018) illuminates the effectiveness of employing a multiscale analysis approach for forecasting crude oil risk. This has succeeded by investigating the dynamic interaction between crude oil price volatility and risk across various time scales. The results offer valuable insights for stakeholders, including practitioners and policymakers, engaged in managing risk within the oil market.

Yingying Xu and Donald Lien (2013) investigate the prediction of volatility in crude oil and gas assets by analyzing three distinct models: GAS (Generalized Autoregressive Score), GARCH (Generalized Autoregressive Conditional Heteroskedasticity), and EGARCH (Exponential GARCH). The research assesses the performance of these models in forecasting volatility within the context of the energy market through an empirical analysis.

Ahmed and Shabri's (2017) study enhances comprehension of the dynamics of crude oil prices by applying GARCH models to spot price data. The research delivers insights into the patterns and features of volatility in crude oil prices. The results emphasize the significance of utilizing suitable econometric models to capture precisely the complexities of crude oil price tendencies.

Furthermore, Kang and Yoon (2009) conclude that the asymmetric CGARCH and FIGARCH models are suitable for forecasting crude oil price volatility. Whereas Dritsaki (2018) found that the hybrid ARIMA-GARCH models provide optimal forecasting results.

Given these considerations, it becomes evident why methods for forecasting oil returns have not just evolved but have experienced rapid advancements in recent years. The data generated from these crude oil returns forecasts holds significant importance, particularly in informing governments and businesses' strategic decision-making and resource allocation. For instance, the European Central Bank (ECB) utilizes futures price data to forecast oil prices, a practice that significantly impacts inflation-related indicators (ECB, 2015). Similarly, global organizations such as the International Monetary Fund (IMF) and the Federal Reserve Board, at the forefront of innovation, rely on futures prices.

CHAPTER 4: FUNDAMENTAL CONCEPTS AND CHARACTERISTICS

4.1 DESCRIPTIVE STATISTICS OF THE DATA

The current stage represents the descriptive statistic characteristics used to process the data in the selected sample of the current research. Subsequently, various visual aids such as charts, graphs, and tables facilitate the presentation of statistical data. The tools employed in this process include statistical tables and formulas, which provide a comprehensive description of the essential characteristics of the data (Hatjinikolou, 2002).

The **arithmetic mean** is the quotient of the variable's set of values divided by the number of observations.

The **median** is the variable's value, which divides the population in half.

The **predominant value** is the value that exhibits the highest frequency within the sample.

The **range** is the difference between the maximum and minimum values.

Variance represents the mean of the squared differences between the values of our variable and their arithmetic mean. Variance is measured in squares and not in the same units as our variable:

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(X_i - \mu)^2 \qquad (1)$$

Standard deviation is the most critical measure of the distribution, i.e., the positive square root of the variance, with the difference being that it is estimated in the same measurement units as our variable.

Skewness quantifies the degree of asymmetry in the series distribution relative to its mean.

$$\alpha_3 = \frac{\frac{1}{N}\sum_{i=1}^{k} f(X_i - \mu)^3}{\sigma^3} \qquad (2)$$

Kurtosis is a metric that evaluates the curve's steepness in the distribution.

$$\alpha_{4=} \frac{\frac{1}{N}\sum_{i=1}^{k} f(X_i - \mu)^4}{\sigma^4} \qquad (3)$$

Jarque-Bera is a statistical test which checks whether the series is normally distributed. It measures the difference between the skewness and kurtosis of the series and those from the normal distribution. Calculated as:

$$Jarque - Bera = \frac{N}{6}({\alpha_3}^2 + \frac{(\alpha_4 - 3)^2}{4} \qquad (4)$$

4.2 TIME SERIES CHARACTERISTICS

Past experiences and predictions of forthcoming events typically inform an entity's economic functions. The significance of forecasting as a framework for making knowledgeable judgments in planning actions has been widely acknowledged. Future predictions rely heavily on historical statistical data. Time Series Analysis, a statistical methodology, gathers past data to forecast future trends. A time series represents a sequence of values of a variable observed over time, typically at regular intervals (Karageorgios et al., 1997).

Continues and Discrete Intertemporal Changes

The observations within a time series form an intertemporal variable, which can be either continuous or discrete. However, categorising a variable as discrete does not inherently imply temporal discontinuity. For instance, a country's population, categorised as a discrete variable, is recorded at all time points, thus generating a continuous variable over time. Conversely, a continuous variable may not always exhibit continuity about time. For instance, the quantity of a commodity, considered a continuous variable, becomes temporally discontinuous when the market is closed and prices are not recorded (Karageorgios et al., 1997).

Time Series Components

Continuous monitoring of time series data reveals that four components contribute to forming variable values. The first component is technological advancements, typically leading to increased output or population growth. Technological advancements are forces that also influence the trend of a time series. The second component is cyclical variation, inherent in all economic activities. Systematic and seasonal fluctuations around the trend line necessitate the observation of monthly, quarterly, weekly, and even daily data for analysis and inference. Finally, another component is random movement, characterised by unpredictable events or unknown factors, lacking any discernible pattern (Karageorgios et al., 1997).

Stationarity

A fundamental concept in time series analysis involves distinguishing between stationary and non-stationary data. This distinction is not just a technicality but a crucial factor that can significantly impact the accuracy and reliability of our analysis. Stationary data refer to those with a consistent average level over time, while non-stationary data exhibit variability over time. Autocorrelation signifies the coefficient measuring the correlation between two time series elements at different time intervals. However, autocorrelation is meaningful only within the context of stationary time series (Box et al., 2008).

4.3 FORECAST

All companies need forecasts to reduce risk and uncertainty about the future. Returns forecasting is a process used to make predictions based on past and present data. Forecasts are never accurate, and thus, short-term forecasting is preferred because we cannot accurately calculate the variables that affect returns in the long term. Long-term forecasts are affected by random factors, such as the recent pandemic.

There are two forecasting methods: quantitative and qualitative methods. The qualitative method deals with estimation using expert judgment.

On the other hand, the quantitative method uses numerical analysis. This method gets information from various consultants and experts about future results. In contrast, the quantitative forecasting method collects and analyses historical data to infer future trends. In this thesis, we are more interested in the quantitative method.

4.3.1 QUANTITATIVE METHODS OF FORECASTS

Quantitative methods necessitate the fulfilment of the following three conditions:

1. They rely on information retrievable from past data.
2. The information must be amenable to numerical representation.
3. It assumes that the underlying data pattern will persist into the future.

Two primary types of models are employed in quantitative methods: explanatory models, which seek to establish relationships between multiple variables, and time series models, which utilize historical trends to forecast future outcomes (Hyndman, 1998). This thesis focuses on analyzing forecasts derived from time series models.

4.3.2 BASIC STEPS IN THE FORECAST PROCESS

According to Hyndman (1998) and Hymans (2020), there are six basic procedures we follow for forecasting.

1. Identification of the problem: Problem identification involves determining the recipients of this service, its potential utility, and the specific manner in which the process can be beneficial.
2. Gathering information: Information gathering entails acquiring numerical data from historical records to facilitate prediction. It also involves incorporating human judgment as a valuable component.
3. Preliminary information analysis: Preliminary information analysis involves examining the data collected using tools such as graphs and statistical indicators like the mean and dispersion. These techniques assess the stability of observed patterns and identify outliers. This process is crucial as it informs the subsequent stage of model selection.
4. Selection of forecast model: Based on the findings identified in stage three, we determine the most suitable model for the analysis.
5. Analysis of the information: Our data analysis involves examining the data through the model chosen at a prior stage.
6. Model evaluation and efficiency assessment: Upon completion of the preceding stages, this phase involves comparing the obtained values with the actual data to identify potential adjustments to the model in instances where significant disparities are observed.

4.3.3 EVALUATION OF FORECASTS

According to Howrey et al. (1991), researchers highlight the importance of selecting a suitable model for enhancing the accuracy of forecasts. Various methods are available to assess the performance of a model. Initially, we will address the prediction error, which can be expressed mathematically as follows:

$$\hat{\varepsilon}_t = y_t - \hat{Y}_t \quad (5)$$

Where $\hat{\varepsilon}_t$: estimated error in time t

Y_t: observed value at time t

$\hat{y}_t$: estimated price in time t

Measuring Forecast Accuracy

The prevalent measures include:

Mean absolute error: $MAE = \frac{1}{n}$ (6)

$\sum_{t=1}^{n} |E_t|$ (7)

Root mean square error: $RMSE = \sqrt{\frac{\sum_{t=1}^{n} Et^2}{n}}$ (8)

The mean absolute error (MAE) is relatively straightforward to comprehend; however, it fails to account for extreme errors and does not provide information regarding the direction of the errors. MAE represents the average absolute difference between actual and estimated values. Conversely, the mean squared error (MSE) is commonly used in estimations despite being more challenging to interpret. Root mean squared error (RMSE) quantifies the magnitude of the error, calculated as the square root of the average of the squared differences between actual and estimated values.

MAE and RMSE differences

The mean absolute error (MAE) is relatively straightforward to comprehend; however, it fails to account for extreme errors and does not provide information regarding the direction of the errors. MAE illustrates the average absolute difference between actual and estimated values. Conversely, the mean squared error (MSE) is commonly used in estimations despite being more challenging to interpret. Root mean squared error (RMSE) quantifies the magnitude of the error, calculated as the square root of the average of the squared differences between actual and estimated values.

Percentage Error

The percentage error serves as a metric for evaluating the predictive performance across the dataset.

Mean absolute percentage error: $$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{E_t}{y_t}\right| 100\% \qquad (9)$$

The Mean Absolute Percentage Error (MAPE) represents the average absolute error expressed as a percentage. Like the previously mentioned methods, MAPE does not indicate the approach of errors. This method is independent of the scale of the data.

Scaling Errors

This approach, introduced by statisticians Rob J. Hyndman and Anne B. Koehler in 2006, offers an alternative to the MAPE method for evaluating the precision of forecasts in a series. It encloses two variants: one applicable to time series indicating seasonality and the other suitable for those devoid of seasonality. The term expresses the data's independent scale.

The term ε_i denotes the data's independent scale.

Absence of seasonality: $$\varepsilon_i = \frac{\mu i_i}{\frac{1}{T-1}\sum_{T=2}^{T}|Y_T - y_{T-1}|} \quad (10)$$

Existence of seasonality: $$\varepsilon_i = \frac{\mu i_i}{\frac{1}{T-M}\sum_{T=M+1}^{T}|Y_T - y_{T-M}|} \quad (11)$$

The mean absolute standard error is:

$$MASE = mean(|\varepsilon_i|) \quad (12)$$

This metric offers forecasting accuracy without the limitations observed in other metrics. It is preferred over the aforementioned methods due to its independence from the data scale, symmetry in handling negative and positive forecast errors, and alignment with the Diebold-Mariano (DM) method. The DM method aids in selecting the optimal prediction method and identifying the most minor measurement error. Specifically, it facilitates a more profound analysis by assessing the significance of numerical differences.

Theil index

$$U = \frac{RMSE}{\sqrt{\frac{1}{n}\sum_{t=1}^{n}(Yt)^2 + \frac{1}{n}\sum_{t=1}^{n}(\widehat{Yt})^2}} \quad (13)$$

The Theil index is a statistical tool for assessing whether a time series of estimated values aligns with a time series of observed values. When the resulting numerical value "U" is closer to 0, it indicates higher prediction accuracy, whereas if it is approaching 1, it signifies greater prediction inaccuracy.

CHAPTER 5: INTERPRTATIVE METHODOLOGY

Econometric Software: Eviews 9

The econometric analysis in this study utilized the statistical software Eviews 9. Eviews is a well-known tool for analyzing time series data. Eviews 9 offers a comprehensive array of tools for both basic statistical analysis and advanced econometric modelling. Moreover, it enables the creation of econometric models and facilitates forecasting without complex commands, requiring only familiarity with the analytical procedures for conducting our analysis.

This chapter is dedicated to enlightening the key concepts crucial for understanding the modelling and forecasting of stochastic time series. Stochastic processes delineate the arrangement of observations within a sequence through a model. Specifically, we employ stochastic models grounded in the attributes of white noise, assuming that the time series under examination originates from independent residuals.

After that, the Box-Jenkins methodology, renowned for its effectiveness in identifying and constructing suitable models for modelling and forecasting time series, is outlined below. A significant aspect of model development involves the assumption of stationarity regarding the time series under scrutiny.

Unit Root Test

Verifying whether the time series exhibits stationarity is compulsory before investigating regression or forecasting models in time series analysis. There are two primary methods for diagnosing stationarity. The first involves subjective judgment, while the second utilizes statistical tests to detect the presence of a unit root.

Statistically, a time series is deemed stationary when its key descriptors—such as mean, variance, covariance, and standard deviation—remain constant over time. Should a time series lack stationarity, regression outcomes become unreliable and nonsensical. Thus, it becomes essential to explore potential transformations that induce stationarity.

One of the most well-known tests for identifying the presence of a unit root is the Dickey-Fuller (DF) test, along with its augmented counterpart (ADF). ADF holds greater power as it can handle more intricate models than DF. Another test based on DF is the Phillips-Perron test, which employs a similar estimation approach but adjusts for autocorrelation and dynamic heteroscedasticity in statistical behaviour.

The ADF test was employed in the paper's econometric analysis. Dickey and Fuller, in their Monte Carlo experiments, identified a suitable skewed distribution for testing the hypothesis H_0 : ρ = 1. While the DF test employs the t-student distribution, accepting or rejecting H_0 is assessed using MacKinnon's critical values. The MacKinnon method has been integrated into Eviews 9 software (MacKinnon, 2002).

The following assumptions make the Dickey-Fuller tests for unit root:

$$H_0\text{: } the\ time\ series\ is\ not\ stationary(epresence\ of\ a\ unit\ root)$$

$$H_1\text{: } the\ time\ series\ is\ stationaty\ (absence\ of\ a\ unit\ root)$$

Once the test results are obtained, it is crucial to examine both the t-statistics and p-values. If the t-statistics value falls below the critical values at the chosen significance levels, and the significance levels surpass the p-value, we reject the null hypothesis (H_0) and accept the alternative hypothesis (H_1) that the time series is stationary. Conversely, if the p-value exceeds the significance levels, the time series contains a unit root, indicating instability.

Identification

After conducting the stationary test for the time series, the initial step of the Box-Jenkins methodology ensues. This step involves identifying an integrated autoregressive moving average model, denoted as ARIMA(p,d,q) (Johnston, 1997).

d = the number of times the raw observations are differenced to allow the time series to become stationary.

p = the number of lag observations in the model

q = the order of the moving average

The parameter d typically assumes values of 0, 1, or, in uncommon instances, 2. In this study, d is set to 0 as the time series is confirmed to be stationary. Once stationarity is established, it becomes imperative to investigate trends and seasonality by examining autocorrelation and partial autocorrelation. Each ARMA(p,q) model is associated with a distinct pair of autocorrelation function (ACF) and partial autocorrelation function (PACF) that determine the model's order. Searching for appropriate values of p and q involves a three-step process, as Anderson (1977) outlined, defining our ARIMA model. Presented below is an analysis of fundamental stationary processes.

Autoregressive models AR(p)

$$y_t = \mu + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \cdots + \alpha_p y_{t-p} + u_t \qquad (14)$$

The autoregressive process AR(p) is a statistical model that predicts future values based on past values. Autoregressive models acknowledge the influence of past values on present ones, making this statistical method widely used to analyse dynamic phenomena in natural, economic, and other temporal processes. Within the AR(p) model, the terms exhibit a consistent variance and a zero mean. The parameter p signifies the order of the autoregressive process, dictating the duration of the lag, with y_{t-1} ,y_{t-2},…,y_{t-p} representing the lagged values of the time series.

Moving Average MA(q)

The Moving Average processes take the following format:

$$y_t = \mu + u_t - \theta_1 u_{t-1} - \theta_2 u_{t-2} - \cdots - \theta_q u_{t-q} \qquad (15)$$

Determining the moving average of a commodity assists in stabilizing the return data by establishing an ongoing average return. Consequently, this reduces the influence of random, short-term fluctuations on the returns within a designated timeframe. Here, the variable y_t is contingent on q lags of u_t. The Moving Average Process maintains its stationarity as it's defined as the finite summation of white noise terms. White noise represents a time series comprising independent random variables.

Autoregressive Integrated Moving Average Prediction Model ARIMA(p,d,q)

An autoregressive integrated moving average model functions as a regression analysis technique aimed at assessing the relationship between a dependent variable and other fluctuating variables. Its objective is to forecast future movements in securities or financial markets by scrutinizing the variances within the series rather than focusing solely on the actual values.

Estimation

The second phase of the Box-Jenkins methodology entails the estimation of parameters for the autoregressive (AR) and moving average (MA) models. The prevailing techniques for parameter estimation are least squares and maximum likelihood methods, both relying on the examination of autocorrelation and partial autocorrelation functions within the series. In evaluating the estimation outcomes, it is crucial to consider the R-squared value, indicating the proportion of estimated values dependent on the analyzed time series.

Diagnostic verification

Considering the accuracy limitations of the models, it is crucial to emphasise the significance of utilising diagnostic tests to validate the suitability of specimens for analysis. Following the identification and estimation processes, an estimated ARMA model is generated, and the next step is the diagnostic test of the equation obtained from the previous steps. Consequently, aggregate assessments are administered to scrutinise the coefficients of the model. These assessments evaluate the statistical significance and stability of the coefficients, examining the residuals' properties and assessing the model's predictive efficacy.

Autocorrelation Test

Johnston and DiNardo (1997) claim that the tests aim to estimate the extent to which the residuals exhibit some autocorrelation. Autocorrelation refers to a similarity between a specific time series and a delayed iteration of itself across consecutive periods. It quantifies the association between a variable's present value and its prior values. Analysts utilize autocorrelation to assess the extent to which historical returns fluctuations influence forthcoming returns dynamics, providing investors with insights to anticipate future returns trends.

The Box-Pierce-Ljung autocorrelation test

The Ljung-Box test is a method for determining the presence of serial autocorrelation up to a designated lag. This test relies on the squares residuals and aims to prove whether the residuals are independent and identically distributed, akin to white noise. Fundamentally, it measures model adequacy: minimal autocorrelation in the residuals suggests a 'lack of significant fit'.

In the analysis conducted using the statistical software Eviews 9, the diagnostic procedure incorporates the Ljung-Box test. The Box-Pierce-Ljung autocorrelation test evaluates the following hypotheses:

$$H_0: absence\ of\ autocorrelation$$

$$H_1: existance\ of\ autocorrelation$$

In the results, we observe the Q-statistic and p-value values at the level of significance we are interested in. If the p-value is less than the level of significance, the null hypothesis H_0 is rejected, and the hypothesis H_1 is accepted; therefore, the sample shows autocorrelation.

Normality Test

Numerous methods exist to evaluate a distribution's normality, including the Jarque-Bera test, which is commonly employed to verify normality. This test assesses the conformity of the data's skewness and kurtosis to those of a normal distribution, which has a skewness of zero and a kurtosis of three.

Heteroskedasticity Test

According to Johnston and DiNardo (1997), testing for heteroscedasticity becomes desirable when least squares estimators are no longer efficient. Heteroskedasticity specifies nonconstant volatility related to the prior period's volatility. This violates the assumptions for linear regression modelling and can impact the validity of econometric analysis.

The ARCH—LM heteroscedasticity control of the ARCH family has been analyzed to detect the presence of dynamic heteroscedasticity in the existing analysis.

The ARCH – LM test defines as:

$$H_0 = absence\ of\ heteroskedasiticy\ (homoskedastic\ residuals)$$

$$H_1 = existance\ of\ heteroskedasticity$$

Within the findings, we discern the F-statistic and Obs*R-squared figures. Should the p-value fall below the predetermined significance level, the null hypothesis H_0 is rejected, leading to the acceptance of the alternate hypothesis H_1 , which indicates heteroscedasticity within the sample. Consequently, it necessitates identifying, estimating, and diagnosing procedures to ascertain the suitable model within the ARCH family (Autoregressive Conditional Heteroscedasticity Model).

GARCH (p,q) Model

The Generalized ARCH (p,q) model is of the form:

$$y_t = \mu + \varepsilon_t \quad (16)$$

$$\varepsilon_t \,\middle|\, I_{t-1} \sim D(0, \sigma_t^2) \quad (17)$$

Where, $\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \cdots + \alpha_q \varepsilon_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 \sigma_{t-2}^2 + \cdots + \beta_p \sigma_{t-p}^2$ (18)

Generalized AutoRegressive Conditional Heteroskedasticity (GARCH) represents a statistical modelling approach employed in forecasting the volatility of returns concerning financial assets or commodities. It scrutinizes time series data in which the variance error is suspected to exhibit serial autocorrelation. This means that the variance of the error term is not constant, signifying dynamic heteroskedasticity. This term delineates the unpredictable variation in an error term or variable within a statistical model. A variant of the GARCH model is the EGARCH (p,q).

EGARCH (p,q) Model

The exponential general autoregressive conditional heteroskedastic (EGARCH) model presents an alternative variation of the GARCH framework. Introduced by Nelson (1991), the EGARCH model addresses a limitation in GARCH's treatment of financial time series, explicitly aiming to accommodate asymmetric effects evident between positive and negative asset returns. A notable advantage of the EGARCH model lies in its capability to capture the differential impact of positive and negative changes within the series on volatility, a feature absent in the traditional GARCH model. Unlike the GARCH model, which illustrates the conditional variance solely as a function of the squared values of past innovations, the EGARCH model allows for considering both positive and negative changes.

T-GARCH (p,q) Model

The threshold GARCH (TGARCH) model is a volatility model frequently employed to address leverage effects. This model characterizes the conditional variance through a linear function operating across distinct intervals or sets. TGARCH is predicated on the assumption of independent and identically distributed (IID) innovations and examines how negative and positive returns influence the dynamics of conditional volatility.

APGARCH (p,q) Model

The APARCH or APGARCH(p,q) model is an adaptation of the T-GARCH model, incorporating asymmetry in return volatility. It demonstrates that volatility tends to escalate to a greater extent in response to negative returns compared to positive returns of equivalent magnitude. Similar to the GARCH model, the APARCH model encapsulates stylised facts observed in time series, such as volatility clustering, where high volatility at time t is more probable if it was also high at time $t-1$.

Following the estimation of the GARCH family of models, the subsequent phase involves model selection for forecasting purposes. Two widely utilized criteria for evaluating models are the Akaike Information Criteria (AIC) and the Schwarz Information Criteria (SIC). The selection of the most appropriate model entails a comparison of these criteria, with preference given to the model exhibiting the most minor indices. Additionally, it is imperative to conduct checks for dynamic heteroscedasticity. These checks serve to confirm the absence of autocorrelation and dynamic heteroscedasticity, thereby facilitating accurate forecasts. Furthermore, the three phases of the Box-Jenkins methodology have been executed, along with the development of four hybrid models, namely ARMA-GARCH and ARMA-EGARCH, as well as ARMA-T-GARCH and ARMA-APGARCH models.

Forecasting

Forecasting entails utilizing historical data as inputs to formulate informed estimations, which serve as predictive tools for discerning future trends. Following the assessment and selection of the most suitable model, the prediction of future values for the time series ensues. The forecasting process was executed by utilizing observations from the in-sample period. The comprehensive forecasting output provided by Eviews 9.0 includes graphical representations of returns and variance forecasts and a reliability assessment of the model's predictive capacity. Notably, key performance indicators such as the mean absolute error (MAE), root mean squared error (RMSE), mean absolute percentage error (MAPE), and the Theil inequality coefficient serve as crucial metrics for evaluating forecasting accuracy, The aforementioned indicators were elaborated in greater detail in the preceding section.

CHAPTER 6: ECONOMETRIC ANALYSIS

6.1 DATA AND DESCRIPTIVE STATISTICS

The analysis includes weekly Brent crude oil prices, retrieved from the Federal Reserve Bank of St. Louis (FRED). U.S. Energy Information Administration (EIA) calculated weekly prices from daily data by taking the average of the daily closing prices for a given comodity over the specified time. On a separate note, the missing values of the sample have been treated with the "End of Period Aggregation Method". When converting from daily to weekly values, there might be a missing value due to public holidays; the end-of-period aggregation method will use the value of the day before and repeat it for the day with the missing value. If both Thursday and Friday had missing values, the end-of-period aggregation method would use the value from Wednesday.

More specifically, the data belong to 29/12/2014 – 31/12/2019 and have been divided into two sections. The first part belongs to the period 29/12/2014 - 31/12/2018, is called the in-sample period and serves the estimation process of the model, while the second part belongs to the period 01/01/2019 - 31/12/2019 is called -the of-sample period and serves to forecast oil returns. In total, the sample consists of 262 observations.

The data range has been chosen based on the fact that forecasts inherently entail uncertainty, as unforeseen events may transpire. A recent example is the unforeseen onset of the coronavirus pandemic, which exerted unprecedented effects on various sectors, notably leading to a significant decline in Brent crude oil returns. Such events underscore the inherent limitations of forecasting accuracy. Moreover, the pandemic-induced shock in demand significantly impacted crude oil markets. In light of this the chosen data sample stops 31 December 2019 as a safeguard to avoid misspecification of the model when we ignore the presence of this type of data.

The following results have been calculated with the help of the statistical program Eviews 9.0.

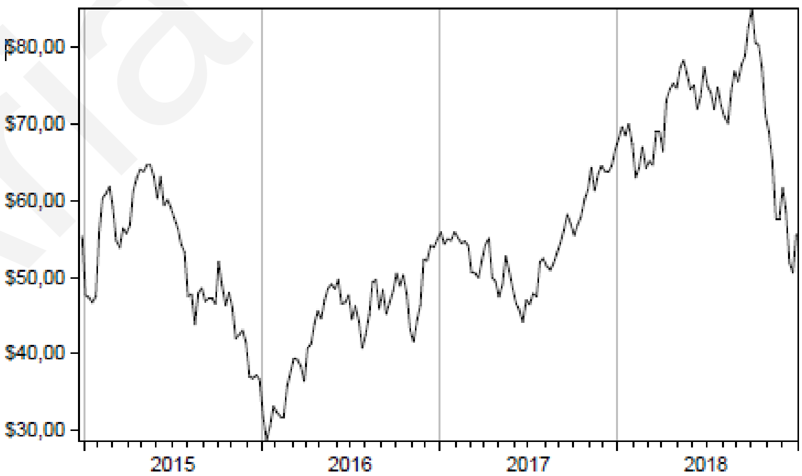


Figure 1: Graph of the weekly Brent Crude Oil spot prices 31/12/2014 - 31/12/2018

The data used is from 01 January 2015 until 31 December 2018. Looking at the chart above, we see a wide range of fluctuations in oil prices. Between mid-2014 and early 2016, the global economy faced one of modern history's most significant oil price declines.

The initial drop in oil prices from mid-2014 to early 2015 was driven by supply factors, including booming U.S. oil production, receding geopolitical concerns, and shifting OPEC policies. However, declining demand also played a noteworthy role from 2015 until the first months of 2016. Rather than raising global growth, the oil price drop was accompanied by a deceleration in 2015 and 2016. A sharp downshift in oil-exporting economies dragged global economic activity down (World Oil Market Chronology From 2003, 2023).

According to the Global Economic prospect performed by the World Bank Group, in December 2015, Brent crude oil fell as low as $36.35 a barrel, the lowest price since summer 2004. OPEC countries met on November 30 and agreed to limit crude oil output for the first time since 2008. As a result, Brent crude oil went over $50, the highest in a month. Behind a similar agreement to limit production between Russia and other countries not part of OPEC, Brent crude oil prices increased. Despite other countries' promises of lower output, evidence of changes still needed to be seen. As a result, Brent crude oil prices fluctuated around the same levels until January 2017. Brent crude oil prices rose 40% from June to October 2017 as oil producers were expected to continue lower production, with an increase of 20% in the third quarter. The increase would have been more, but Turkey did not act on a thread against Kurdistan's vote for independence (World Oil Market Chronology From 2003, 2023).

During the last week of 2017, Brent crude oil prices passed $67 for the first time since May 2015 due to pipeline problems in Libya and the North Sea, which led to production cuts by OPEC and Russia. In January 2018, U.S. production increased, and demand was predicted to go down when winter was over. During 2018, Brent crude oil prices were also affected by threats to supply from Libya and proposed sanctions on countries importing oil from Iran. At the end of 2018, higher U.S. interest rates, more active U.S. oil rigs, higher U.S. crude production, and lower expected worldwide demand did not cancel out proposed production cuts by OPEC nations; therefore, Brent crude oil prices went down 20 per cent

(Global Economic Prospects, January 2018: Broad-Based Upturn, but for How Long?, 2017)

The modelling process requires the calculation of excess returns of Brent crude oil spot prices. The computation is given below:

$$return_t = (\log(price_t) - (\log(price_{t-1}))) * 100 \qquad (19)$$

Where $price_{t-1}$ is the stock price of the previous period, and $price_t$ is the stock price of the current period. The equation $\log(price_t) - (\log(price_{t-1}))$ defines the percentage price change. Thus, $return_t$ is the percentage return. The econometric analysis of this thesis is based on the percentage returns of the weekly Brent crude oil prices.

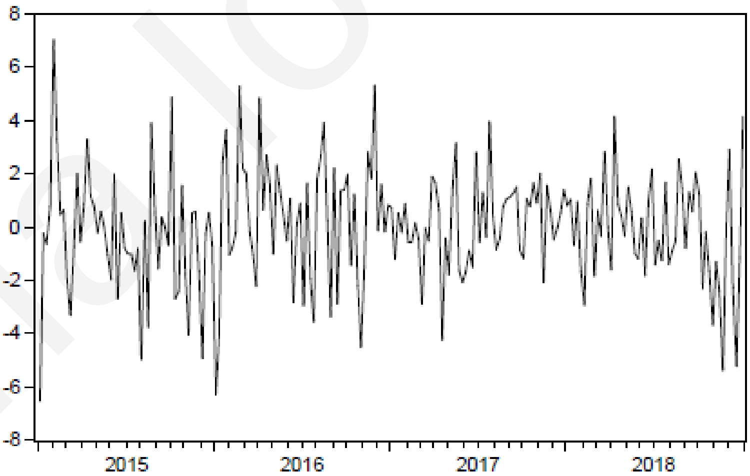


Figure 2: Graph of the weekly Brent Crude Oil returns 31/12/2014 - 31/12/2018

Figure 2 shows the graph of the percentage returns of the weekly prices of Brent crude oil for the period 1 January 2015 to 31 December 2018.

Mean	0.000973
Median	0.006713
Maximum	7.037997
Minimum	-6.538120
Std. Dev.	2.157921
Skewness	-0.120534
Kurtosis	3.764995
Jarque-Bera	5.602341
Probability	0.060739
Sum	0.203417
Sum Sq. Dev.	968.5779
Observations	209

Table 1: Descriptive Statistics

Table 1 presents the descriptive statistics of the return time series of the weekly returns of Brent crude oil. Based on these statistics, the mean and median are close in value, which suggests the data is roughly symmetric. The skewness is slightly negative (-0.1205), indicating a slight left skew. The kurtosis is higher than 3 (3.765), indicating slightly heavier tails compared to a normal distribution. Then, we interpret the value of the standard deviation. Standard deviation compares each data point to the mean of all data points, describing whether the data points are nearby or spread out. Outliers have a heavier impact on standard deviation. Any standard deviation above or equal to 2 can be considered high, which means that in this analysis, the time series data are spread out. While the skewness and kurtosis are not far from zero and three, respectively, suggesting some deviation from normality, it's important to remember that normality is a matter of degree. Depending on your specific context and requirements, this distribution may be considered approximately normal for many practical purposes, especially if the deviations are not extreme. However, for rigorous statistical analysis, it's often advisable to conduct formal normality tests.

Therefore, to assess whether the series is normally distributed, we can utilize the Jarque-Bera test, which is a test of the null hypothesis that the data follows a normal distribution based on skewness and kurtosis. The test statistic Jarque-Bera and its associated p-value are provided:

Jarque-Bera: 5.602341

Probability (p-value): 0.060739

Typically, if the p-value is less than a chosen significance level (e.g., 0.05), we reject the null hypothesis, indicating that the data significantly deviates from a normal distribution.

In this case, the p-value is approximately 0.0607, which is greater than the typical significance level of 0.05. Therefore, we fail to reject the null hypothesis at the 0.05 significance level. This suggests that, based on the Jarque-Bera test, there is not strong evidence to conclude that the series significantly deviates from a normal distribution.

Next follows the Box–Jenkins methodology through which we will develop an ARIMA model. The method is carried out based on the following steps:

First step: Identification

Before the modelling process begins, the unit root test is executed by performing the Augmented Dickey-Fuller (ADF) test to determine whether a unit root exists in the time series of the analysis.

Null Hypothesis: DCOILBRENTEU has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=15)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.629663	0.4660
Test critical values:	1% level	-3.455289	
	5% level	-2.872413	
	10% level	-2.572638	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(DCOILBRENTEU)
Method: Least Squares
Date: 05/14/24 Time: 19:49
Sample (adjusted): 1/11/2015 1/05/2020
Included observations: 261 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
DCOILBRENTEU(-1)	-0.022162	0.013599	-1.629663	0.1044
C	1.318580	0.792346	1.664147	0.0973

R-squared	0.010150	Mean dependent var	0.052490
Adjusted R-squared	0.006328	S.D. dependent var	2.522973
S.E. of regression	2.514977	Akaike info criterion	4.690038
Sum squared resid	1638.204	Schwarz criterion	4.717352
Log likelihood	-610.0499	Hannan-Quinn criter.	4.701017
F-statistic	2.655803	Durbin-Watson stat	1.796343
Prob(F-statistic)	0.104388		

Table 2: Augmented Dickey-Fuller Stationarity test performed for Brent crude oil prices

Null Hypothesis: DCOILBRENTEU LOGRETURN has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=14)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-13.02731	0.0000
Test critical values:	1% level	-3.461783	
	5% level	-2.875262	
	10% level	-2.574161	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(DCOILBRENTEU LOGRETURN)
Method: Least Squares
Date: 02/03/24 Time: 17:35
Sample (adjusted): 1/12/2015 12/31/2018
Included observations: 208 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
DCOILBRENTEU LOGRETURN(-1)	-0.889710	0.068296	-13.02731	0.0000
C	0.034504	0.146061	0.236227	0.8135

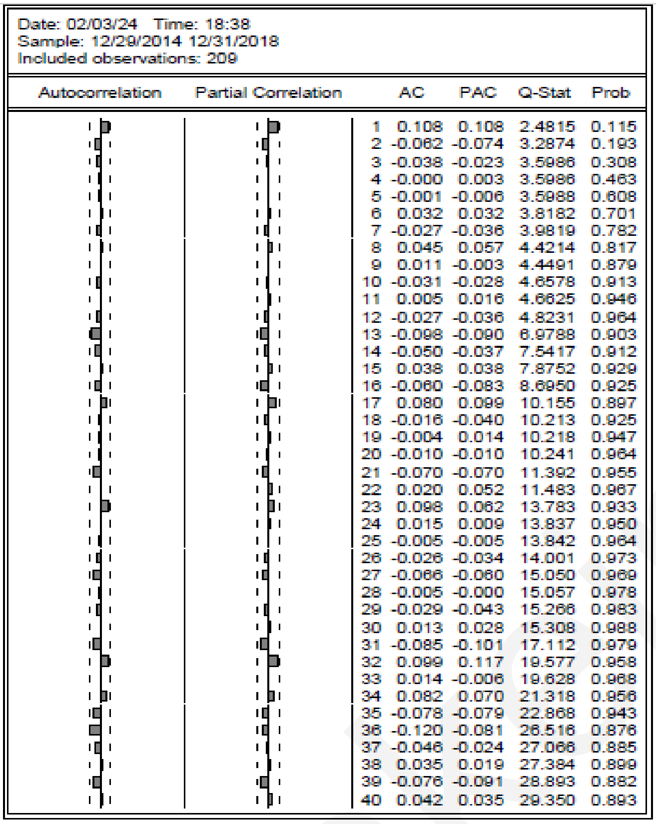
R-squared	0.451706	Mean dependent var	0.051382
Adjusted R-squared	0.449044	S.D. dependent var	2.837860
S.E. of regression	2.106441	Akaike info criterion	4.337445
Sum squared resid	914.0412	Schwarz criterion	4.369537
Log likelihood	-449.0943	Hannan-Quinn criter.	4.350422
F-statistic	169.7107	Durbin-Watson stat	1.956368
Prob(F-statistic)	0.000000		

Table 3: Augmented Dickey-Fuller Stationarity test performed for Brent crude oil returns

In accordance with the Box-Jenkins process, we should focus on testing the stationarity of the time series we intend to model. We will begin by testing whether the time series of prices is stationary. Table 2 represents the first attempt at the Augmented Dickey-Fuller Stationarity test performed for Brent crude oil prices. Based on the results, we identify the existence of the unit root in the time series of Brent crude oil prices. This non-stationarity is often due to inflation or other structural breaks.

Consequently, we need to convert the data to fulfil the stationarity criterion. A common transformation is differencing the prices and generating the returns of the commodity tested. Therefore, in Table 3, we examine the stationarity of the time series of Brent crude oil returns. Upon the testing, we determined that the null hypothesis, denoted as H_0 , which posits the existence of stagnation, is rejected at the 5% significance level. Specifically, the probability value (0.0000), being lower than the 0.05 significance level, coupled with the t-statistic value (-13.02731), falling below the Test Critical Values corresponding to the 1%, 5% and 10% significance levels, leads to the conclusion that the time series can be deemed stationary at significance levels of 1%, 5%, and 10%.

With the attainment and validation of time series stationarity, we are poised to ascertain the p, d, q values essential for defining the ARMA model. Initially, leveraging Figure 3, we will discern the value of p, signifying the autoregressive (AR) process order, by scrutinizing the autocorrelation function (ACF) values. Subsequently, we will reduce the value of q, representing the moving average (MA) process order, through examination of the partial autocorrelation function (PAC) values.



Date: 02/03/24 Time: 18:38
Sample: 12/29/2014 12/31/2018
Included observations: 209

	AC	PAC	Q-Stat	Prob
1	0.108	0.108	2.4815	0.115
2	-0.062	-0.074	3.2874	0.193
3	-0.038	-0.023	3.5986	0.308
4	-0.000	0.003	3.5986	0.463
5	-0.001	-0.006	3.5988	0.608
6	0.032	0.032	3.8182	0.701
7	-0.027	-0.036	3.9819	0.782
8	0.045	0.057	4.4214	0.817
9	0.011	-0.003	4.4491	0.879
10	-0.031	-0.028	4.6578	0.913
11	0.005	0.016	4.6625	0.946
12	-0.027	-0.036	4.8231	0.964
13	-0.098	-0.090	6.9788	0.903
14	-0.050	-0.037	7.5417	0.912
15	0.038	0.038	7.8752	0.929
16	-0.060	-0.083	8.6950	0.925
17	0.080	0.099	10.155	0.897
18	-0.016	-0.040	10.213	0.925
19	-0.004	0.014	10.218	0.947
20	-0.010	-0.010	10.241	0.964
21	-0.070	-0.070	11.392	0.955
22	0.020	0.052	11.483	0.967
23	0.098	0.062	13.783	0.933
24	0.015	0.009	13.837	0.950
25	-0.005	-0.005	13.842	0.964
26	-0.026	-0.034	14.001	0.973
27	-0.066	-0.060	15.050	0.969
28	-0.005	-0.000	15.057	0.978
29	-0.029	-0.043	15.266	0.983
30	0.013	0.028	15.308	0.988
31	-0.085	-0.101	17.112	0.979
32	0.099	0.117	19.577	0.958
33	0.014	-0.006	19.628	0.968
34	0.082	0.070	21.318	0.956
35	-0.078	-0.079	22.868	0.943
36	-0.120	-0.081	26.516	0.876
37	-0.046	-0.024	27.066	0.885
38	0.035	0.019	27.384	0.899
39	-0.076	-0.091	28.893	0.882
40	0.042	0.035	29.350	0.893

Matrix 1:Graphical representation of ACF and PAC) of the time series

After conducting an exploratory analysis, during which alternative models were evaluated, it has been determined that the ARMA (36,0) and ARMA (0,1) models are the most effective. To substantiate this conclusion, the subsequent step in the Box Jenkins methodology, namely Estimation, is pursued.

Second step: Estimation

This step will estimate the p parameters of the autoregressive model and the q parameters of the moving average model. With the help of the EViews 9.0 statistical program, we have estimated two models and got the following results:

Dependent Variable: RETURNS
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 03/02/24 Time: 13:10
Sample: 1/05/2015 12/31/2018
Included observations: 209
Convergence achieved after 4 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.000478	0.169135	-0.002827	0.9977
MA(1)	0.131885	0.075741	1.741261	0.0831
SIGMASQ	4.567421	0.392406	11.63952	0.0000

R-squared	0.014441	Mean dependent var	0.000973
Adjusted R-squared	0.004872	S.D. dependent var	2.157921
S.E. of regression	2.152658	Akaike info criterion	4.385618
Sum squared resid	954.5909	Schwarz criterion	4.433594
Log likelihood	-455.2971	Hannan-Quinn criter.	4.405015
F-statistic	1.509185	Durbin-Watson stat	1.961462
Prob(F-statistic)	0.223524		
Inverted MA Roots	-.13		

Table 4: Estimation Equation MA(1) model

Dependent Variable: RETURNS
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 03/02/24 Time: 13:07
Sample: 1/05/2015 12/31/2018
Included observations: 209
Convergence achieved after 6 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.014599	0.132117	0.110497	0.9121
AR(36)	-0.153041	0.075785	-2.019400	0.0447
SIGMASQ	4.530784	0.384455	11.78494	0.0000

R-squared	0.022346	Mean dependent var	0.000973
Adjusted R-squared	0.012854	S.D. dependent var	2.157921
S.E. of regression	2.144007	Akaike info criterion	4.381563
Sum squared resid	946.9339	Schwarz criterion	4.429539
Log likelihood	-454.8733	Hannan-Quinn criter.	4.400960
F-statistic	2.354260	Durbin-Watson stat	1.752281
Prob(F-statistic)	0.097514		

Inverted AR Roots	.95+.08i	.95-.08i	.92+.25i	.92-.25i
	.86-.40i	.86+.40i	.78+.54i	.78-.54i
	.67-.67i	.67+.67i	.54-.78i	.54+.78i
	.40-.86i	.40+.86i	.25+.92i	.25-.92i
	.08+.95i	.08-.95i	-.08-.95i	-.08+.95i
	-.25+.92i	-.25-.92i	-.40-.86i	-.40+.86i
	-.54-.78i	-.54+.78i	-.67+.67i	-.67+.67i
	-.78-.54i	-.78+.54i	-.86-.40i	-.86+.40i
	-.92+.25i	-.92-.25i	-.95+.08i	-.95-.08i

Table 5: Estimation Equation AR(36) model

The estimation outputs of the MA(1) and AR(36) models are sequentially presented, revealing that, with the exception of the constant coefficient c, all other parameters exhibit statistical significance at significance levels of 10% and 5%, respectively, leading to rejection of the null hypothesis. Subsequently, the models are compared utilizing three information criteria: Akaike, Hannan-Quinn, and Schwartz. Based on the aforementioned results, the model with the minimal value across these criteria in Tables 4 and 5 is selected. It is noted that all three criteria consistently advocate for the AR(36) model.

Following the model estimation, the analysis proceeds with the final phase of the Box-Jenkins methodology.

Third step: Diagnostic verification

In this phase, we assess the appropriateness of the model for our dataset. Specifically, we conduct a diagnostic check to ascertain the presence of autocorrelation in the residuals and to evaluate whether they adhere to a white noise process. This evaluation is facilitated through the application of the Ljung-Box test, which examines the presence of autocorrelation (serial correlation) across the entire set of autocorrelations in the sample. Here, the null hypothesis H_0 posits the absence of autocorrelation, while the alternative hypothesis H_1 suggests the presence of autocorrelation.

Date: 02/11/24 Time: 20:03
Sample: 12/29/2014 12/31/2018
Included observations: 208

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.414	-0.414	36.201	0.000
		2	-0.043	-0.258	36.584	0.000
		3	-0.036	-0.213	36.856	0.000
		4	0.034	-0.123	37.101	0.000
		5	-0.033	-0.124	37.330	0.000
		6	0.061	-0.022	38.138	0.000
		7	-0.086	-0.102	39.733	0.000
		8	0.050	-0.043	40.283	0.000
		9	0.002	-0.018	40.285	0.000
		10	-0.036	-0.062	40.565	0.000
		11	0.042	0.002	40.965	0.000
		12	0.015	0.030	41.012	0.000
		13	-0.063	-0.038	41.914	0.000
		14	-0.023	-0.093	42.032	0.000
		15	0.109	0.048	44.732	0.000
		16	-0.149	-0.128	49.766	0.000
		17	0.147	0.039	54.682	0.000
		18	-0.072	-0.014	55.880	0.000
		19	0.009	-0.015	55.898	0.000
		20	0.032	0.041	56.131	0.000
		21	-0.086	-0.097	57.851	0.000
		22	0.024	-0.047	57.982	0.000
		23	0.064	-0.004	58.940	0.000
		24	-0.015	0.030	58.992	0.000
		25	-0.007	0.033	59.004	0.000
		26	0.015	0.034	59.057	0.000
		27	-0.070	-0.047	60.254	0.000
		28	0.056	-0.004	61.017	0.000
		29	-0.044	-0.066	61.480	0.000
		30	0.073	0.039	62.804	0.000
		31	-0.150	-0.134	68.337	0.000
		32	0.154	0.009	74.250	0.000
		33	-0.076	-0.007	75.701	0.000
		34	0.123	0.099	79.520	0.000
		35	-0.078	0.059	81.061	0.000
		36	-0.048	-0.020	81.647	0.000

Matrix 2:Graphical Representation (Correlogram) of ACF and PACF

The purpose at this stage is to decide whether the model we have estimated is finally suitable. If not, then we need to suggest modifications to the template. The decision on the appropriateness of the model is made after considering the autocorrelation function of the residuals, as well as some cross-correlation functions between the white noise process and the residuals. When autocorrelation is observed in the residuals, a wrong model choice has been made. However, a correct choice is made when we observe the convergence of the estimates with the actual values in the population, resulting in the residuals approaching the random errors of white noise.

With the help of the following Ljung-Box test, we will conclude whether there is autocorrelation in the estimated model.

Carrying out the assessment of the following cases:

$$H_0 = there\ is\ no\ autocorrelation$$

$$H_1 = there\ is\ autocorrelation$$

In Figure 4, we notice that the residuals of the first differences of the autocorrelations (ACF) and partial autocorrelations (PACF) time series move at low values and close to zero. Nevertheless, looking at the Q-Statistic and p-value values, we see that the p-value results are all zero (p-value < 0.05), so the null hypothesis H_0 (no autocorrelation) is rejected at the 5% significance level, and the alternative hypothesis H_1 is accepted which defines the existence of autocorrelation in the residuals of the sample.

To summarise the Box-Jenkins process, the model was identified as an AR(36) model, transforming the time series to stationary. Then, the estimation of the model was carried out, from which it was shown that the parameter AR(p) is significant and that the estimated values have a moderate dependence on the time series we are considering. Finally, a diagnostic check was carried out on the model, which shows that there is autocorrelation between the residuals and that they do not behave as a white noise process. Consequently, more than this model is needed as a predictive tool.

Overall, the AR (36) model is unsuitable for forecasting the weekly Brent crude oil returns returns; therefore, we should continue to build a suitable model for this time series.

Heteroskedasticity Test

The completion of the Box-Jenkins methodology analysis involves the examination of dynamic heteroskedasticity within the sample residuals.

This method is employed to identify serial autocorrelation of any magnitude and does not presume the absence of time lags in the dependent variable, as with explanatory variables.

The ARCH – LM test defines the following hypothesis:

$$H_0 = Lack\ of\ Heteroskedasticity\ (homoskedastic\ residuals)$$

$$H_1 = Existance\ of\ Heteroskedasticity$$

Via the statistical software employed in this analysis, we interpreted the subsequent findings:

Heteroskedasticity Test: ARCH

F-statistic	0.109745	Prob. F(3,202)	0.9543
Obs*R-squared	0.335207	Prob. Chi-Square(3)	0.9533

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 03/02/24 Time: 15:03
Sample (adjusted): 1/26/2015 12/31/2018
Included observations: 206 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	4.457261	0.724373	6.153269	0.0000
RESID^2(-1)	-0.009528	0.070863	-0.134457	0.8932
RESID^2(-2)	0.024202	0.070838	0.341651	0.7330
RESID^2(-3)	-0.029399	0.067185	-0.437577	0.6622

R-squared	0.001627	Mean dependent var	4.390896
Adjusted R-squared	-0.013200	S.D. dependent var	7.072778
S.E. of regression	7.119306	Akaike info criterion	6.782724
Sum squared resid	10238.27	Schwarz criterion	6.847343
Log likelihood	-694.6206	Hannan-Quinn criter.	6.808858
F-statistic	0.109745	Durbin-Watson stat	1.966406
Prob(F-statistic)	0.954334		

Table 6: Dynamic Heteroskedasticity test (ARCH-LM test) of ARMA model (36,0,0) with three (3) lags

The F statistic assesses the collective statistical significance of the temporal lags in the residuals (omitted variables test). Should the probability value (Prob(obs-R^2)) exceed 0.01 or 0.05, we fail to reject the null hypothesis indicating the absence of autocorrelation. Regardless of the chosen level of statistical significance, we are unable to reject the null hypothesis that there is no serial autocorrelation in the residuals up to the second temporal lag.

Heteroskedasticity Test: ARCH			
F-statistic	5.573880	Prob. F(4,200)	0.0003
Obs*R-squared	20.56084	Prob. Chi-Square(4)	0.0004

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 03/02/24 Time: 15:26
Sample (adjusted): 2/02/2015 12/31/2018
Included observations: 205 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	3.029446	0.754308	4.016193	0.0001
RESID^2(-1)	0.000150	0.067643	0.002224	0.9982
RESID^2(-2)	0.026526	0.067592	0.392441	0.6951
RESID^2(-3)	-0.013831	0.068511	-0.201877	0.8402
RESID^2(-4)	0.301185	0.064102	4.698546	0.0000

R-squared	0.100297	Mean dependent var	4.409630
Adjusted R-squared	0.082303	S.D. dependent var	7.084967
S.E. of regression	6.787152	Akaike info criterion	6.692028
Sum squared resid	9213.085	Schwarz criterion	6.773077
Log likelihood	-680.9328	Hannan-Quinn criter.	6.724810
F-statistic	5.573880	Durbin-Watson stat	1.904701
Prob(F-statistic)	0.000283		

Table 7: Dynamic Heteroskedasticity test (ARCH-LM test) of ARMA model (36,0,0) with four (4) lags

Thus, we revisit the initial hypothesis concerning the absence of ARCH-type dynamic heteroscedasticity in the residuals up to the i-th temporal lag. We specify the desired number of time lags, denoted as i. Given that the probability value (Prob(obs-R^2)) in the 4th lag does not exceed 0.01 or 0.05, we fail to accept the null hypothesis of no dynamic heteroscedasticity. Consequently, at the 5% level of statistical significance, we reject the null hypothesis positing the absence of dynamic heteroscedasticity in the residuals.

The above results lead us to the conclusion that there is an indication of dynamic heteroscedasticity in the time series, so we should continue the modelling process with a model of the ARCH (Autopalindromic Bound Heteroscedasticity Model) family.

ARCH-GARCH models capture volatility clustering but not the leverage effect. They assume that future values of $\sigma_t{}^2$ depend only on the magnitude and not the sign (positive or negative) of u_t. Stationary conditions and positivity constraints can cause difficulties during the model estimation process.

6.2 GARCH (Generalized Autoregressive Conditional Heteroskedasticity)

In the preceding section, the presence of dynamic heteroskedasticity was observed in the time series under examination, attributed to volatility in the returns of the weekly spot price returns of Brent crude oil. Subsequently, an endeavour is made to characterize the volatility of the time series utilizing models from the ARCH (Autoregressive Conditional Heteroskedasticity) family.

At first, the models will undergo evaluation. Subsequently, the most appropriate one will be selected based on the AIC (Akaike's Information Criterion) and SIC (Schwarz's Information Criterion) model selection criteria. Specifically, the estimation will be performed for the hybrid ARMA-GARCH, ARMA-T-GARCH, and ARMA-APARCH models. Following the model estimation outcomes analysis, a comparison of the AIC and SIC indices will be conducted to determine the most suitable model for progression to the subsequent prediction phase.

6.2.1 ESTIMATION

Estimation of ARMA-GARCH model

We determine the order of the GARCH (p,q). The selection of the ARCH order corresponds to the value of q, while the GARCH order corresponds to the value of p. Using the EViews 9.0 statistical software, we have estimated two models and obtained the subsequent results:

Dependent Variable: RETURNS
Method: ML ARCH - Normal distribution (OPG - BHHH / Marquardt steps)
Date: 05/14/24 Time: 20:21
Sample (adjusted): 1/05/2015 12/31/2018
Included observations: 209 after adjustments
Convergence achieved after 27 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.090393	0.129360	0.698767	0.4847
AR(36)	-0.154555	0.068930	-2.242214	0.0249
Variance Equation				
C	0.472305	0.340829	1.385752	0.1658
RESID(-1)^2	0.083665	0.059550	1.404961	0.1600
GARCH(-1)	0.804307	0.106117	7.579438	0.0000

R-squared	0.020854	Mean dependent var	0.000973
Adjusted R-squared	0.016123	S.D. dependent var	2.157921
S.E. of regression	2.140454	Akaike info criterion	4.344099
Sum squared resid	948.3796	Schwarz criterion	4.424059
Log likelihood	-448.9583	Hannan-Quinn criter.	4.376427
Durbin-Watson stat	1.749780		

Inverted AR Roots	.95-.08i	.95+.08i	.92-.25i	.92+.25i
	.86+.40i	.86-.40i	.78+.54i	.78-.54i
	.67-.67i	.67+.67i	.54-.78i	.54+.78i
	.40-.86i	.40+.86i	.25-.92i	.25+.92i
	.08-.95i	.08+.95i	-.08+.95i	-.08-.95i
	-.25+.92i	-.25-.92i	-.40-.86i	-.40+.86i
	-.54-.78i	-.54+.78i	-.67-.67i	-.67+.67i
	-.78-.54i	-.78+.54i	-.86-.40i	-.86+.40i
	-.92+.25i	-.92-.25i	-.95+.08i	-.95-.08i

Table 8: Parameter Estimation, ARIMA-GARCH (1,1)

Dependent Variable: RETURNS
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 03/02/24 Time: 16:11
Sample (adjusted): 1/05/2015 12/31/2018
Included observations: 209 after adjustments
Convergence achieved after 55 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1) + C(6)*GARCH(-2)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.110594	0.135598	0.815603	0.4147
AR(36)	-0.187817	0.070928	-2.648008	0.0081
Variance Equation				
C	0.136474	0.030164	4.524337	0.0000
RESID(-1)^2	0.028888	0.013494	2.140841	0.0323
GARCH(-1)	1.802347	0.041715	43.20651	0.0000
GARCH(-2)	-0.858900	0.032779	-26.20240	0.0000

R-squared	0.020942	Mean dependent var	0.000973
Adjusted R-squared	0.016213	S.D. dependent var	2.157921
S.E. of regression	2.140357	Akaike info criterion	4.305600
Sum squared resid	948.2935	Schwarz criterion	4.401552
Log likelihood	-443.9352	Hannan-Quinn criter.	4.344394
Durbin-Watson stat	1.755601		

Inverted AR Roots	.95+.08i	.95-.08i	.92-.25i	.92+.25i
	.87-.40i	.87+.40i	.78+.55i	.78-.55i
	.68+.68i	.68-.68i	.55-.78i	.55+.78i
	.40-.87i	.40+.87i	.25-.92i	.25+.92i
	.08+.95i	.08-.95i	-.08-.95i	-.08+.95i
	-.25-.92i	-.25+.92i	-.40-.87i	-.40+.87i
	-.55+.78i	-.55-.78i	-.68-.68i	-.68+.68i
	-.78+.55i	-.78-.55i	-.87-.40i	-.87+.40i
	-.92+.25i	-.92-.25i	-.95-.08i	-.95+.08i

Table 9: Parameter Estimation, ARIMA-GARCH (1,2)

Dependent Variable: RETURNS
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 03/02/24 Time: 16:12
Sample (adjusted): 1/05/2015 12/31/2018
Included observations: 209 after adjustments
Failure to improve likelihood (non-zero gradients) after 87 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1) + C(6)*GARCH(-2) + C(7)*GARCH(-3)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.154834	0.110820	1.397166	0.1624
AR(36)	-0.250555	0.066548	-3.765017	0.0002
Variance Equation				
C	0.040817	0.009255	4.410032	0.0000
RESID(-1)^2	0.010634	0.002307	4.609513	0.0000
GARCH(-1)	2.609877	0.004302	606.6094	0.0000
GARCH(-2)	-2.325024	0.000954	-2437.267	0.0000
GARCH(-3)	0.696759	0.002768	251.6768	0.0000

R-squared	0.016162	Mean dependent var	0.000973
Adjusted R-squared	0.011410	S.D. dependent var	2.157921
S.E. of regression	2.145575	Akaike info criterion	4.293040
Sum squared resid	952.9233	Schwarz criterion	4.404985
Log likelihood	-441.6227	Hannan-Quinn criter.	4.338300
Durbin-Watson stat	1.764940		

Inverted AR Roots	.96+.08i	.96-.08i	.93+.25i	.93-.25i
	.87-.41i	.87+.41i	.79+.55i	.79-.55i
	.68-.68i	.68+.68i	.55-.79i	.55+.79i
	.41-.87i	.41+.87i	.25+.93i	.25-.93i
	.08+.96i	.08-.96i	-.08-.96i	-.08+.96i
	-.25+.93i	-.25-.93i	-.41-.87i	-.41+.87i
	-.55+.79i	-.55-.79i	-.68+.68i	-.68+.68i
	-.79-.55i	-.79+.55i	-.87-.41i	-.87+.41i
	-.93+.25i	-.93-.25i	-.96+.08i	-.96-.08i

Table 10: Parameter Estimation, ARIMA-GARCH (1,3)

Dependent Variable: RETURNS
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 03/02/24 Time: 16:13
Sample (adjusted): 1/05/2015 12/31/2018
Included observations: 209 after adjustments
Convergence achieved after 79 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1) + C(6)*GARCH(-2) + C(7)*GARCH(-3) + C(8)*GARCH(-4)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.150496	0.119686	1.257417	0.2086
AR(36)	-0.168978	0.068753	-2.457755	0.0140
Variance Equation				
C	0.383847	0.083300	4.608027	0.0000
RESID(-1)^2	0.084168	0.025664	3.279629	0.0010
GARCH(-1)	0.853711	0.040471	21.09441	0.0000
GARCH(-2)	-0.207406	0.012714	-16.31349	0.0000
GARCH(-3)	1.036707	0.022556	45.96248	0.0000
GARCH(-4)	-0.852060	0.040258	-21.16475	0.0000

R-squared	0.017955	Mean dependent var	0.000973
Adjusted R-squared	0.013211	S.D. dependent var	2.157921
S.E. of regression	2.143620	Akaike info criterion	4.267942
Sum squared resid	951.1870	Schwarz criterion	4.395878
Log likelihood	-437.9999	Hannan-Quinn criter.	4.319667
Durbin-Watson stat	1.746721		

Inverted AR Roots	.95+.08i	.95-.08i	.92+.25i	.92-.25i
	.86-.40i	.86+.40i	.78+.55i	.78-.55i
	.67-.67i	.67+.67i	.55-.78i	.55+.78i
	.40-.86i	.40+.86i	.25+.92i	.25-.92i
	.08+.95i	.08-.95i	-.08-.95i	-.08+.95i
	-.25+.92i	-.25-.92i	-.40-.86i	-.40+.86i
	-.55-.78i	-.55+.78i	-.67+.67i	-.67+.67i
	-.78-.55i	-.78+.55i	-.86-.40i	-.86+.40i
	-.92+.25i	-.92-.25i	-.95+.08i	-.95-.08i

Table 11: Parameter Estimation, ARIMA-GARCH (1,4)

Dependent Variable: RETURNS
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 03/02/24 Time: 16:12
Sample (adjusted): 1/05/2015 12/31/2018
Included observations: 209 after adjustments
Convergence achieved after 67 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*RESID(-2)^2 + C(6)
*GARCH(-1) + C(7)*GARCH(-2)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.056087	0.126029	0.445036	0.6563
AR(36)	-0.134193	0.057702	-2.325614	0.0200
Variance Equation				
C	0.387833	0.137226	2.826228	0.0047
RESID(-1)^2	-0.072479	0.036293	-1.997058	0.0458
RESID(-2)^2	0.165403	0.053690	3.080681	0.0021
GARCH(-1)	1.497648	0.128807	11.62704	0.0000
GARCH(-2)	-0.675620	0.120540	-5.604964	0.0000

R-squared	0.020642	Mean dependent var	0.000973
Adjusted R-squared	0.015911	S.D. dependent var	2.157921
S.E. of regression	2.140685	Akaike info criterion	4.312624
Sum squared resid	948.5846	Schwarz criterion	4.424568
Log likelihood	-443.6692	Hannan-Quinn criter.	4.357884
Durbin-Watson stat	1.747260		

Inverted AR Roots	.94+.08i	.94-.08i	.91-.24i	.91+.24i
	.86-.40i	.86+.40i	.77+.54i	.77-.54i
	.67+.67i	.67-.67i	.54-.77i	.54+.77i
	.40-.86i	.40+.86i	.24-.91i	.24+.91i
	.08+.94i	.08-.94i	-.08-.94i	-.08+.94i
	-.24-.91i	-.24+.91i	-.40-.86i	-.40+.86i
	-.54-.77i	-.54+.77i	-.67-.67i	-.67+.67i
	-.77+.54i	-.77-.54i	-.86-.40i	-.86+.40i
	-.91+.24i	-.91-.24i	-.94-.08i	-.94+.08i

Table 12: Parameter Estimation, ARMA-GARCH (2,2)

The estimation outputs of the ARMA-GARCH(1,1), ARMA-GARCH(1,2), ARMA-GARCH(1,3), ARMA-GARCH(1,4),and ARMA-GARCH(2,2), models are sequentially presented, revealing that, with the exception of the ARMA-GARCH (1,1) and the constant coefficient c, all other parameters exhibit statistical significance at significance levels of 10% and 5%, respectively, leading to rejection of the null hypothesis. Subsequently, the models are compared utilizing three information criteria: Akaike, Hannan-Quinn, and Schwartz. Based on the aforementioned results, the model with the minimal value across these criteria in Tables 8 -12 is selected. It is noted that all three criteria consistently advocate for the ARMA-GARCH(1,4) model.

6.2.2 RESIDUAL DIAGNOSTICS/ARCH LM TEST

Heteroskedasticity Test: ARCH				
F-statistic	1.425853	Prob. F(4,200)		0.2267
Obs*R-squared	5.683910	Prob. Chi-Square(4)		0.2240

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 03/02/24 Time: 16:21
Sample (adjusted): 2/02/2015 12/31/2018
Included observations: 205 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.159148	0.182828	6.340112	0.0000
WGT_RESID^2(-1)	-0.105910	0.070478	-1.502729	0.1345
WGT_RESID^2(-2)	-0.050793	0.070520	-0.720270	0.4722
WGT_RESID^2(-3)	-0.083328	0.070925	-1.174870	0.2414
WGT_RESID^2(-4)	0.080691	0.070617	1.142659	0.2545

R-squared	0.027726	Mean dependent var	1.001729
Adjusted R-squared	0.008281	S.D. dependent var	1.348303
S.E. of regression	1.342709	Akaike info criterion	3.451343
Sum squared resid	360.5735	Schwarz criterion	3.532392
Log likelihood	-348.7627	Hannan-Quinn criter.	3.484126
F-statistic	1.425853	Durbin-Watson stat	1.966556
Prob(F-statistic)	0.226723		

Table 13: Dynamic Heteroskedasticity test (ARCH-LM test) of ARMA-GARCH (1,4) model

The null hypothesis of homoscedasticity of the residuals cannot be rejected at the 5% level of statistical significance.

6.3 THRESHOLD GARCH:

6.3.1 ESTIMATION

A Threshold GARCH model comprises a threshold component that helps compute asymmetries regarding negative and positive shocks. This means that the models treat the good news and bad news asymmetrically.

After conducting an exploratory analysis, during which alternative models were evaluated, it has been determined that the ARMA -T- GARCH (1,1) model is the most effective.

Dependent Variable: RETURNS
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 03/02/24 Time: 17:47
Sample (adjusted): 1/05/2015 12/31/2018
Included observations: 209 after adjustments
Convergence achieved after 46 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*RESID(-1)^2*(RESID(-1)<0) + C(6)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.118861	0.108630	1.094185	0.2739
AR(36)	-0.149777	0.047552	-3.149765	0.0016
Variance Equation				
C	0.279166	0.073150	3.816360	0.0001
RESID(-1)^2	-0.194331	0.040483	-4.800269	0.0000
RESID(-1)^2*(RESID(-1)<0)	0.374097	0.083548	4.477619	0.0000
GARCH(-1)	0.927968	0.032884	28.21926	0.0000

R-squared	0.019190	Mean dependent var	0.000973
Adjusted R-squared	0.014452	S.D. dependent var	2.157921
S.E. of regression	2.142271	Akaike info criterion	4.186505
Sum squared resid	949.9904	Schwarz criterion	4.282457
Log likelihood	-431.4897	Hannan-Quinn criter.	4.225299
Durbin-Watson stat	1.746210		

Inverted AR Roots	.95+.08i	.95-.08i	.92-.25i	.92+.25i
	.86-.40i	.86+.40i	.78+.54i	.78-.54i
	.67+.67i	.67-.67i	.54-.78i	.54+.78i
	.40-.86i	.40+.86i	.25-.92i	.25+.92i
	.08+.95i	.08-.95i	-.08-.95i	-.08+.95i
	-.25-.92i	-.25+.92i	-.40-.86i	-.40+.86i
	-.54-.78i	-.54+.78i	-.67-.67i	-.67+.67i
	-.78+.54i	-.78-.54i	-.86+.40i	-.86-.40i
	-.92+.25i	-.92-.25i	-.95-.08i	-.95+.08i

Table 14:Parameter Estimation, ARMA-T-GARCH (1,1)

Table 14 displays the outcomes of the mean equation in the upper section, the variance equation in the middle segment, and the principal statistical findings of the regression utilizing the residuals from the mean equation. Upon examination of the analysis of variance results, it is observed that the coefficient C(5) holds a positive value. Furthermore, the p-value associated with the parameter coefficient is zero, indicating the significance of the coefficient at a 5% significance level and its positive correlation. These findings suggest the presence of a leverage effect, wherein negative returns from past prices exert a more pronounced influence on the future volatility of the time series under investigation compared to positive returns from past prices.

6.3.2 RESIDUAL DIAGNOSTICS/ARCH LM TEST

Heteroskedasticity Test: ARCH

F-statistic	0.051645	Prob. F(1,206)	0.8205
Obs*R-squared	0.052133	Prob. Chi-Square(1)	0.8194

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 03/02/24 Time: 17:49
Sample (adjusted): 1/12/2015 12/31/2018
Included observations: 208 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.976306	0.108089	9.032450	0.0000
WGT_RESID^2(-1)	-0.015740	0.069260	-0.227254	0.8205

R-squared	0.000251	Mean dependent var	0.961053
Adjusted R-squared	-0.004603	S.D. dependent var	1.219097
S.E. of regression	1.221899	Akaike info criterion	3.248259
Sum squared resid	307.5658	Schwarz criterion	3.280351
Log likelihood	-335.8189	Hannan-Quinn criter.	3.261235
F-statistic	0.051645	Durbin-Watson stat	1.985025
Prob(F-statistic)	0.820451		

Table 15: Dynamic Heteroskedasticity test (ARCH-LM test) for ARIMA-TGARCH (1,1) model

The null hypothesis of homoscedasticity of the residuals cannot be rejected at the 5% level of statistical significance.

6.4 ASYMMETRIC POWER GARCH:

6.4.1 ESTIMATION

The Asymmetric Power GARCH (APGARCH) model is a type of Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model that allows for asymmetric volatility responses to shocks in financial time series data. In traditional GARCH models, volatility responds symmetrically to positive and negative shocks. However, in APGARCH models, the response of volatility to negative shocks can be different from the response to positive shocks, capturing asymmetry in volatility dynamics. They allow for a more flexible and realistic representation of volatility dynamics compared to symmetric GARCH models.

Dependent Variable: RETURNS
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 03/02/24 Time: 18:36
Sample (adjusted): 1/05/2015 12/31/2018
Included observations: 209 after adjustments
Failure to improve likelihood (non-zero gradients) after 86 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
@SQRT(GARCH)^C(9) = C(3) + C(4)*(ABS(RESID(-1)) - C(5)*RESID(-1))^C(9) + C(6)*@SQRT(GARCH(-1))^C(9) + C(7)*@SQRT(GARCH(-2))^C(9) + C(8)*@SQRT(GARCH(-3))^C(9)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.026822	0.118676	0.226008	0.8212
AR(36)	-0.164673	0.045185	-3.644399	0.0003
Variance Equation				
C(3)	0.101598	0.047451	2.141111	0.0323
C(4)	0.059370	0.030894	1.921767	0.0546
C(5)	0.994278	0.125911	7.896684	0.0000
C(6)	0.710095	0.118923	5.971060	0.0000
C(7)	0.909564	0.045678	19.91265	0.0000
C(8)	-0.724153	0.119248	-6.072670	0.0000
C(9)	0.622300	0.405655	1.534059	0.1250

R-squared	0.022842	Mean dependent var	0.000973
Adjusted R-squared	0.018122	S.D. dependent var	2.157921
S.E. of regression	2.138279	Akaike info criterion	4.249932
Sum squared resid	946.4534	Schwarz criterion	4.393860
Log likelihood	-435.1179	Hannan-Quinn criter.	4.308123
Durbin-Watson stat	1.754806		

Inverted AR Roots	.95+.08i	.95-.08i	.92+.25i	.92-.25i
	.86-.40i	.86+.40i	.78+.55i	.78-.55i
	.67-.67i	.67+.67i	.55-.78i	.55+.78i
	.40-.86i	.40+.86i	.25+.92i	.25-.92i
	.08+.95i	.08-.95i	-.08+.95i	-.08-.95i
	-.25+.92i	-.25-.92i	-.40-.86i	-.40+.86i
	-.55-.78i	-.55+.78i	-.67+.67i	-.67+.67i
	-.78-.55i	-.78+.55i	-.86-.40i	-.86+.40i
	-.92+.25i	-.92-.25i	-.95+.08i	-.95-.08i

Table 16: Parameter Estimation, ARIMA-APGARCH (1,1)

Table 17 displays the outcomes of the mean equation in the upper section, the variance equation in the middle segment, and the principal statistical findings of the regression utilizing the residuals from the mean equation. Upon examination of the analysis of variance results, it is observed that the coefficient C(6) holds a positive value. Furthermore, the p-value associated with the parameter coefficient is zero, indicating the significance of the coefficient at a 5% significance level and its positive correlation. These findings suggest the presence of a leverage effect, wherein negative returns from past prices exert a more pronounced influence on the future volatility of the time series under investigation compared to positive returns from past prices.

6.4.2 RESIDUAL DIAGNOSTICS/ARCH LM TEST

Heteroskedasticity Test: ARCH

F-statistic	1.362914	Prob. F(4,200)	0.2482
Obs*R-squared	5.439673	Prob. Chi-Square(4)	0.2451

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 03/02/24   Time: 18:43
Sample (adjusted): 2/02/2015 12/31/2018
Included observations: 205 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.243304	0.184772	6.728857	0.0000
WGT_RESID^2(-1)	-0.103895	0.070540	-1.472852	0.1424
WGT_RESID^2(-2)	-0.092314	0.070610	-1.307373	0.1926
WGT_RESID^2(-3)	-0.084174	0.070562	-1.192918	0.2343
WGT_RESID^2(-4)	0.047372	0.070288	0.673972	0.5011

R-squared	0.026535	Mean dependent var	1.010374
Adjusted R-squared	0.007066	S.D. dependent var	1.307510
S.E. of regression	1.302882	Akaike info criterion	3.391123
Sum squared resid	339.5005	Schwarz criterion	3.472172
Log likelihood	-342.5901	Hannan-Quinn criter.	3.423905
F-statistic	1.362914	Durbin-Watson stat	1.951183
Prob(F-statistic)	0.248167		

Table 17: Dynamic Heteroskedasticity test (ARCH-LM test), ARIMA-APGARCH (1,3) model

The null hypothesis of homoscedasticity of the residuals cannot be rejected at the 5% level of statistical significance.

6.5 MODEL SELECTION

After assessing the above models and analysing their results, the most appropriate model will be selected by comparing the AIC and SIC criteria indices. The most suitable model will be the one with the lowest indicators.

Criteria	GARCH (1,4)	T-GARCH (1,1)	APGARCH (1,3)
AIC	4,2679	4.1865	4.2499
SIC	4.3958	4.2824	4.3938

Table 18 :Indices of Criteria AIC and SIC per GARCH model

Upon examination of the information criteria, we ascertain that the ARMA-T-ARCH (1,1) model is the most suitable choice, as it exhibits the lowest AIC and SIC criteria.

Consequently, the ARMA-T-ARCH (1,1) model will be employed to forecast the values of the time series under scrutiny, subject to preliminary diagnostic checks.

Upon selecting the model, a diagnostic assessment becomes imperative to validate its appropriateness. This procedure involves two sequential steps. The first step involves testing for autocorrelation in the residuals of the examined time series, followed by a subsequent test for dynamic heteroscedasticity. The diagnostic check aims to confirm the absence of the aforementioned characteristics, enabling the progression to the prediction of the returns of the weekly returns of Brent crude oil. Below, the assessment for autocorrelation is conducted by visually inspecting the standardized squared residuals using the statistical software EViews 9.0.

Date: 03/02/24 Time: 19:03
Sample: 12/29/2014 12/31/2018
Included observations: 209
Q-statistic probabilities adjusted for 1 ARMA term

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.035	0.035	0.2549	
		2	0.025	0.024	0.3889	0.533
		3	0.028	0.027	0.5618	0.755
		4	0.010	0.008	0.5833	0.900
		5	0.005	0.003	0.5892	0.964
		6	0.023	0.021	0.7030	0.983
		7	-0.016	-0.018	0.7594	0.993
		8	0.018	0.018	0.8285	0.997
		9	-0.034	-0.035	1.0766	0.998
		10	-0.081	-0.079	2.5280	0.980
		11	0.026	0.033	2.6830	0.988
		12	-0.025	-0.022	2.8235	0.993
		13	-0.074	-0.069	4.0605	0.982
		14	-0.016	-0.011	4.1173	0.990
		15	0.019	0.027	4.2024	0.994
		16	-0.108	-0.105	6.8783	0.961
		17	0.121	0.130	10.251	0.853
		18	-0.066	-0.071	11.264	0.843
		19	0.087	0.092	13.000	0.792
		20	-0.047	-0.066	13.513	0.811
		21	-0.099	-0.093	15.832	0.727
		22	-0.008	-0.004	15.846	0.778
		23	0.038	0.025	16.179	0.807
		24	0.008	0.024	16.193	0.847

*Probabilities may not be valid for this equation specification.

Matrix 3: Correlogram of standardized residuals squared, ARIMA-T-GARCH (1,1)

In Matrix 3, we observe the graphical representation of the autocorrelations (ACF) and the partial autocorrelations (PACF) of the residuals, whose values approach zero. Also, observing the Q-Statistic and p-value values, we see that the outcomes of the p-value values are more significant than the 5% significance level; therefore, the null hypothesis Ho is accepted, which means that there is no evidence of autocorrelation in the time series examination at the 5% significance level.

Subsequently, the examination proceeds with the test for dynamic heteroskedasticity. Upon scrutinizing Table 19, we discern the values of the F-Statistic and Obs*R-squared alongside their respective probabilities at the 5% significance level.

Heteroskedasticity Test: ARCH				
F-statistic	0.051645	Prob. F(1,206)		0.8205
Obs*R-squared	0.052133	Prob. Chi-Square(1)		0.8194
Test Equation:				
Dependent Variable: WGT_RESID^2				
Method: Least Squares				
Date: 03/02/24 Time: 17:49				
Sample (adjusted): 1/12/2015 12/31/2018				
Included observations: 208 after adjustments				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.976306	0.108089	9.032450	0.0000
WGT_RESID^2(-1)	-0.015740	0.069260	-0.227254	0.8205
R-squared	0.000251	Mean dependent var		0.961053
Adjusted R-squared	-0.004603	S.D. dependent var		1.219097
S.E. of regression	1.221899	Akaike info criterion		3.248259
Sum squared resid	307.5658	Schwarz criterion		3.280351
Log likelihood	-335.8189	Hannan-Quinn criter.		3.261235
F-statistic	0.051645	Durbin-Watson stat		1.985025
Prob(F-statistic)	0.820451			

Table 19: Dynamic Heteroskedasticity test (ARCH-LM test), ARIMA-TGARCH (1,1) model

Notably, the computed p-values surpass the predetermined significance threshold, leading to the acceptance of the null hypothesis (Ho).

If the residuals in a time series of Brent Crude oil returns are homoskedastic, it suggests that the variance of the residuals is constant over time. In other words, there is no systematic pattern of variability in the residuals as the time series progresses.

Homoskedasticity in the residuals is a desirable property in many statistical models, including regression models and time series models, because it implies that the model's errors have a consistent level of variability and do not exhibit patterns of increasing or decreasing variance over time.

In the context of financial time series like Brent Crude oil returns, homoskedasticity in the residuals indicates that there is no evidence of changing volatility or clustering of volatility in the data. This can make the modeling and forecasting process more straightforward and reliable, as assumptions about the constant variance of the residuals are met.

This outcome suggests the absence of dynamic heteroscedasticity within the scrutinized time series at the 5% significance level. With the diagnostic test concluded, we deduce that the ARMA-T-GARCH (1,1) model remains suitable for progression to the forecasting stage, given the absence of autocorrelation and dynamic heteroscedasticity within the time series data.

6.7 FORECASTING – FORECASTING EVALUATION

In this section, we will conduct the forecasting for the returns of weekly Brent crude during the out-of-sample period (01/01/2019 – 31/12/2019) utilizing the ARMA-T-GARCH model (1,1) with the assistance of the statistical software Eviews 9.0.

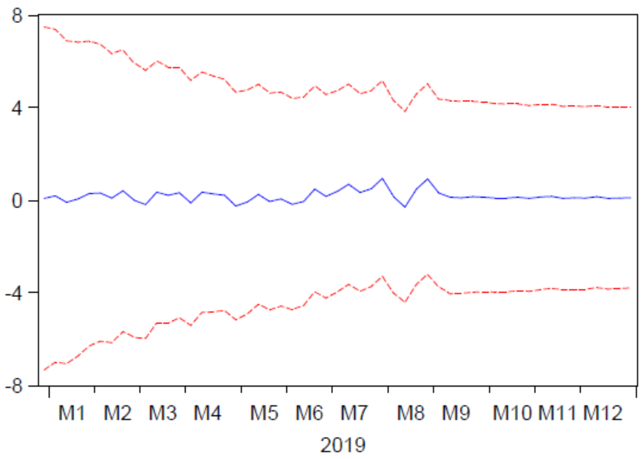


Figure 3:Prediction of percentage returns from 01/01/2019 - 31/12/2019

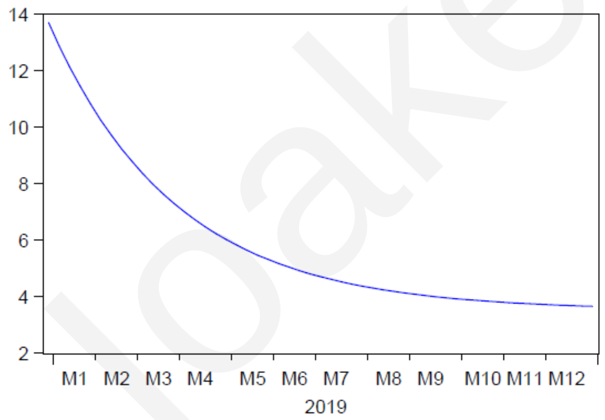


Figure 4: Prediction of variance (forecast period 01/01/2019 - 31/12/2019)

Figure 3 delineates the comprehensive outcomes of the prediction procedure. Initially, the forecast of the percentage returns graph is presented, where the blue line depicts the forecasts. In contrast, the red lines delineate the forecast values and the associated error margin. It is observed that the output values spanning the period from 01/01/2019 to 12/31/2019 fluctuate within the range of -0.5 to 0.5. Subsequently, the volatility forecast chart (Figure 4) is displayed, wherein the blue line illustrates the forecasted volatility across the considered timeframe. Notably, a discernible downward trend in the variance value is apparent, commencing from value 14 and concluding at value 4.

Preliminary information analysis

Preliminary information analysis involves examining the data collected using graphs and statistical indicators like the mean and dispersion. These techniques assess the models' ability to forecast future return volatility, observe patterns, and identify outliers. This process is crucial as it informs the subsequent stage of model selection.

The current forecast evaluation comprises the following indicators: Root Mean Square Error, Mean Absolute Error, Mean Absolute Percentage Error (MAPE), and Theil's inequality index.

Tables 20 - 22 represent the forecast evaluation values for the GARCH models created in this analysis.

Real Forecast: Percentage Returns of the weekly Brent Crude Oil Prices

Forecast Sample: 01 January 2019 - 31 December 2019 Παρατηρήσεις: 52							
Indicator	Root Mean Squared Error	Mean Absolute Error	Mean Absolute % Error	Theil Inequality Coef.	Bias Proportion	Varianc. Proportion	Covariance Proportion
Total	1.6240	1.2527	98.0909	0.8483	0.0233	0.6275	0.3490

Table 20: Assessing the reliability of the forecasting the ARMA-GARCH (1,4) model

Real Forcast: Percentage Returns of the weekly Brent Crude Oil Prices

Forecast Sample: 01 January 2019 - 31 December 2019 **Παρατηρήσεις: 52**							
Indicator	Root Mean Squared Error	Mean Absolute Error	Mean Absolute % Error	Theil Inequality Coef.	Bias Proportion	Varianc. Proportion	Covarianc. Proportion
Total	1.5995	1.2214	105.5216	0.8372	0.0024	0.7103	0.2872

Table 21:Assessing the reliability of the forecasting the ARMA-T- GARCH (1,1) model

Real Forcast: Percentage Returns of the weekly Brent Crude Oil Prices

Forecast Sample: 01 January 2019 - 31 December 2019 **Παρατηρήσεις: 52**							
Indicator	Root Mean Squared Error	Mean Absolute Error	Mean Absolute % Error	Theil Inequality Coef.	Bias Proportion	Varianc. Proportion	Covarianc. Proportion
Total	1.6180	1.2445	96.8420	0.8495	0.0180	0.6443	0.3375

Table 22: Assessing the reliability of the forecasting the ARMA-APGARCH (1,3) model

RMS measures the standard deviation of the residuals. Residuals are prediction errors measuring the distance between the data points from the regression line, influenced by outliers. Therefore, RMSE is a representation of how spread out these residuals are. Comparing the RMSE of all three models, we can observe that the value of the specific index is relatively low, which indicates the reliability of the model's predictive ability—the lower our index, the better. The model with the lowest residual spread around the line of best fit is ARMA – T-GARCH.

The MAE index helps determine forecast accuracy. In particular, the index is the average of errors in absolute value. This indicator shows the magnitude of errors without considering whether they are overestimations or underestimations. The smaller the MAE, the better the model's predictions align with the data. An MAE of 0 would mean a perfect prediction, but in most cases, achieving such perfection is unlikely, and thus, lower values are better. Unlike other metrics, MAE is less sensitive to the data's extreme values (outliers). In this analysis, all models indicate that the mean absolute error is near one (1), which is also low and indicates the accuracy of the prediction results.

The subsequent metric under consideration is the Mean Absolute Percentage Error (MAPE) index. MAPE is the average absolute percentage error equal to 98.0909, 105.5216 and 96.8420 (refer to the Tables 20-22 respectively) confirming the forecasts' reliability based on the time series model where it was followed.

Finally, we will observe the Theil inequality coefficient, which always lies between 0 and 1. When the Theil Inequality Coefficient is close to 1, it indicates a high level of inequality within the dataset. The Theil coefficient is a measure used to assess inequality within a distribution. It takes values from 0 to 1. Therefore, a value close to 1 suggests a significant disparity among the values in the dataset, with some values dominating others, resulting in a highly unequal distribution. The Theil index breaks up into three ratios of inequality, such that bias + variance + covariance = 1. Whatever the value of the Theil index, we prefer the model with a bias indicator close to 0. However, if the variance is large, the actual series has fluctuated broadly, whereas the forecast has not. This proportion measures unsystematic error. The outcomes show that the relative differences among forecasting evaluation criteria are minor. Thus, the ARMA - T- GARCH model is chosen as the most preferred among all the other models to study the volatility behaviour and the corresponding forecasting of returns.

Upon concluding the forecasting procedure for the weekly returns of Brent crude oil, the ARMA-T-GRACH (1,1) model demonstrated reliability in its predictive capacity.

CHAPTER 7: CONCLUSION

This analysis involved applying econometric standards to determine the most suitable forecasting model. Upon achieving this thesis's objective, a methodical approach was employed to synthesize the econometric models' results, conclusions, and analyses.
Beginning with the initial phase of the Box Jenkins methodology, namely identification, the stationarity of the time series was verified through the Augmented Dickey-Fuller (ADF) test. Subsequently, the diagnostic test was conducted as the third stage of the Box Jenkins methodology to evaluate the autoregressive models. It was determined that the ARMA model (36,0) exhibited superior adaptability compared to the alternative ARMA model.
Two conditional generalized constrained dynamic heteroscedasticity models were then estimated where, again, using appropriate diagnostic tests, we concluded that the most appropriate model for forecasting Brent Crude oil returns is ARMA-T-GARCH(1,1) to which we have fitted the autoregressive ARMA model (36,0). T-GARCH consider the tendency of volatility clustered in time, meaning that periods of high volatility are often followed by additional periods of high volatility. This is a common phenomenon observed in financial time series data. T-GARCH can capture the asymmetric response of volatility to positive and negative shocks, known as leverage effects. This feature is essential for accurately modelling

the behaviour of financial assets, where adverse shocks often result in higher volatility compared to positive shocks.

Therefore, forecasting was conducted for the period spanning from 01 January 2019 to 31 December 2019. Moreover, the forecast outcomes were deemed notably satisfactory. Subsequently, metrics were applied to assess the reliability of the model predictions, utilizing the Root Mean Square Error (RMSE), Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE), and Theil indices. Considering the 52 observations within our sample, the RMSE index exhibited a relatively low value, indicative of the optimal alignment of our data. Furthermore, the MAE index, at 1.2214, reflected a negligible disparity between the forecasted and actual returns, affirming the precision of the prediction results. Notably, the MAE index serves as an indicator of predictive accuracy. However, the Theil Inequality Coefficient is close to 1. This means there is a significant disparity among the values in the dataset, with some values dominating others, resulting in a highly unequal distribution.

Referring to Figure 1, where we identified a wide range of fluctuations and spikes in Brent crude oil prices, it is highlighted that outliers could lead to a misspecification of the model when we ignore the presence of this type of data. Despite that, the model's parameters governing volatility dynamics are biased when we do not consider outliers, regardless of the trading environment (calm or noisy periods).

As per the literature reviewed in chapter three, forecasting models may be of two types: symmetric models, including ARCH and GARCH, and asymmetric models, comprised of EGARCH, T-GARCH, and APGARCH. The difference between these two models is that the asymmetric models capture leverage effects in the time series.

Upon comparing the outcomes of our analysis with the existing literature, a notable consensus emerges. Most studies, in line with our findings, assert that asymmetric GARCH models are well-suited for forecasting returns. This aligns with the study of Ng’ang’a, F. W.,

& Oleche, M. (2017), which outlines that the IGARCH model is the most suitable model out of the five asymmetric models of the GARCH family. In addition, Kang and Yoon (2009) conclude that the CGARCH and FIGARCH models are suitable for forecasting crude oil price volatility. Dritsaki (2018) found that the hybrid ARIMA-GARCH models provide optimal forecasting results. Thus, the combination of ARIMA and GARCH family models provides biased results on handling the volatility of oil returns. This makes hybrid models the most suitable for analyzing and forecasting time series of commodities.

Having conducted the aforementioned analysis utilizing the statistical software Eviews 9.0, it becomes evident that this statistical tool is appropriate for modelling and forecasting time series data.

This thesis delineates the methodology employed in modelling time series and elucidates the process of forecasting future commodity returns. Mitigating investment risks is imperative for businesses to attain profitability. Thus, employing predictive models can substantially mitigate risks and enhance performance. However, it is essential to acknowledge that forecasts inherently entail uncertainty, as unforeseen events may transpire.

A recent example is the unforeseen onset of the coronavirus pandemic, which exerted unprecedented effects on various sectors, notably leading to a significant decline in Brent crude oil returns. Such events underscore the inherent limitations of forecasting accuracy. Moreover, the pandemic-induced shock in demand significantly impacted crude oil markets.

7.1 SUGGESTIONS FOR FUTURE RESEARCH

This paper focuses its analysis on searching for the most suitable model to estimate and forecast the volatility and return values of the weekly spot prices of Brent crude oil. Future research could extend the analysis of this paper and make predictions by applying all the models developed here to compare and evaluate their results.

Another suggestion for future research is to compare the forecast results of the weekly returns of WTI crude oil with the BRENT crude oil counterparts since these two types of crude oil are used as benchmarks in the oil industry. It would be interesting to analyze the results of this comparison.

Finally, a suggestion for future research is to compare the forecast results between weekly returns of Brent-type crude oil and future weekly returns of the same type of oil. We know that futures price trends in the stock market play an important role in investors' expectations of future oil prices.

REFERENCES

Greek Literature

ΔΕΠΑ εμπορίας Α.Ε. Διαθέσιμο από https://www.depa.gr/ [Πρόσβαση 7 Ιανουαρίου 2021].

Καραγεώργος, Δ., Κόκλα, Α. και Παπακωνσταντίνου, Ε. (1997) Στατιστική επιχειρήσεων. Αθήνα: Εκδόσεις Διοφαντος.

Ντέμος,Α. (2019) *Χρηματοοικονομική Οικονομετρία*. Αθήνα: ΟΠΑ

Παπανικολάου Κ., (2018), Αγορά Ενέργειας: Το Φυσικό Αέριο και η Σύνδεση της Τιμής του με την Τιμή του Πετρελαίου, Διπλωματική Εργασία, Διατμηματικό Πρόγραμμα Μεταπτυχιακών Σπουδών, Πληροφορική και Διοίκηση, Τμήματα Πληροφορική και Οικονομικών Επιστημών, Α.Π.Θ., Θεσσαλονίκη.

Χατζηνικολάου, Δ. (2002) *Στατιστική για οικονομολόγους*. 2η Έκδοση. Ιωάννινα: Ιδιωτική Έκδοση.

English Literature

Ahmed, R. A., & Shabri, A. B. (2017). Fitting GARCH Models to Crude Oil Spot Price Data. *International Journal of Energy Economics and Policy*, 7(4), 67-74. Available at: http://www.lifesciencesite.com [Accessed 27 April 2024].

Andreas Breitenfellner & Jesús Crespo Cuaresma & Catherine Keppel, 2009. "*Determinants of Crude Oil Prices: Supply, Demand, Cartel or Speculation?"*. Monetary Policy & the Economy, Oesterreichische Nationalbank (Austrian Central Bank). 4, pp. 111-136. Available at: https://ideas.repec.org/a/onb/oenbmp/y2009i4b6.html

Apergis, N. and Payne, J.E. (2010) Natural gas consumption and economic growth: a panel investigation of 67 countries, *Applied Energy*, August, 87(8), pp. 2759-2763. [Online]. Available at Science Direct. https://doi.org/10.1016/j.apenergy.2010.01.002 [Accessed 5 January 2021].

Berrisch, J. and Ziel, F. (2020) Modeling and Probababilistic Forecasting of Natural Gas Prices. Essen. [Online]. Available at: Research Gate [Accessed 15 December 2020].

Box, G.E.P, Jenkins,G.M. and Reinsel, G.C. (2008). Time series analysis forcasting and control. 4th ed. New Jersey: John Wiley & Sons, Inc., New Jersey

Chen, J. (2019) *Henry Hub*. [Online]. Available at: Investopedia https://www.investopedia.com/ [Accessed 12 January 2021].

Christopoulos, A., Kalantonis, P., Katsampoxakis, I., Katsampoxakis, I., & Vergos, K. (2021). COVID-19 and the Energy Price Volatility. *Energies*, 14(20), 6496.

Copinschi, P. and Smedley, M. (2016) Sub-saharan Africa: a future global gas player? in Colombo, S., El Harrak, M. and Sartori, N. (eds.) *The future of natural gas. Market and geopolitics*. The Netherlands: Lenthe/European Energy Review, pp. 181–203.

Daly, M. (2001) *Economic impact of rising natural gas prices*. [Online]. Available from: Federal Reserved Bank of San Francisco [Accessed 12 January 2021].

Dritsaki, C., 2018. The performance of hybrid ARIMA-GARCH modeling and forecasting oil price. *International Journal of Energy Economics and Policy,* 8(3), pp.14-21. Available at: chrome-extension://efaidnbmnnnibpcajpcglclefindmkaj/https://www.zbw.eu/econisarchiv/bitstream/11159/2094/1/1028123450.pdf

ECB (2015) "Forecasting the Price of Oil", *Economic Bulletin*, 4. [Online]. Available at: https://www.ecb.europa.eu/pub/pdf/other/art03_eb201504.en.pdf [Accessed 27 April 2024]

Garcia,R.C., Contreras, J.,van Akkeren, M.and Garcia, J.B.C. (2005) A GARCH Forecasting Model to Predict Day-Ahead Electricity Prices, *IEEE Transactions On Power Systems*, 20 (2), pp.867-874.[Online]. Available at: Institute of Electrical and Electronics Engineers. 10.1109/TPWRS.2005.846044 [Accessed 9 December 2020].

Global Economic Prospects, January 2018: Broad-Based Upturn, but for How Long? (2017). Washington, DC: World Bank. doi:https://doi.org/10.1596/978-1-4648-1163-0. Available at: https://doi.org/10.1596/978-1-4648-1163-0. [Accessed 20 March 2024]

Gotham, D.J., Nderitu, D.G., Giraldo, J.S. and Preckel, P.V., 2013. Natural Gas Market Study.

Hamilton, J. D. (1983). Oil and the macroeconomy since World War II. *Journal of political economy*, *91*(2), 228-248. Available from: https://doi.org/10.1086/261140.

He, K., Tso, G. K. F., Zou, Y., & Liu, J. (2018). Crude oil risk forecasting: New evidence from multiscale analysis approach. *Energy Economics*, 74, 262-269. Available at: https://doi.org/10.1016/j.eneco.2018.10.001 . [Accessed 27 April 2024]

Herrera, G.P., Constantino, M., Tabak, B.M., Pistori, H., Su, J., Athula Naranpanawa, A., 2019, Data on forecasting energy prices using machinelearning, *Data in brief*, August, 25, pp. 1-5. [Online]. Available at: Science Direct. https://doi.org/10.1016/j.dib.2019.104122 [Accessed 15 December 2020].

Howrey, E. Philip, Saul H. Hymans, and Michael R. Donihue. (1991) Merging Monthly and Quarterly Forecasts: Experience with MQEM, *Journal of Forecasting*, May, 10, pp: 255–268. [Online]. Available at: https://doi.org/10.1002/for.3980100303 [Accessed 13 January 2021].

Hyndman, R., 1998. Forecasting: Methods And Applications. [online] Robjhyndman.com. Available at: <ttps://robjhyndman.com/files/solutions.pdf> [Accessed 16 September 2020].

Hymans, S., 2020. Forecasting And Econometric Models - Econlib. [online] Econlib. Available at: https://www.econlib.org/library/Enc/ForecastingandEconometricModels.html> [Accessed 11 September 2020].

International Energy Agency (IEA). World energy outlook. Paris: IEA/OECD; 2018.

James D. Hamilton, 2009. "Understanding Crude Oil Prices," *The Energy Journal, International Association for Energy Economics*, vol. 30(2), pages 179-206. Available at: https://doi.org/10.3386/w14492. [Accessed 27 April 2024]

Hamilton, J. D. (2013). Historical oil shocks. In *Routledge handbook of major events in economic history* (pp. 239-265). Routledge. Available at: https://doi.org/10.4324/9780203067871

Jin,J. and Kim,J. (2015) Forecasting Natural Gas Prices Using Wavelets, Time Series, and Artificial Neural Networks,*Plos One*,November,10(11).[Online].Available at: https://doi.org/10.1371/journal.pone.0142064 [Accessed 9 December 2020].

Johnston, J. and DiNardo, J. (1997) Econometric Methods. 4th ed. New York: McGraw Hill.

Kang, S.H., Kang, S.-M. and Yoon, S.-M. (2009). Forecasting volatility of crude oil markets. *Energy Economics*, 31(1), pp.119–125. Available at doi:https://doi.org/10.1016/j.eneco.2008.09.006. [Accessed 23 May 2024]

Levin, S., Taylor, G., Arthur, D. and Tolleth, M. (2014) Understanding Crude Oil and Product Markets, American Petroleum Institute. Available at: doi: 10.1002/ejoc.201200111

Lin, B. and Wesseh Jr., P.K., (2013), `What causes price volatility and regime shifts in the natural gas market, *Energy*, April, 55, pp.553-563. [Online]. Available at: ScienceDirect. https://doi.org/10.1016/j.energy.2013.03.082 [Accessed 13 December 2020].

Lv, X. and Shan, X. (2013) Modeling natural gas market volatility using GARCH with different distributions, *Physica A*, July, 329(22), pp.5685-5699. [Online]. Available at: Science Direct. https://doi.org/10.1016/j.physa.2013.07.038 [Accessed 16 December 2020].

MacKinnon, J.G., 2002. Computing numerical distribution functions in econometrics. In High Performance Computing Systems and Applications (pp. 455-471). Springer, Boston, MA.

Ng'ang'a, F. W., & Oleche, M. (2017). Modelling and Forecasting of Crude Oil Price

Volatility: Comparative Analysis of Volatility Models. *International Journal of Energy Economics and Policy*, 7(2), 93-102. Available at: https://doi.org/10.4236/jfrm.2022.111008 [Accessed 27 April 2024]

Pustišek, A. and Karasz, M. (2017) Natural gas: a commercial perspective Switzerland: *Springer International Publishing AG.*

Ramberg, D.J. and Parsons, J.E., (2012) The weak tie between natural gas and oil prices. *Energy Journal*, 33 (2), pp.13–35. [Online]. Available at: IAEE. DOI**:** 10.5547/01956574.33.2.2 [Accessed 16 December 2020].

Ratti, R. A., & Vespignani, J. L. (2015). OPEC and non-OPEC oil production and the global economy. *Energy Economics.* Available at: https://doi.org/10.1016/j.eneco.2014.12.001 [Accessed 27 April 2024]

Salehnia, N., Falahi, M. A., Seifi, A. and Adeli, M. H. M., (2013) Forecasting natural gas spot prices with nonlinear modeling using Gamma test analysis, *Journal of Natural Gas Science and Engineering*, 14 (2013), pp.238 – 249.[Online] Available at: ScienceDirect. https://doi.org/10.1016/j.jngse.2013.07.002 [Accessed 9 December 2020].

Saltik, O., Degirmen, S. and Ural, M., (2016) Volatility Modelling In Crude Oil and Natural Gas Prices, *Procedia Economics and Finance*, 38, pp.476 – 491.[Online] Available at: SienceDirect. https://doi.org/10.1016/S2212-5671(16)30219-2 [Accessed 9 December 2020]

Statistical Review of World Energy 69th edition. (2020) BP. [Online]. Available at: https://www.bp.com/ [Accessed 5 January 2021].

The international association for natural gas. (2020) *Cedigaz*. [Online]. Available at: https://www.cedigaz.org/ [Accessed 5 January 2021].

U.S Energy Information Administration (2020) Factors affecting natural gas prices. [Online]. Available at https://www.eia.gov/ [Accessed 13 January 2021].

Wooldridge, J.M. (2013) *Εισαγωγή στην οικονομετρία μια σύγχρονη προσέγγιση*. 4η Έκδοση. Αθήνα: Εκδόσεις Παπαζήση

Xu, Y., & Lien, D. (2013). Forecasting volatilities of oil and gas assets: A comparison of GAS, GARCH, and EGARCH models. *Energy Economics*, 40, 523-529, Available at: https://doi.org/10.1002/for.2812, Accessed: 24 April 2024.

Zhenfeng, W., Zhipeng, S., Jitian, Z., Minggang, G. and Rufeng, J. (2015) Natural gas geological characteristics and great discovery of large gas fields in deep-water area of the western South China Sea, *Natural Gas Industry B*, December, 2(6), pp. 489-498, Available at: https://doi.org/10.1016/j.ngib.2016.03.001, Accessed January 2021.